Nonlinear Climate Dynamics: from Deterministic Behavior to Stochastic Excitability and Chaos

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Glacial-interglacial cycles are global climatic changes which have characterised the last 3 million years. The eight latest glacial-interglacial cycles represent changes in sea level over 100 m, and their average duration was around 100 000 years. There is a long tradition of modelling glacial-interglacial cycles with low-order dynamical systems. In one view, the cyclic phenomenon is caused by non-linear interactions between components of the climate system: The dynamical system model which represents Earth dynamics has a limit cycle. In an another view, the variations in ice volume and ice sheet extent are caused by changes in Earth's orbit, possibly amplified by feedbacks. This response and internal feedbacks need to be non-linear to explain the asymmetric character of glacial-interglacial cycles and their duration. A third view sees glacial-interglacial cycles as a limit cycle synchronised on the orbital forcing.

The purpose of the present contribution is to pay specific attention to the effects of stochastic forcing. Indeed, the trajectories obtained in presence of noise are not necessarily noised-up versions of the deterministic trajectories. They may follow pathways which have no analogue in the deterministic version of the model. Our purpose is to demonstrate the mechanisms by which stochastic excitation may generate such large-scale oscillations and induce intermittency. To this end, we consider a series of models previously introduced in the literature, starting by autonomous models with two variables, and then three variables. The properties of stochastic trajectories are understood by reference to the bifurcation diagram, the vector field, and a method called stochastic sensitivity analysis. We then introduce models accounting for the orbital forcing, and distinguish forced and synchronised ice-age scenarios, and show again how noise may generate trajectories which have no immediate analogue in the deterministic model.