Linear and nonlinear waves in three-dimensional stratified rotating astrophysical flows in the Boussinesq approximation

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Magnetohydrodynamic waves in a stratified rotating plasma in a gravitational field in the Boussinesq approximation are studied. The theory of flows on the f-plane, on the non-traditional f-plane (taking into account the horizontal component of the Coriolis force), on the beta-plane, and on the non-traditional beta-plane is developed. In each considered case linear solutions of systems of three-dimensional magnetohydrodynamic equations in the Boussinesq approximation are obtained in form of magnetic inerto-gravity waves, magnetostrophic waves, and magneto-Rossby waves. For equations of a rotating stratified plasma without taking into account sphericity (in the approximation of the f-plane and the non-traditional f-plane), dispersion relations describe three-dimensional magnetic inerto-gravity waves and three-dimensional magnetostrophic waves. In the case of propagating only along the vertical component of the wave vector, their dispersion relations describe two types of magnetic waves, the first of which is a special case of magnetic inerto-gravity waves propagating only vertically, and the second is a special case of magnetostrophic waves propagating only vertically. In addition, it was found that dispersion relations describing wave propagation taking into account sphericity in a first approximation (on the beta-plane and on the non-traditional beta-plane) along the vertical component of the wave vector have a similar particular form. In the case of wave propagation in a horizontal plane, magnetic inerto-gravity waves turn into Alfvén waves, and magnetostrophic waves turn into magnetogravitational waves. In addition, for waves on a non-traditional f-plane, the influence of the horizontal component of the Coriolis force on the existence of various types of three-wave interactions is shown. For equations of a rotating stratified plasma on the beta-plane and on the non-traditional beta-plane dispersion relations for horizontal waves are found in form of magnetogravitational waves (similar to waves on the f-plane) and various types of magneto-Rossby waves. In addition, the equivalence of the low-frequency mode of the magneto-Rossby wave in the Boussinesq approximation and in the magnetohydrodynamic shallow water approximation was shown. The dispersion curves of all the detected wave types are qualitatively analyzed to identify the fulfillment of the synchronism condition, which ensures the presence of three-wave interactions. A system of amplitude equations for interacting waves and the increments of two types of instability that occur in the system (decay and amplification) are obtained using the method of multiscale expansions. The difference in the coefficients and differential operators in the three-wave interaction system is shown for each of the found types of three-wave interactions.
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