Hyperbolic geometry of earthquake networks

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We examine the space-time-magnitude distribution of earthquakes using the Gromov hyperbolic property of metric spaces. The Gromov $\delta$-hyperbolicity quantifies the curvature of a metric space via so-called four-point condition, which is a computationally convenient analog of the famous thin triangle property. We estimate the standard and scaled values of the $\delta$-parameter for the observed earthquakes of Southern California during 1981 – 2017 according to the catalog of Hauksson et al. [2012], the global seismicity according to the NCEDC catalog during 2000 – 2015, and synthetic seismicity produced by the ETAS model with parameters fit for Southern California. In this analysis, a set of earthquakes is represented by a point field in space-time-energy domain $\mathcal{D}$. The Baiesi-Paczuski asymmetric proximity $\eta$, which has been shown efficient in applied cluster analysis of natural and human-induced seismicity and acoustic emission experiments, is used to quantify the distances between the earthquakes. The analyses performed in the earthquake space $(\mathcal{D},\eta)$ and in the corresponding proximity networks show that earthquake field is strongly hyperbolic, i.e. it is characterized by small values of $\delta$. We show that the Baiesi-Paczuski proximity is a natural approximation to a proper hyperbolic metric in the space-time-magnitude domain of earthquakes, with the $b$-value related to the space curvature. We discuss the hyperbolic properties in terms of the examined earthquake field. The results provide a novel insight into the geometry and dynamics of seismicity and expand the list of natural processes characterized by underlying hyperbolicity.