Composable, Scalable Solvers on Staggered Grids

Patrick Sanan\textsuperscript{1} and Dave May\textsuperscript{2}
\textsuperscript{1}Institute of Geophysics, ETH Zurich, Zurich, Switzerland
\textsuperscript{2}Department of Earth Sciences, University of Oxford, Oxford, United Kingdom

Scalable preconditioners for saddle point problems are essential to the solution of problems in geodynamics and beyond. Recent years have produced a wealth of research into efficient solvers for finite element methods. These solvers are also effective, however, for orthogonal-grid finite volume discretizations of saddle point problems, also known as "staggered grid" or "marker and cell (MAC)" methods. Perhaps, ironically, due to the highly-structured nature of these discretizations, the use of advanced solvers is stymied due to the lack of a uniform topological abstraction, which is required for most scalable solvers, such as geometric multigrid. We present new software to allow experimentation with and composition of these advanced solvers. We focus on variable-viscosity Stokes problems with discontinuous coefficient jumps. In particular, we attempt to demonstrate how the important known robust preconditioners may be employed, and how new variants may be experimented with. Important solvers are compositions of block factorizations and multigrid cycles. We demonstrate as many of these as possible, including triangular block preconditioners with nested multigrid solves, and monolithic multigrid solves with cellwise (Vanka) or field-based (Distributed Gauss-Seidel, Braess-Sarazin) smoothers. Implementations are provided as part of the PETSc library, using the new DMStag component, and examples from the StagBL library are also shown where appropriate. These tools are intended to help break down the barrier between cutting-edge research into advanced solvers (which is only becoming more complex, as multi-phase problems are further explored) and practical usage in geophysical research and production codes.