Intrinsic non-uniqueness of the acoustic full waveform inverse problem

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In the context of seismic imaging, the full waveform inversion (FWI) is more and more popular. Because of its lower numerical cost, the acoustic approximation is often used, especially at the exploration geophysics scale, both for tests and for real data. Moreover, some research domains such as helioseismology face true acoustic medium for which FWI can be useful. In this work, we show that the general acoustic inverse problem based on limited frequency band data is intrinsically non-unique, making any general acoustic FWI impossible. Our work is based on two tools: particle relabelling and homogenization. On the one hand, the particle relabelling method shows it is possible to deform a true medium based on a smooth mapping into a new one without changing the signal recorded at seismic stations. This is a potentially strong source of non-uniqueness for an inverse problem based a seismic data. Nevertheless, in the elastic case, the deformed medium loses the elastic tensor minor symmetries and, in the acoustic case, it implies density anisotropy. It is therefore not a source of non-uniqueness for elastic or isotropic acoustic inverse problems, but it is for the anisotropic acoustic case. On the other hand, the homogenization method shows that any fine-scale medium can be up-scaled into an effective medium without changing the waveforms in a limited frequency band. The effective media are in general anisotropic, both in the elastic and acoustic cases, even if the true media are isotropic at a fine scale. It implies that anisotropy is in general present and needs to be inverted. Therefore, acoustic anisotropy can not be avoided in general. We conclude, based on a particle relabelling and homogenization arguments, that the acoustic FWI solution is in general non-unique. We show, in 2-D numerical FWI examples based on the Gauss-Newton iterative scheme, the effects of this non-uniqueness in the local optimization context. We numerically confirm that the acoustic FWI is in general non-unique and that finding a physical solution is not possible.