Log-log linearity of the asymptotic distribution - a valid indicator of multi-fractality?

Peter Bossew  
BfS (German Federal Office for Radiation Protection), Berlin, Germany (pbossew@bfs.de)

The asymptotic shape of the marginal frequency distribution of geochemical variables has been proposed as indicator of multi-fractality. Transition into a certain statistical regime as inferred from the distribution function may be considered as criterion to delineate geochemical anomalies, including mineral resources or pollutants such as radioactive fallout or geogenic radon.

The argument is that asymptotic linearity in log-log scale, \( \log(F(z)) = a - b \log(z) \) as \( z \to \infty \), \( b > 0 \) a constant, indicates multi-fractality.

We discuss this with respect to two issues:

1. What are the consequences of estimating the slope \( b \) for non-ergodic, in particular non-representative and preferential sampling schemes, as often the case in geochemical or pollution surveys?

2. Frequently in geochemistry, multiplicative cascades are considered valid generators of multifractal fields, corroborated by observed \( f(\alpha) \) functions and variograms (Matérn or power, for low lags). This generator leads to marginally asymptotically (high cascade orders) log-normal distributions, which in log-log scale are asymptotically (high \( z \)) parabolic, not linear.

Theoretical aspects are addressed as well as examples given.