Precipitation is an important meteorological variable which is critical for weather risk assessment. For instance, intense but short precipitation events can lead to flash floods and landslides. Most statistical modelling studies assume that the occurrence of precipitation events is based on a Poisson process with exponentially distributed waiting times while precipitation intensities are typically described by a gamma distribution or a mixture of two exponential distributions. Here, we show by using hourly precipitation data over the United States that the waiting time between precipitation events is non-exponentially distributed and best described by a fractional Poisson process. A systematic model selection procedure reveals that the hourly precipitation intensities are best represented by a two-distribution model for about 90% of all stations. The two-distribution model consists of (a) a generalized Pareto distribution (GPD) model for bulk precipitation event sizes and (b) a power-law distribution for large and extreme events. Finally, we analyse regional climate model output to evaluate how the climate models represent the high-frequency temporal structure of U.S. precipitation. Our results reveal that these regional climate models fail to accurately reproduce the power-law behaviour of intensities and severely underestimate the long durations between events.