The Lorenz convection model's random attractor (LORA) and its robust topology

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Chekroun et al. (\textit{Physica D}, 240, 2011) studied the global random attractor associated with the Lorenz (1963) model driven by multiplicative noise; they dubbed this time-evolving attractor LORA for short. The present talk examines the topological structure of the snapshots that approximate LORA's evolution in time.

Sciamarella & Mindlin (\textit{Phys. Rev. Lett.}, 82, 1999; \textit{Phys. Rev. E}, 64, 2001) introduced the methodology of Branched Manifold Analysis through Homologies (BraMAH) to the study of chaotic flows. Here, the BraMAH methodology is extended for the first time, to the best of our knowledge, from deterministically chaotic flows to nonlinear noise-driven systems.

The BraMAH algorithm starts from a cloud of points given by a large number of orbits and it builds a rough skeleton of the underlying branched manifold on which the points lie. This construction is achieved by local approximations of the manifold that use Euclidean closed sets; the nature of these sets depends on their topological dimension, e.g., segments or disks. The skeleton is mathematically expressed as a complex of cells, whose algebraic topology is analyzed by computing its homology groups.

The analysis is performed for a fixed realization of the driving noise at different time instants. We show that the topology of LORA changes in time and that it is quite distinct from the time-independent one of the classical Lorenz (1963) convection model, for the same values of the parameters. Topological tipping points are also studied by varying the parameter values from the classical ones.