An improved monolithic Newton-Raphson scheme for solving plastic flow with nonlinear flow laws

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Brittle-plastic flows where the yield strength is a decreasing, non-linear function of plastic strain are thought to be commonplace in the Earth, and responsible for some of its most catastrophic events. Recent work [1] has highlighted again the computational benefit of an iterative Newton-Raphson scheme that contains a linearization of the plastic flow problem that is consistent with its time discretization. However, such a consistent linearization requires a nested set of iterations to converge on a yield strength if it is governed by a law that is non-linear in strain (or strain rate).

Eckert and co-authors [2] have shown that the construction of a consistent linearization can be avoided altogether, including these inner iterations, though at the considerable cost of including the full plastic strain tensor as an objective variable alongside the displacement vector. The resulting system is therefore larger, but as it can be expressed directly, possesses the quality that it may be linearized automatically, cheaply, and accurately by finite-differencing the non-linear residual with respect to the solution variables. Their algorithm naturally incorporates predictor and corrector polynomials that are second-order accurate in time, contrasting with traditional methods that are often derived using a Backward Euler time integrator. We present a modification to this algorithm that suppresses the cost of operating it significantly by replacing the symmetric second-order plastic strain tensor with a single effective plastic strain scalar objective variable, cutting the number of unknowns by 40\% (2D) and 55\% (3D) This makes it computationally more on par with existing schemes that employ a consistent tangent modulus.

We demonstrate this improved algorithm with test cases of non-linear strain softening laws relevant to Earth scientists, that include regularization by both Kelvin visco-plasticity [3] and non-local measures of effective plastic strain [4]. In addition, we analyse performance of this scheme with respect to existing algorithms.

References