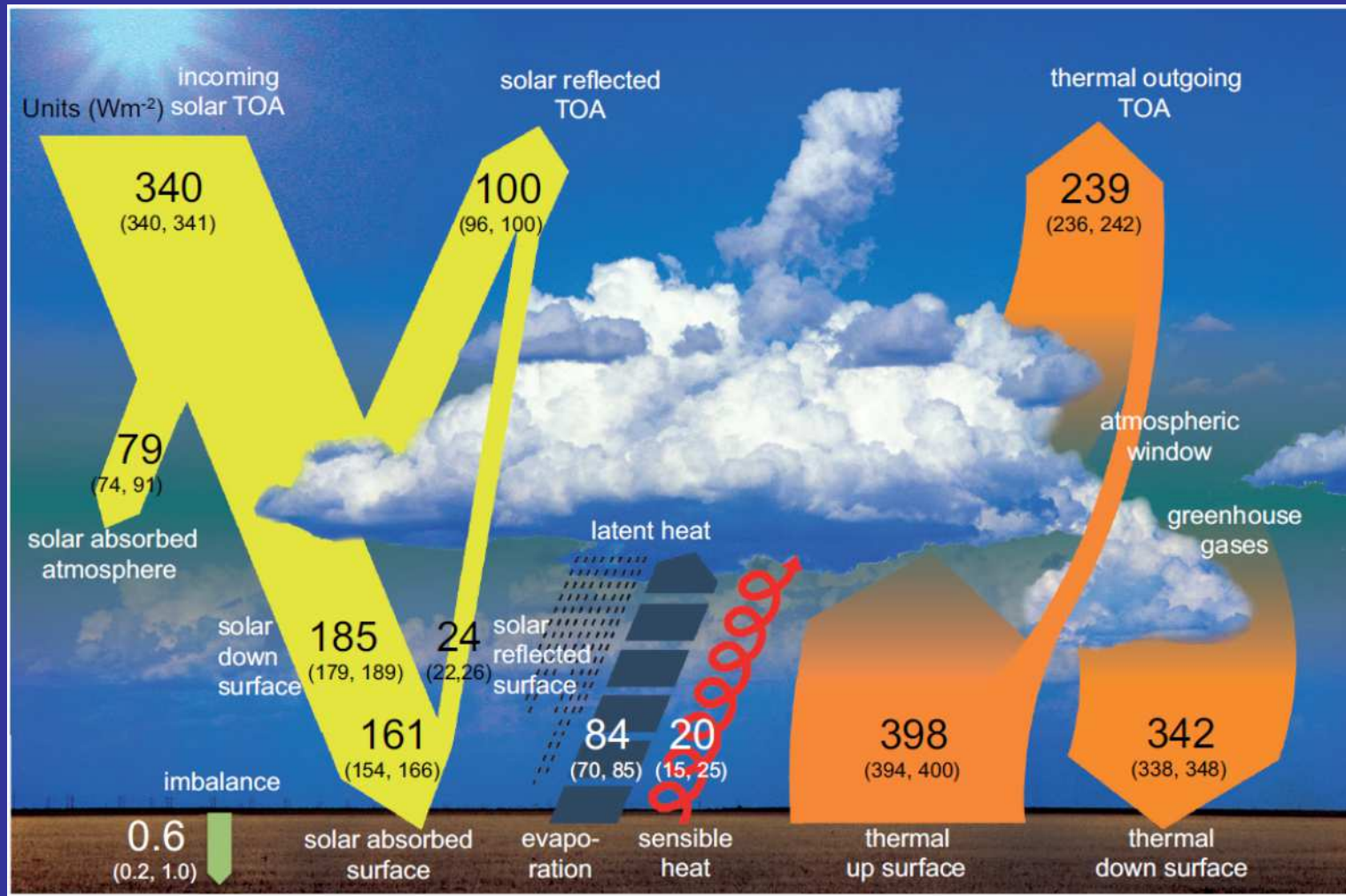
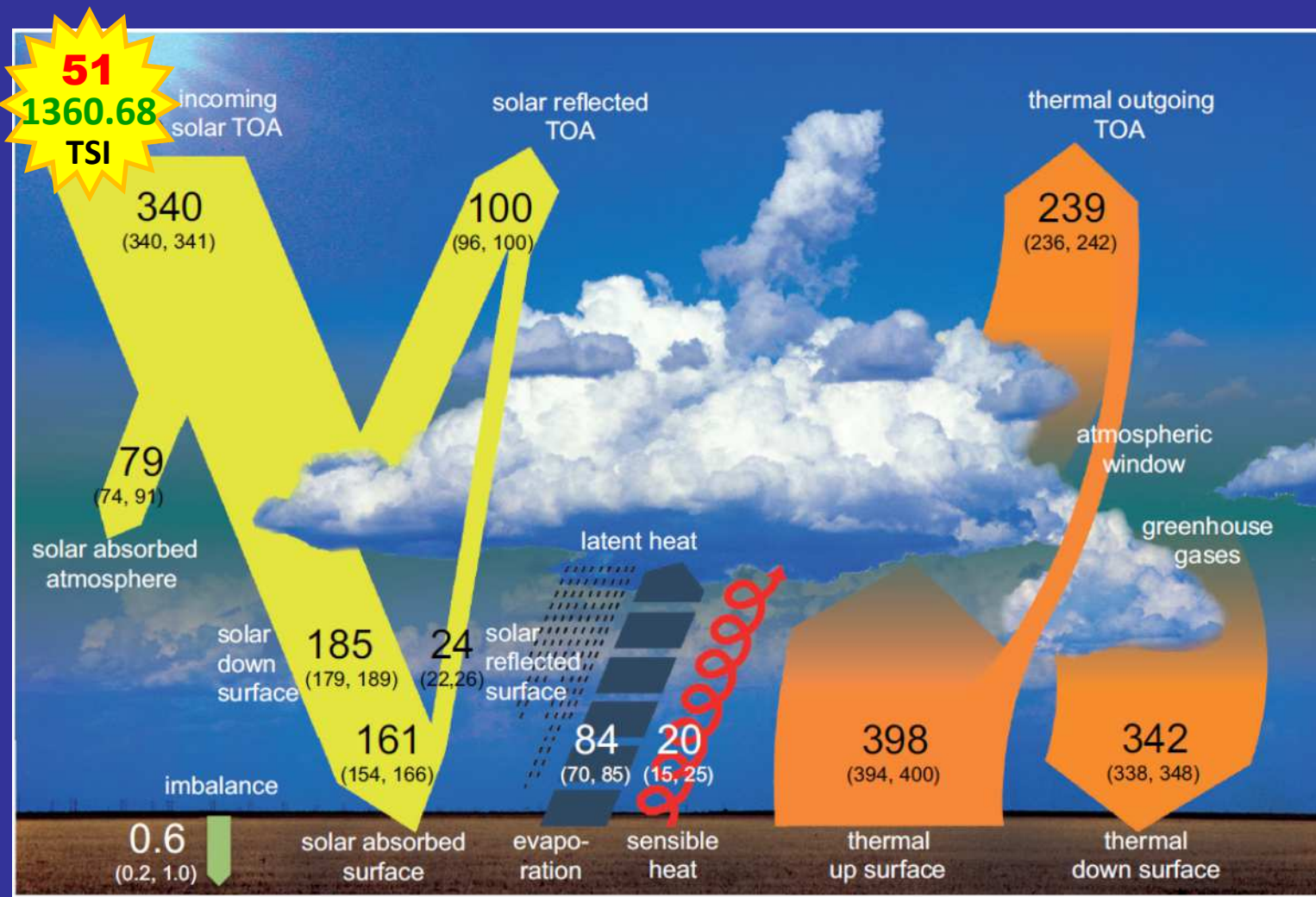


