Linear response theory for macroscopic observables in high-dimensional systems: when is it valid and when not?

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The long-term average response of observables of chaotic systems to dynamical perturbations can often be predicted using linear response theory, but not all chaotic systems possess a linear response. Macroscopic observables of complex dissipative chaotic systems, however, are widely assumed to have a linear response even if the microscopic variables do not, but the mechanism for this is not well-understood.

We present a comprehensive picture for the linear response of macroscopic observables in high-dimensional coupled deterministic dynamical systems, where the coupling is via a mean field and the microscopic subsystems may or may not obey linear response theory. We derive stochastic reductions of the dynamics of these observables from statistics of the microscopic system, and provide conditions for linear response theory to hold in finite dimensional systems and in the thermodynamic limit. In particular, we show that for large systems of finite size, linear response is induced via self-generated noise.

We present examples in the thermodynamic limit where the macroscopic observable satisfies LRT, although the microscopic subsystems individually violate LRT, as well a converse example where the macroscopic observable does not satisfy LRT despite all microscopic subsystems satisfying LRT when uncoupled. This latter, maybe surprising, example is associated with emergent non-trivial dynamics of the macroscopic observable. We provide numerical evidence for our results on linear response as well as some analytical intuition.