



## On the link between Beamforming and Kernel-based Source Inversion

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Beamforming and backprojection methods offer a data-driven approach to image noise sources, but provide no opportunity to account for prior information or iterate through an inversion framework. In contrast, recent methods have been developed to locate ambient noise sources based on cross-correlations between stations and the construction of finite-frequency kernels, allowing for inversions over multiple iterations (i.e., Tromp et al., 2010, Ermert et al. 2017, Sager et al. 2018). These kernel-based approaches show great promise, both in mathematical rigour and in results, but may remain difficult to understand or implement for the wider community. Here we show that these two different classes of methods, beamforming and kernel-based inversion, are achieving exactly the same result in certain circumstances. This means existing beamforming and backprojection methods can also incorporate prior information in a mathematically correct manner.

We start with a description of a relatively simple beamforming or backprojection algorithm, based on time-domain shifting and measurement of waveform coherence. Only by changing the order of steps, we begin to resemble the kernel-based approaches. By adding a physical model for the distribution of noise sources, and therefore synthetic correlation functions, we can extend backprojection to an iterative, gradient-based inversion scheme. Adjoint methods and a direct simulation of correlation wavefields can later be used to increase computational efficiency, but we stress that these are not needed to understand the approach.

Given the equivalence of these approaches between these two communities, both sides can benefit from bridging the gap. For example, for kernel-based inversion schemes, a current challenge lies in defining the misfit and time window over which a correlation will be scored; a windowing function based on beamform images offers a more intuitive way to identify significant contributions in the noise wavefield, exploiting more than just the direct surface-wave arrivals.