Challenging CMIP6 Model Predictions



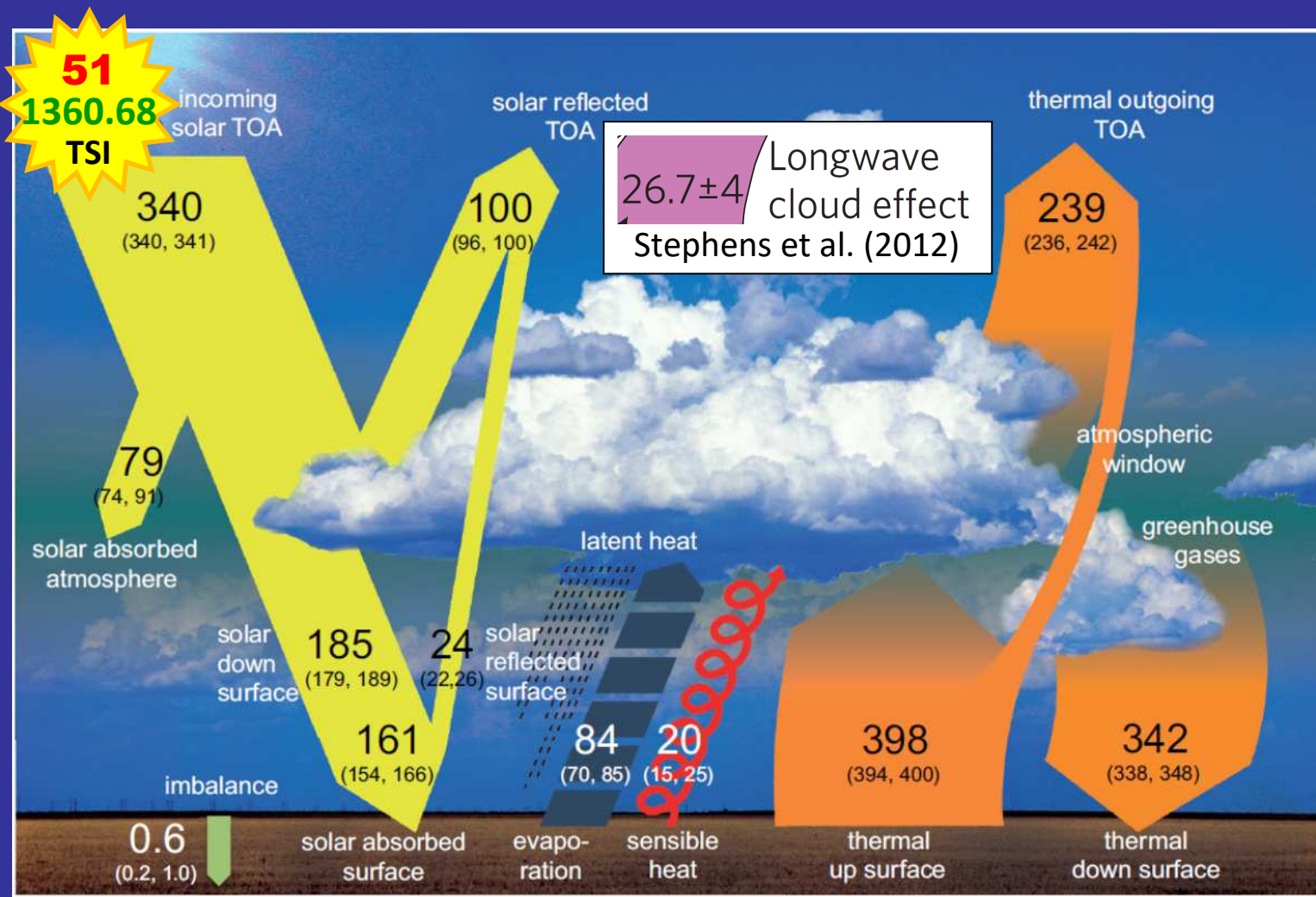
Challenging CMIP6 Model Predictions



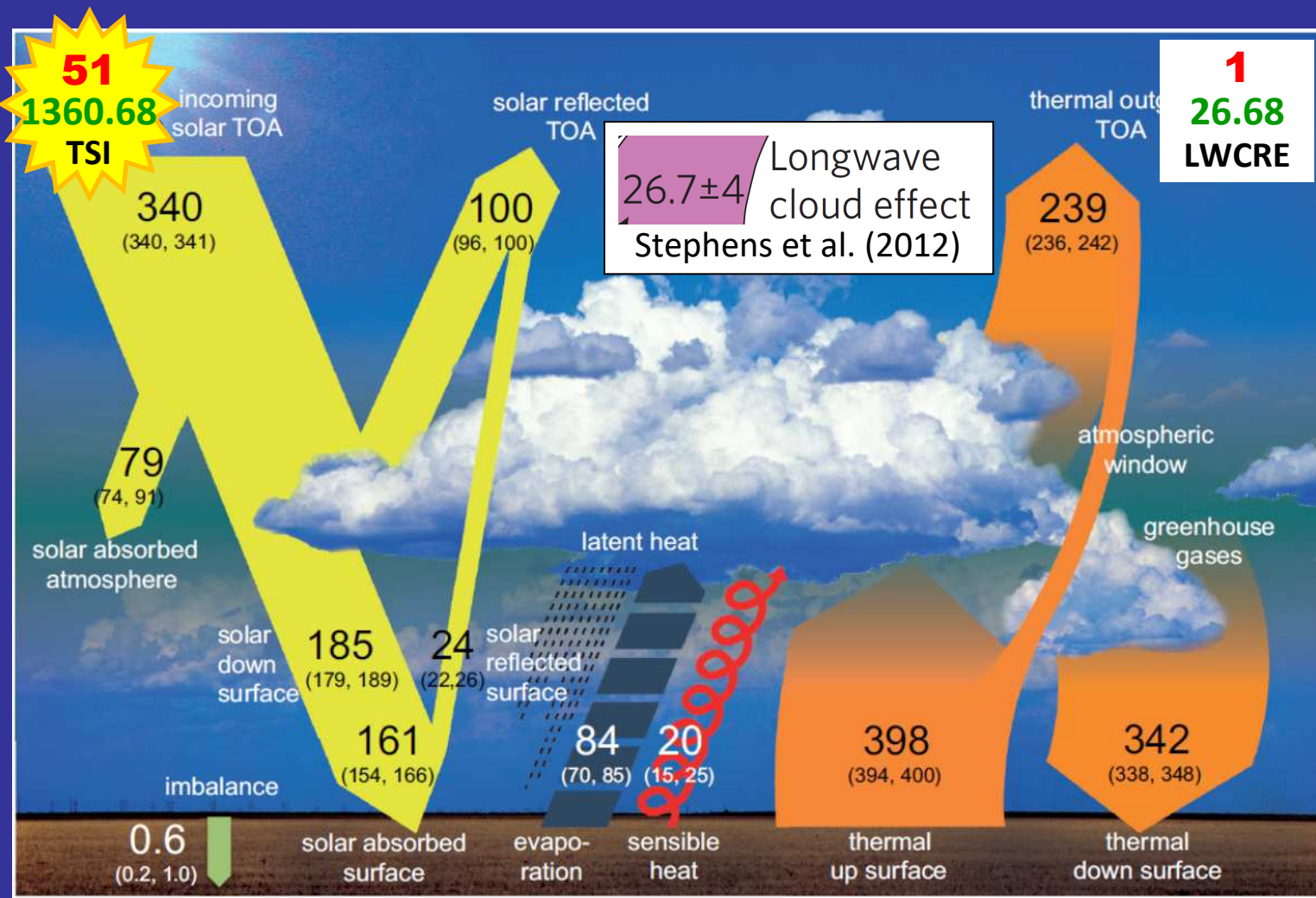
Miklos Zagoni — Session CL2.1, Abstract EGU2020-1



