Using the fractional energy balance equation for accurate temperature projections through 2100

Roman Procyk, Shaun Lovejoy, and Lenin Del Rio Amador
McGill University, Montreal, Canada

The conventional energy balance equation (EBE) is a first order linear differential equation driven by solar, volcanic and anthropogenic forcings. The differential term accounts for energy storage usually modelled as one or two “boxes”. Each box obeys Newton's law of cooling, so that when perturbed, the Earth's temperature relaxes exponentially to a thermodynamic equilibrium.

However, the spatial scaling obeyed by the atmosphere and its numerical models implies that the energy storage process is a scaling, power law process, a consequence largely of turbulent, hierarchically organized oceans currents but also hierarchies of land ice, soil moisture and other processes whose rates depend on size.

Scaling storage leads to power law relaxation and can be modelled via a seemingly trivial change - from integer to fractional order derivatives - the Fractional EBE (FEBE): with temperature derivatives order $0 < H < 1$ rather than the EBE value $H = 1$. Mathematically the FEBE is a past value problem, not an initial value problem. Recent support for the FEBE comes from [Lovejoy, 2019a] who shows that the special $H = 1/2$ case (close to observations), the “Half-order EBE” (HEBE), can be analytically obtained from classical Budyko-Sellers energy balance models by improving the boundary conditions.

The FEBE simultaneously models the deterministic forced response to external (e.g. anthropogenic) forcing as well as the stochastic response to internal forcing (variability) [Lovejoy, 2019b]. We directly exploit both aspects to make projections based on historical data estimating the parameters using Bayesian inference. Using global instrumental temperature series, alongside CMIP5 and CMIP6 standard forcings, the basic FEBE parameters are $H \approx 0.4$ with a relaxation time $\approx 4$ years.

This observation-based model also produces projections for the coming century with forcings prescribed by the CMIP5 Representative Concentration Pathways scenarios and the CMIP6 Shared Socioeconomic Pathways.

We compare both generations of General Circulation Models (GCMs) outputs from CMIP5/6 alongside with the projections produced by the FEBE model which are entirely independent from GCMs, contributing to our understanding of forced climate variability in the past, present and future. When comparing to CMIP5 projections, we find that the mean projections are about 10-
15% lower while the uncertainties are roughly half as large. Our global temperature projections are therefore within the CMIP5 90% confidence limits and thus give them strong, independent support.

References
