A new model for the three-dimensional propagation of water waves in the presence of vertically sheared, horizontally varying currents

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In coastal areas, steep bathymetries and strong currents are often observed. Among several causes, the presence of cliffs, rocky beds, or human structures may cause strong variations of the sea bed, while oceanic circulation, tides, wind action or wave breaking can be responsible for the generation of strong currents. For both coastal safety and engineering purposes, there are many interests in providing efficient models predicting the nonlinear, phase resolved behavior of water waves in such areas. The difficulty is known to be important, and many models achieving that goal are described in the related literature.

Recently, it was established that beneath the influence of vertically uniform currents, the vorticity involved in depth varying mean flows could have significant impact on the propagation of water waves (Rey et al. 2014). This gave rise to new derivations of equations aimed to describe this interaction. First, an extended mild slope equation was obtained (Touboul et al. 2016). Then, the now classical coupled mode theory was introduced in the system to obtain a set of coupled equations, which could be compared to the system derived by Belibassakis et al (2011) but considering currents which may present constant shear with depth (Belibassakis et al. 2017, Belibassakis et al., 2019). In these works, the currents were assumed to vary linearly with depth, presenting a constant shear. However, this approach was recently extended to more general configurations (Belibassakis & Touboul, 2019; Touboul & Belibassakis, 2019).

In this work, we extend this model to three dimensional configurations. It is emphasized that the model is able to describe rotational waves, as expected, for example, when water waves propagate with a non-zero angle with respect to the current direction (see e.g. Ellingsen, 2016).