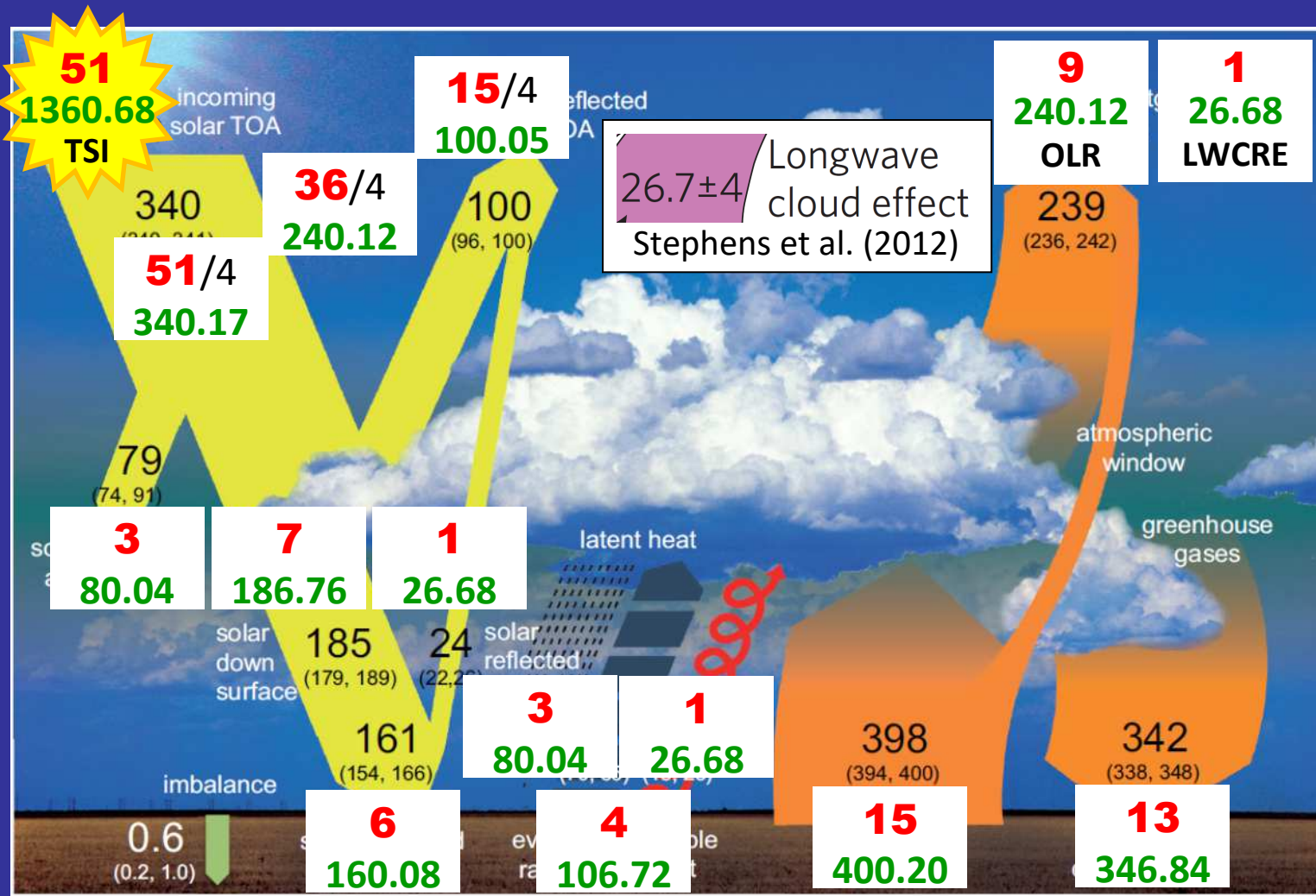
Challenging CMIP6 Model Predictions



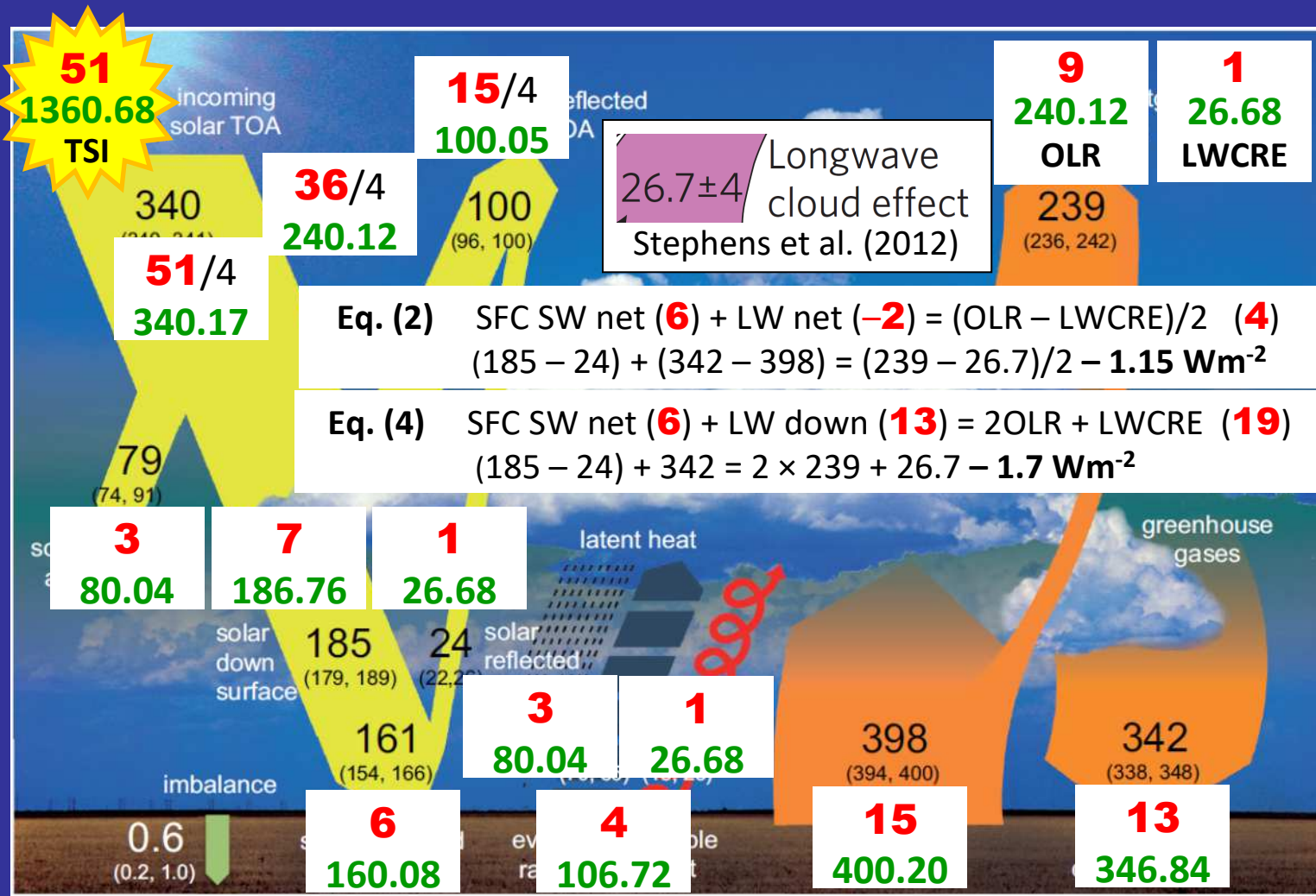
Challenging CMIP6 Model Predictions



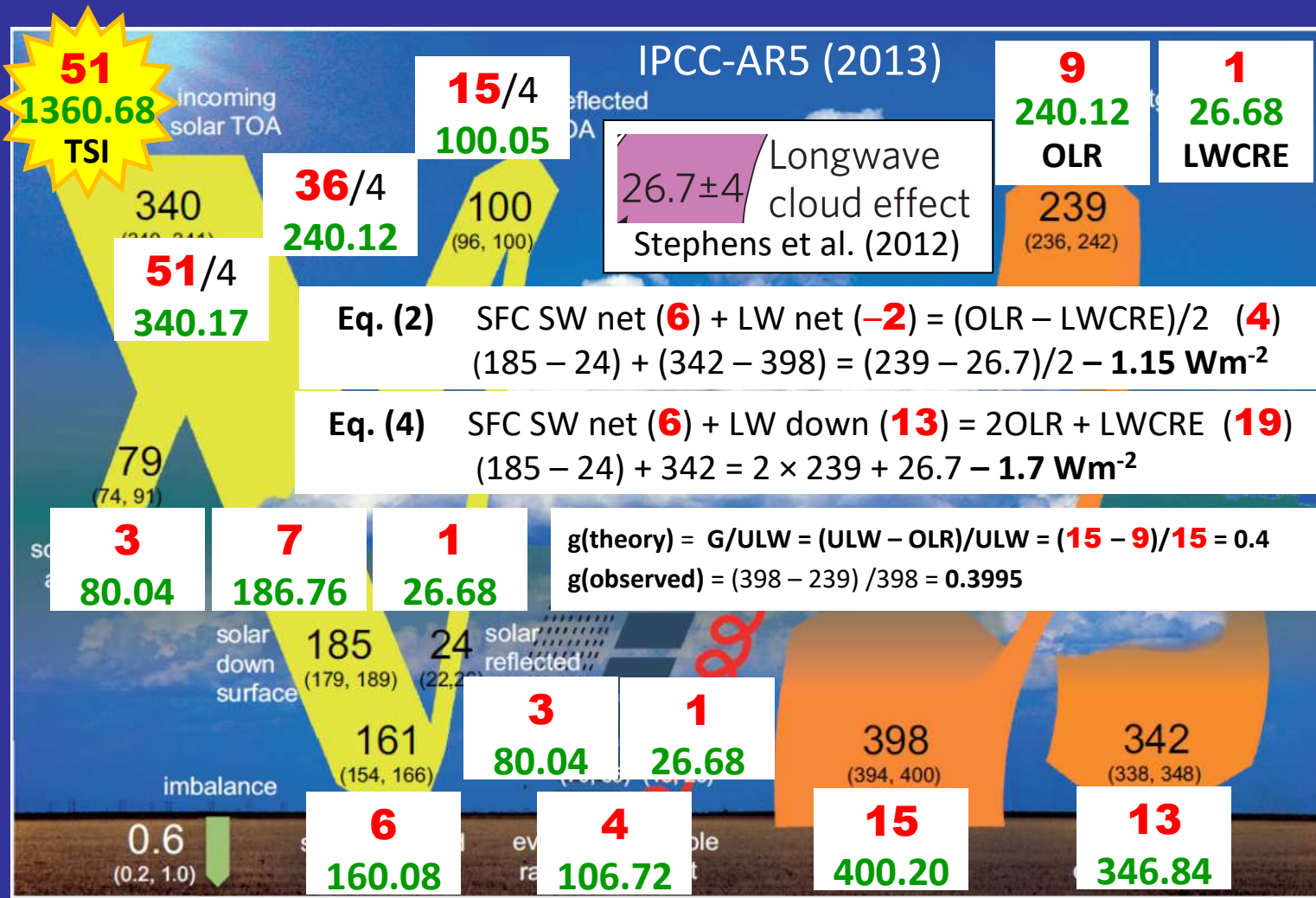
Challenging CMIP6 Model Predictions



Challenging CMIP6 Model Predictions



Challenging CMIP6 Model Predictions



Radiative forcing / Radiative equilibrium

„**Arrhenius** (1896) made the quantitative connection to estimate the surface temperature increase due to increases in CO₂. Arrhenius' systematic investigation and inferences have proven to be pivotal in *shaping the modern-day thinking and computational modeling of the climate effects due to CO₂ radiative forcing*.“ (Ramaswamy et al. 2019, Am Met Soc Monographs: **Radiative forcing of climate**)

