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Fundamental Notions in Relativistic Geodesy - physics of a timelike Killing vector field

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The Earth's geoid is one of the most important fundamental concepts to provide a gravity field-related height reference in geodesy and associated sciences. To keep up with the ever-increasing experimental capabilities and to consistently interpret high-precision measurements without any doubt, a relativistic treatment of geodetic notions within Einstein's theory of General Relativity is inevitable.

Building on the theoretical construction of isochronometric surfaces we define a relativistic gravity potential as a generalization of known (post-)Newtonian notions. It exists for any stationary configuration and rigidly co-rotating observers; it is the same as realized by local plumb lines and determined by the norm of a timelike Killing vector. In a second step, we define the relativistic geoid in terms of this gravity potential in direct analogy to the Newtonian understanding. In the respective limits, it allows to recover well-known results. Comparing the Earth's Newtonian geoid to its relativistic generalization is a very subtle problem. However, an isometric embedding into Euclidean three-dimensional space can solve it and allows an intrinsic comparison. We show that the leading-order differences are at the mm-level. In the next step, the framework is extended to generalize the normal gravity field as well. We argue that an exact spacetime can be constructed, which allows to recover the Newtonian result in the weak-field limit. Moreover, we comment on the relativistic definition of chronometric height and related concepts.

In a stationary spacetime related to the rotating Earth, the aforementioned gravity potential is of course not enough to cover all information on the gravitational field. To obtain more insight, a second scalar function can be constructed, which is genuinely related to gravitomagnetic contributions and vanishes in the static case. Using the kinematic decomposition of an isometric observer congruence, we suggest a potential related to the twist of the worldlines therein. Whilst the first potential is related to clock comparison and the acceleration of freely falling corner cubes, the twist potential is related to the outcome of Sagnac interferometric measurements. The combination of both potentials allows to determine the Earth's geoid and equip this surface with

coordinates in an operational way. Therefore, relativistic geodesy is intimately related to the physics of timelike Killing vector fields.