Analyzing the Effects of Dirichlet and Neumann Boundary Conditions on Reduced-Order Modeling of Groundwater Flow through Heterogeneous Porous Media

Saumava Dey\(^1\) and Anirban Dhar\(^2\)

\(^1\)Indian Institute of Technology Kharagpur, Civil Engineering, India (dey.saumava@gmail.com)
\(^2\)Indian Institute of Technology Kharagpur, Civil Engineering, India (anirban@civil.iitkgp.ac.in)

Reduced-order modeling is an emerging technique for cutting down the computational expenses incurred in terms of CPU time and usage associated with repetitive simulation of flow dynamics of natural aquifer systems. Identifying the patterns related to the evolution of aquifer response with time is the key to model order reduction methodology. However, the accuracy of reduced-order groundwater models is dependent on several factors. It has been observed that the accuracy decreases while accounting for random heterogeneity of natural aquifers. Besides, the imposition of different boundary conditions also tends to influence the accuracy of reduced-order models. In this work, we study the effects of Dirichlet and Neumann boundary conditions on reduced-order modeling of groundwater flow through randomly distributed heterogeneous porous media. For low dimensional modeling, we have performed Singular Value Decomposition of the ‘Snapshot Matrix’ to obtain a set of orthonormal basis functions. The ‘Snapshot Matrix’ is formed from the solution of a Finite Volume Method based full system groundwater flow model at some exponentially distributed time instants. The governing groundwater flow equation is then projected onto the reduced sub-space of orthonormal basis functions via Galerkin Projection to obtain the solution at each time-step. We have carried out the study on a two-dimensional square-shaped synthetic heterogeneous aquifer with multiple pumping wells operating simultaneously within the domain. Four illustrative case studies have been performed with the aquifer being subjected to: (1) Dirichlet condition on all boundaries; (2) Dirichlet condition on three boundaries while the remaining boundary is impermeable; (3) Dirichlet condition on two parallel boundaries while the other two boundaries are impermeable; (4) Three impermeable boundaries and Dirichlet condition on the remaining boundary. The study shows that the accuracy of the reduced-order model is maximum when all the four boundaries of the aquifer are subjected to a constant specified head (Dirichlet) boundary condition. The accuracy starts to go down as we start introducing impermeable boundaries withdrawing the Dirichlet boundaries. The error analysis is performed by comparing the error statistic parameters: Maximum Absolute Error, Mean Absolute Error (MAE), Root Mean Square Error (RMSE) and Normalized Root Mean Square Error (NRMSE) for the four case studies with respect to the results obtained from corresponding full system model runs. However, if we look into the computational expenses, the model takes lesser computation time per iteration as the complexity of boundary conditions increases. Although the reduction in
the accuracy of the reduced-order model is observed with the introduction of impermeable boundaries, the error statistic parameters are within desirable limits. Hence, the proposed reduced-order modeling methodology can potentially be accepted as an accurate and efficient alternative for replication of high-dimensional full system groundwater flow models, and can also be applied for natural aquifers on a watershed scale.