Another line of thinking: **Radiative equilibrium** (Sampson, 1894)

On the Equilibrium of the Sun's Atmosphere (Schwarzschild 1906):

$$(11) \quad E = \frac{A_0}{2}(1 + m), \quad A = \frac{A_0}{2}(2 + m), \quad B = \frac{A_0}{2}m.$$

E : blackbody emission, A : outward radiation, $A_0 = \text{OLR}$
 B inward radiation, m : optische Masse (τ , optical depth)

Ueber das Gleichgewicht der Sonnenatmosphäre

Von

K. Schwarzschild.

Vorgelegt in der Sitzung vom 13. Januar 1906.

Völlig analog folgt für die nach außen gehende Strahlung:

$$(8) \quad \frac{dA}{dh} = -a(E - A).$$

Indem man sich das Absorptionsvermögen a als Funktion der Tiefe h gegeben denkt, bilde man die über der Tiefe h liegende „optische Masse“ der Atmosphäre:

$$(9) \quad m = \int^h a \, dh.$$

Dann lauten die Differentialgleichungen:

$$(10) \quad \frac{dB}{dm} = E - B, \quad \frac{dA}{dm} = A - E.$$

Wir fragen nach einem stationären Zustand der Temperaturverteilung. Derselbe ist bedingt durch die Forderung, daß jede Schicht ebensoviel Energie empfängt, als ausstrahlt, daß also gilt:

$$aA + a \cdot B = 2aE, \quad A + B = 2E.$$

Führt man dieser Bedingung entsprechend die Hilfsgröße γ ein durch:

$$A = E + \gamma, \quad B = E - \gamma,$$

so gehn die Differentialgleichungen durch Addition und Subtraktion über in:

$$\frac{d\gamma}{dm} = 0, \quad \frac{dE}{dm} = \gamma$$

und integriert:

$$\begin{aligned} \gamma &= \text{const}, & E &= E_0 + \gamma m, \\ A &= E_0 + \gamma(1 + m), & B &= E_0 + \gamma(m - 1). \end{aligned}$$

