Unstable Periodic Orbits Sampling in Climate Models

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Unstable periodic orbits (UPOs) have been proved to be a relevant mathematical tool in the study of Climate Science. In a recent paper Lucarini and Gritsun [1] provided an alternative approach for understanding the properties of the atmosphere. Climate can be interpreted as a non-equilibrium steady state system and, as such, statistical mechanics can provide us with tools for its study.

UPOs decomposition plays a relevant role in the study of chaotic dynamical systems. In fact, UPOs densely populate the attractor of a chaotic system, and can therefore be thought as building blocks to construct the dynamic of the system itself. Since they are dense in the attractor, it is always possible to find a UPO arbitrarily near to a chaotic trajectory: the trajectory will remain close to the UPO, but it will never follow it indefinitely, because of its instability. Loosely speaking, a chaotic trajectory is repelled between neighbourhoods of different UPOs and can thus be approximated in terms of these periodic orbits. The characteristics of the system can then be reconstructed from the full set of periodic orbits in this fashion.

The sampling of UPOs is therefore a relevant problem for describing chaotic dynamical systems and can represent an interesting topic for the study of Climate Science. In this work we address this problem and present an algorithm to numerically extract UPOs from the attractor of a simple Climate Model such as Lorenz-63.