Synchronization of traveling waves in coupled dispersive systems

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Coupled KdV-type equations arise in multimodal dispersive models such as the Gear – Grimshaw system which describes weakly nonlinear internal waves in neighboring pycnoclines. Coupling occurs when two or more phase speeds of different modes are close together. This phenomenon of kissing modes is known as the Eckart resonance providing energy transfer between pycnoclines in stratified fluid. Decoupled basic equations generate separated modes of traveling waves with different phase shifts. In this context, synchronization means the existence of coupled phase-shifted solutions which can be constructed from decoupled modes by appropriate perturbation procedure. In the present paper, we consider analytic conditions which provide the existence of periodic solutions describing synchronized cnoidal-type wave trains. Application of the Lyapunov – Schmidt method reduces this problem to the nonlinear system of implicit bifurcation equations for unknown phase shift and wave amplitude. Asymptotic analysis of these equations results in a sufficient condition of synchronization, which involves the Poincare – Pontryagin function depending on coupling nonlinear terms. In addition, we illustrate two different limit cases which lead to the same existence condition. First of them corresponds to a solitary-wave limit for cnoidal waves (i.e. a nonlinear long-wave limit), and the second one is adapted to a small-amplitude limit of coupled harmonic wave packets.

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