Die Integrationskonstanten E_0 und γ wurden dadurch bestimmt, daß an der Außengrenze der Atmosphäre ($m = 0$) keine nach innen wandernde Energie B vorhanden ist und die nach außen wandernde Energie den zu beobachtenden Betrag A_0 hat. Es muß also für $m = 0$:

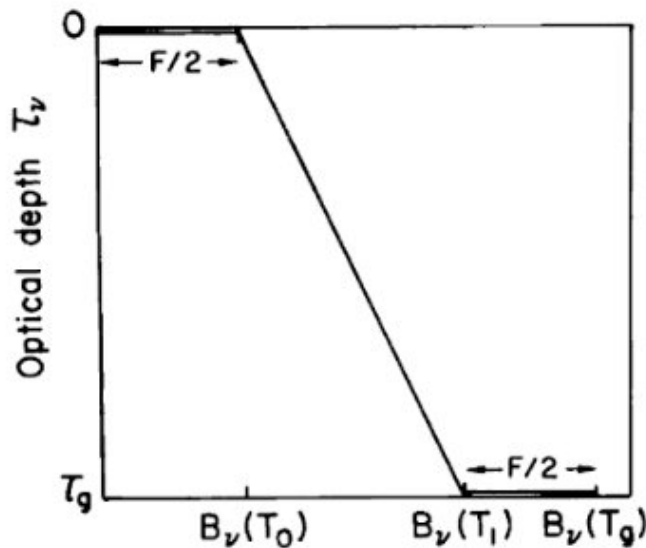
$$B = 0, \quad A = A_0$$

sein. Hiermit ergibt sich das Resultat:

$$(11) \quad E = \frac{A_0}{2}(1 + m), \quad A = \frac{A_0}{2}(2 + m), \quad B = \frac{A_0}{2}m.$$

$$(11) \quad E = \frac{A_0}{2}(1+m), \quad A = \frac{A_0}{2}(2+m), \quad B = \frac{A_0}{2}m.$$

$$A - E = \Delta A = A_0/2 \quad \text{independent of } m$$



Chamberlain (1978)

Theory of Planetary Atmospheres,
Academic Press

Fig. 1.4 The MRE solution for $T(\tau)$, presented as $B_v(T)$ vs. τ_v . Note the discontinuity at the ground and the finite skin temperature at $\tau = 0$.

$$B_g - B_0 = \frac{\phi}{2\pi}$$

Houghton (2002, Eq. 2.13)

$$B_g - B_0 = \Delta B_g = B_{\text{eff}}/2$$

Deduction by E. A. Milne (1930)

Now multiply (84) by $\sin \theta \, d\theta$ and integrate from $\theta = 0$ to $\theta = \frac{1}{2}\pi$. We find

$$\frac{1}{2} \frac{dI}{d\tau} = I - B \quad (88)$$

and similarly

$$\frac{1}{2} \frac{dI'}{d\tau} = B - I'. \quad (89)$$

These equations may be described as the equations of “linear” or “tubular” flow of radiation. They may be derived from first principles by dividing the radiation into an outward and an inward beam, and assuming a coefficient of absorption $2k$ to allow for the mean obliquity of the rays to the direction of the axis. They have proved exceedingly useful in many approximate investigations.

The equation of radiative equilibrium (82) becomes

$$2B = I + I'. \quad (90)$$

The mean flux, from (83) is given by

$$\mathfrak{F} = I - I'. \quad (91)$$

Equation (91) is easily seen to be an integral of equations (88) and (89), when regard is paid to (90): we have simply to subtract (88) and (89).

Clearly

$$I = B + \frac{1}{2}\mathfrak{F}, \quad I' = B - \frac{1}{2}\mathfrak{F}.$$

Adding (88) and (89) and using (90) we have

$$\frac{dB}{d\tau} = \mathfrak{F},$$

the solution of which is

$$B = \mathfrak{F}\tau + B_0$$

where B_0 is a constant, whence

$$I = \mathfrak{F}(\frac{1}{2} + \tau) + B_0$$

$$I' = \mathfrak{F}(-\frac{1}{2} + \tau) + B_0.$$

There is no rad

$$B_0 = \frac{1}{2}\mathfrak{F}$$

nce $I'(0) = 0$. Hence

(92)

whence

$$B = \mathfrak{F}(\frac{1}{2} + \tau)$$

(93)

$$I = \mathfrak{F}(1 + \tau)$$

(94)

$$I' = \mathfrak{F}\tau.$$

(95)

We notice that $I(0)$

20. Boundary temperature. we have always $B = (\sigma/\pi) T^4$. The temperature T_0 at the boundary is therefore given by

$$B_0 = (\sigma/\pi) T_0^4.$$

Since $B_0 = \frac{1}{2}\mathfrak{F}$ and $\mathfrak{F} = (\sigma/\pi) T_1^4$, we have immediately

$$T_0^4 = \frac{1}{2} T_1^4 \quad (96)$$

$$T_0 = T_1 / \sqrt[4]{2} = 0,840 T_1.$$

The complete temperature distribution, from (92), is then given by

$$T^4 = \frac{1}{2} T_1^4 (1 + 2\tau). \quad (96')$$

**Milne
cont'd**



Über Diffusion und Absorption in der Sonnenatmosphäre.

Von K. SCHWARZSCHILD.

(Vorgelegt von Hrn. EINSTEIN am 5. November 1914 [s. oben S. 979].)

§ 1.

Die Absorptions- und Emissionslinien in den Spektren der Sonne und der Sterne sind außerordentlich verschieden in ihrem Aussehen. Von Linien, die sich nur als geringe Abnahme der Intensität des kontinuierlichen Spektrums über einige hundertstel Angström zu erkennen geben, gibt es alle Übergänge zu Linien von mehreren Ångström

Schwarzschild (1914)

as presented by Goody and Yung (1989)

$$-\frac{1}{e_{\nu,\nu}} \frac{dI_{\nu}(P, \mathbf{s})}{ds} = I_{\nu}(P, \mathbf{s}) - J_{\nu}(P, \mathbf{s}). \quad (2.17)$$

Equation (2.17) is known as the *equation of transfer*, and was first given in this form by Schwarzschild. While it sets the pattern of the formalism used in transfer problems, its physical content is very slight.

