Inhomogeneous waves in isotropic anelastic media: explicit expressions for $Q$

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Seismic waves propagating in attenuative materials are generally inhomogenous waves which, unlike homogeneous waves, have different directions of propagation and attenuation. The degree of wave inhomogeneity can be represented by the inhomogeneity parameter $D$ which varies from 0 to infinity (Cerveny & Psencik, 2005). The dissipation ($1/Q$) factors of inhomogeneous waves vary according to the different definitions. Based on the complex energy balance equations (Carcione, 2001) and the mixed specification of the slowness vector (Cerveny & Psencik, 2005), explicit formulas for the dissipation factors of $P$- and SV-waves are developed under the two different definitions, (1) $1/Q_V$, the ratio of the time-averaged dissipated energy density to the time-averaged strain-energy density, and (2) $1/Q_T$, the time-averaged dissipated energy density to the time-averaged energy density. By setting the degree of wave inhomogeneity $D$ as zero, these dissipation factor expressions are reduced to their special case versions as homogeneous waves, i.e., $1/Q_{VH} = \text{Im}(M)/\text{Re}(M)$ and $1/Q_{TH} = 2\alpha v/\omega$, where, $M$ is the wave modulus, $\alpha$ the attenuation coefficient, $v$ the phase velocity and $\omega$ the frequency. An example viscoelastic material is chosen to represent the dissipative features of a reservoir for which $P$-waves are normally more dissipative than $S$-waves. The calculated dissipation factors of $P$-waves under the two definitions (i.e. $1/Q_{PV}$ and $1/Q_{PT}$) decrease with increasing degree of wave inhomogeneity. For the counterpart $S$ waves, $1/Q_{SV}$ is independent of the degree of wave inhomogeneity and $1/Q_{ST}$ shows the trend of increasing with increasing degree of wave inhomogeneity. These findings can be explained by the limiting dissipation factors (defined at the infinite degree of inhomogeneity) which all depend only on the shear modulus. To ensure the correctness of our results, we repeated each step of the investigation in a parallel way based on Buchen's (1971) classic real value energy balance equation, including derivation of explicit formulas for $1/Q_{PV}$ and $1/Q_{PT}$, with inhomogeneity angle $\gamma$ ($-\pi/2 < \gamma < \pi/2$) representing the degree of inhomogeneity of the plane wave. We also obtain the inhomogeneity-independent formula for $1/Q_{SV}$ and exactly the same phase velocity and dissipation factor dispersion results for the example material.

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References
