



A meshfree model for tsunami wave propagation

Zili Zhou¹ and Patrick Lynett²

¹Civil and Environmental Engineering Department, University of Southern California, Los Angeles, U.S.A. (zilizhou@usc.edu)

²Civil and Environmental Engineering Department, University of Southern California, Los Angeles, U.S.A. (plynett@usc.edu)

The nonlinear deformation and run-up of tsunami waves on a plane beach and in a constant depth section are studied numerically and analytically based on a Meshfree Shallow Water Model. Because of the strong nonlinearity on the boundary of the propagation, issues like mesh distortion and discontinuous oscillation easily happen in the traditional mesh-based methods (e.g., FVM, FDM). But in the meshfree method, the drastic nonlinear changes can be well approximated by the high spectrum.

The study region is a rectangle, and the boundary condition is homogeneous, so the model meets the spectral expansion condition. And then, the trigonometric functions of a high order and high frequency can be used to solve mesh distortion and discontinuous oscillation problems. This means that the waves are simulated by multiple overlaid wavelets, making the simulation more similar to actual scenarios. The wave height (H) and horizontal wave speeds (U , V) are described by different trigonometrical series combinations.

Analytical methods including Fourier series expansion are used in the spatial dimension. After the expansion, we have the nonlinear partial differential equations with unknown coefficients, and they are functions of time. The Finite Difference Method is used in the time dimension. We choose the semi-implicit scheme to discretize the equations. This scheme saves much time since the model does not need to calculate the inversion matrix in every time step. Without this time-consuming task, compared to traditional mesh-based methods, the meshfree method can do less work, and the result will still be better, because the meshfree method (spectral method) can still be stable with a relatively big time step, while big time steps can cause inaccurate results in traditional mesh-based methods. Also, even though the numerical method is applied in the time dimension, time is only one dimensional, which makes the results not far away from the exact solutions. Since the series (or kernel, or basis) used to describe H , U , V is the orthogonal set. And All orthogonal sets remain continuous and smooth even when they oscillate strongly at a higher order. In this way, the leading causes of the drastic change problem are reduced to: 1. the calculation error, which means we need to try different integrations and find the optimal one; 2. the time step size is not small enough, which leads to more partial analysis on the boundary.