Stochastic modeling of extreme El-Niño and La Niña events by nonlinearity coupled oscillators

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The El-Niño index behaves as a nonlinear and non-Gaussian stochastic process. A well-known characteristic is its positive skewness coming from the occurrence of stronger episodes of El-Niño than of La Niña. Here, we use the period 1870-2018 of the standardized El-Niño index $x(t)$, sampled in trimesters to analyze the spectral origin of the bicorrelation: $sk(t_1,t_2)=E[x(t)x(t+t_1)x(t+t_2)]$ and skewness $sk(0,0)$. For that, we estimate the two-dimensional Fourier transform of $sk(t_1,t_2)$ or bispectrum $B(f_1,f_2)$. Its sum over bi-frequencies $(f_1,f_2)$ equals the skewness (0.45 in our case). Positive and negative bispectrum peaks are due to phase locking of frequency triplets: $(f_1,f_2,f_1+f_2)$, contributing to extreme El-Niños and La Niñas respectively. Moreover, the most significant positive and/or negative bispectrum regions are rather well localized in the bispectrum domain. Here, we propose a partition of the El Niño signal into a set of band-pass spectrally separated components whose self and cross interactions can explain the broad structure of bispectrum. In the simplest case where the signal is decomposed into a fast and a slow component (with a cutoff frequency of $1/2.56$ cycles/yr.), we verify that slow-slow interactions (or phase locking) explain most of La-Niñas, particularly at the frequency triplet $(1/4.9, 1/15$ and $1/3.7$ cycles/yr) whereas the fast-slow interactions explain most of El Niños, particularly at the frequency triplet $(1/4.9, 1/4.9$ and $1/2.5$ cycles/yr). In order to simulate this stochastic behavior, we calibrate a set of nonlinearly coupled oscillators (Auto-regressive processes, forced by self and cross quadratic component terms), one for each component. In the case of weak cross-component interactions, and thus weak nonlinearity, the coupling coefficients between spectral-band components are proportional to the corresponding cross-skewnesses, which represent good first guesses in the calibration of the model parameters. The predictability of the model is then assessed, in particular for the anticipation of big El Niños and la Niñas. The authors would like to acknowledge the financial support FCT through project \textbf{UIDB/50019/2020 – IDL}.