2.4. Approximate methods for thermal radiation

2.4.1. *The atmospheric problem*

2.4.5. *Approximations for a stratified atmosphere*

For a stratified atmosphere, we set $\partial/\partial\tilde{\tau}_x = \partial/\partial\tilde{\tau}_y = 0$ and $\tilde{\tau}_z = \tau$ in (2.136) to give

$$\frac{d^2F}{d\tau^2} = 3F - 4\pi \frac{dJ}{d\tau}, \quad (2.140)$$

As an illustration, consider the case of radiative equilibrium with black bodies emitting $B^*(0)$ or $B^*(\tau_1)$ at the two boundaries. The third terms on the right-hand side of (2.144) and (2.145) are now zero and

$$F/2\pi = B(0) - B^*(0) = B^*(\tau_1) - B(\tau_1). \quad (2.146)$$

Equation (2.146) requires a discontinuity in the Planck function, implying a discontinuity of temperature, at the boundary.

The class of approximation of which (2.140) is representative is extensive and a large number of different names and terms are used to describe members of the class: the *Schwarzschild–Schuster* approximation, the *Eddington* approximations, *Chandrasekhar's first approximation*, and a variety of *two-stream approximations*.

Goody and Yung (1989)

The Eddington approxima-
tion will generally be employed; while it is not precise it omits no
essential physical principles, provided that the medium is stratified.

Goody (1964)

ATMOSPHERES IN RADIATIVE EQUILIBRIUM

9.1. Introduction

In this chapter we discuss *radiative equilibrium models* of the earth's atmosphere and the closely related *radiative-convective models*, for which small-scale convection is included in a highly parameterized form. In both cases, heat transports by planetary-scale motions are neglected.

$$B(\tau) = \frac{\sigma\theta(\tau)^4}{\pi} = \frac{-F_s(1 + 3\tau/2)}{2\pi} \quad \text{There are discontinuities,} \quad \Delta B = \frac{F_s}{2\pi} \quad (9.5)$$

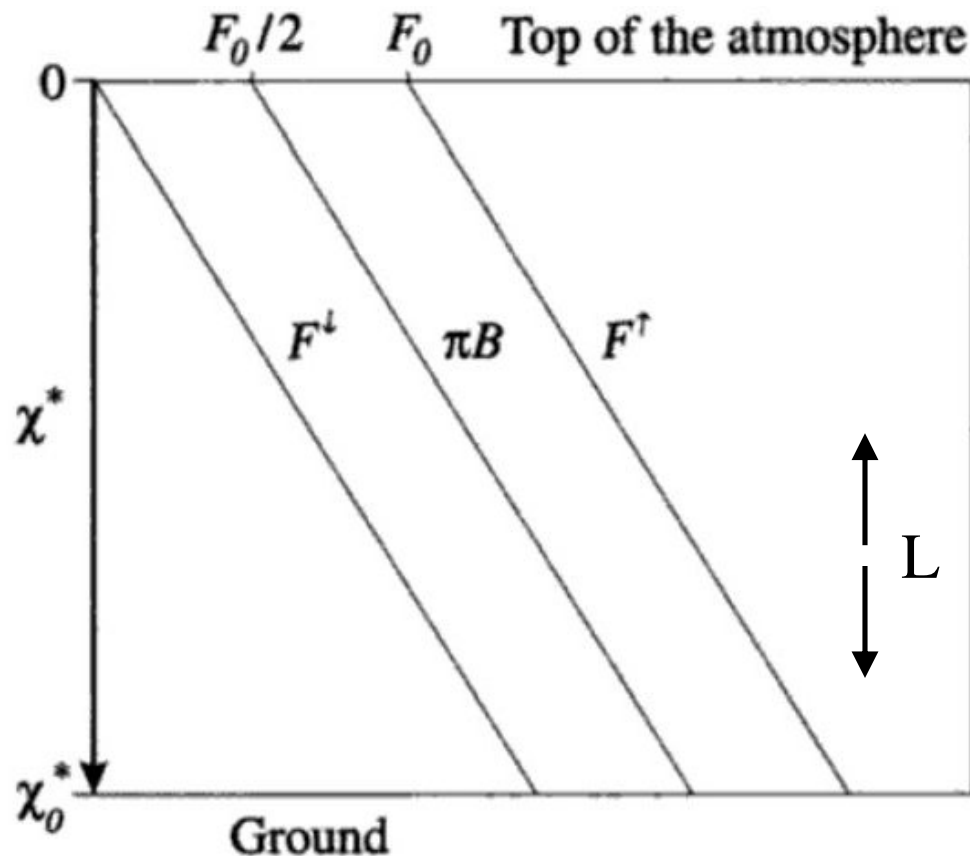
$$B^*(\tau_1) = \frac{\sigma\theta_g^4}{\pi} = \frac{-F_s(2 + 3\tau_1/2)}{2\pi}$$

My Eq. (1): $\Delta B = B_{\text{eff}}/2$

The solution, (9.5), although based upon many simplifications, has features that are instructive for planetary atmospheres.

Andrews (2000)

An Introduction to Atmospheric Physics. Cambridge Univ Press



Eq. (1) (clear-sky)

$$\Delta B = B_{\text{eff}}/2$$

My Eq. (2) (all-sky)

$$\Delta B = (B_{\text{eff}} - L)/2$$

Separating atmospheric radiation from longwave cloud effect (L):

$$\text{Eq. (2): } \Delta B_g = (B_{\text{eff}} - L)/2 \quad (\text{surface net, all-sky})$$

Hartmann (1994)

Global Physical Climatology

Earth's atmosphere is not quite like that, but not so far. Let's try!

