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## The strange instability of the equatorial Kelvin wave

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The Kelvin wave is perhaps the most important of the equatorially trapped waves in the terrestrial atmosphere and ocean, and plays a role in various phenomena such as tropical convection and El Niño. Theoretically, it can be understood from the linear dynamics of a stratified fluid on an equatorial  $\beta$ -plane, which, with simple assumptions about the disturbance structure, leads to wavelike solutions propagating along the equator, with exponential decay in latitude. However, when the simplest possible background flow is added (with uniform latitudinal shear), the Kelvin wave (but not the other equatorial waves) becomes unstable. This happens in an extremely unusual way: there is instability for arbitrarily small nondimensional shear  $\lambda$ , and the growth rate is proportional to  $\exp(-1/\lambda^2)$  as  $\lambda \rightarrow 0$ . This is in contrast to most hydrodynamic instabilities, in which the growth rate typically scales as a positive power of  $\lambda - \lambda_c$  as the control parameter  $\lambda$  passes through a critical value  $\lambda_c$ .

This Kelvin wave instability has been established numerically by Natarov and Boyd, who also speculated as to the underlying mathematical cause by analysing a quantum harmonic oscillator perturbed by a potential with a remote pole. Here we show how the growth rate and full spatial structure of the Kelvin wave instability may be derived using matched asymptotic expansions applied to the (linear) equations of motion. This involves an adventure with confluent hypergeometric functions in the exponentially-decaying tails of the Kelvin waves, and a trick to reveal the exponentially small growth rate from a formulation that only uses regular perturbation expansions. Numerical verification of the analysis is also interesting and challenging, since special high-precision solutions of the governing ordinary differential equations are required even when the nondimensional shear is not that small (circa 0.5).