Multiscale measures of phase-space trajectories

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Several attempts have been made in characterizing the multiscale nature of fluctuations from nonlinear and nonstationary time series. Particularly, the study of their fractal structure has made use of different approaches like the structure function analysis, the evaluation of the generalized dimensions, and so on. Here we report on a different approach for characterizing phase-space trajectories by using the empirical modes derived via the Empirical Mode Decomposition (EMD) method. We show how the derived Intrinsic Mode Functions (IMFs) can be used as source of local (in terms of scales) information allowing us in deriving multiscale measures when looking at the behavior of the generalized fractal dimensions at different scales. This formalism is applied to three pedagogical examples like the Lorenz system, the Henon map, and the standard map. We also show that this formalism is readily applicable to characterize both the behavior of the Earth’s climate during the past 5 Ma and the dynamical properties of the near-Earth electromagnetic environment as monitored by the SYM-H index.