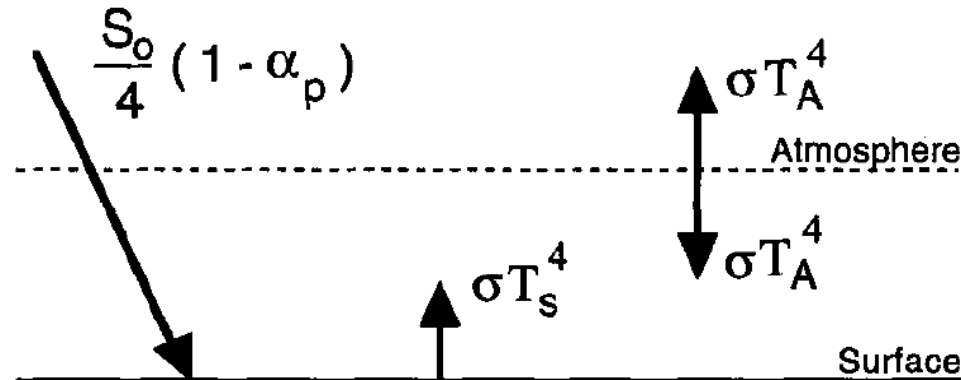


Fig. 2.3 Diagram of the energy fluxes for a planet with an atmosphere that is transparent for solar radiation but opaque to terrestrial radiation.

atmosphere and the surface. The atmospheric energy balance gives

$$\sigma T_s^4 = 2\sigma T_A^4 \Rightarrow \sigma T_s^4 = 2\sigma T_e^4 \quad (2.12)$$

and the surface energy balance is consistent:

$$\frac{S_0}{4} (1 - \alpha_p) + \sigma T_A^4 = \sigma T_s^4 \Rightarrow \sigma T_s^4 = 2\sigma T_e^4 \quad (2.13)$$

Surface gross: $B_g = 2B_{\text{eff}}$; Adding cloud effect: $B_g = 2B_{\text{eff}} + L$

Houghton (2002)

The physics of Atmospheres, Cambridge Univ Press

2.5 The greenhouse effect

Combining (2.12) and (2.13) we find that for the radiative equilibrium atmosphere:

$$B_g = \frac{\phi}{2\pi}(\chi_0^* + 2) \quad (2.15)$$

where χ_0^* is the optical depth at the bottom of the atmosphere. If $\chi_0^* = 0$, $B_g = \phi/\pi$ and the surface temperature is in equilibrium with the incoming and the outgoing radiation, which are both equal to ϕ . If χ_0^* is large, the surface temperature represented by the black-body function B_g will be very considerably enhanced, an illustration of the *greenhouse effect* mentioned in §1.2. It will be considered in more detail in chapter 14.

With optical depth $\chi_0^* = 2$,

My Eq. (3) Surface gross radiation, clear-sky: $\pi B_g = 2\Phi$

My Eq. (4) Adding cloud effect, all-sky: $\pi B_g = 2\Phi + L$

Let be my four equations

Eq. (1) Schwarzschild (1906, Eq. 11), net, clear-sky

$$\mathbf{A - E = \Delta A = A_0 / 2}$$

Eq. (2) Schwarzschild (1906, Eq. 11), incl LWCRE, net, all-sky

$$\mathbf{A - E = \Delta A = (A_0 - L) / 2}$$

Eq. (3) Schwarzschild (1906, Eq. 11), at $\tau = 2$, gross, clear-sky

$$\mathbf{A = 2A_0}$$

Eq. (4) Schwarzschild (1906, Eq. 11), at $\tau = 2$, incl LWCRE, gross, all-sky

$$\mathbf{A = 2A_0 + L}$$

My four equations

Eq. (1): Houghton Eq. (2.13)

Eq. (2): Houghton Eq. (2.13) incl LWCRE

Eq. (3): Houghton Eq. (2.15) at $\chi^*_0 = 2$

Eq. (4): Houghton Eq. (2.15) at $\chi^*_0 = 2$, incl LWCRE

Eq. (1) Surface net, clear-sky: $\Delta B_g = B_g - B_0 = B_{\text{eff}}/2$

Eq. (2) Surface net, all-sky: $\Delta B_g = B_g - B_0 = (B_{\text{eff}} - L)/2$

Eq. (3) Surface gross, clear-sky: $B_g = 2B_{\text{eff}}$

Eq. (4) Surface gross, all-sky: $B_g = 2B_{\text{eff}} + L$

My four equations

Eq. (1): Pierrehumbert (2010, Eq. 4.44, 4.45)

Eq. (2): Pierrehumbert (2010, Eq. 4.44, 4.45) incl LWCRE

Eq. (3): Pierrehumbert (2010, Eq. 4.44) at $\tau = 2$

Eq. (4): Pierrehumbert (2010, Eq. 4.44) at $\tau = 2$ incl LWCRE

Eq. (1) Surface net, clear-sky: $\sigma(T_g^4 - T_0^4) = \text{OLR} / 2$

Eq. (2) Surface net, all-sky: $\sigma(T_g^4 - T_0^4) = (\text{OLR} - \text{LWCRE}) / 2$

Eq. (3) Surface gross, clear-sky: $\sigma T_g^4 = 2\text{OLR}$

Eq. (4) Surface gross, all-sky: $\sigma T_g^4 = 2\text{OLR} + \text{LWCRE}$

Solution to the four equations

Eq. (1) Surface net, clear-sky: $\Delta B_g = B_g - B_0 = B_{\text{eff}}/2$

Eq. (2) Surface net, all-sky: $\Delta B_g = B_g - B_0 = (B_{\text{eff}} - L)/2$

Eq. (3) Surface gross, clear-sky: $B_g = 2B_{\text{eff}}$

Eq. (4) Surface gross, all-sky: $B_g = 2B_{\text{eff}} + L$

Solution:

Clear-sky: $\Delta B_g = \mathbf{5}$, $B_{\text{eff}} = \mathbf{10}$, $B_0 = \mathbf{15}$, $B_g = \mathbf{20}$; $G = B_0 - B_{\text{eff}} = \mathbf{5}$

All-sky: $\Delta B_g = \mathbf{4}$, $B_{\text{eff}} = \mathbf{9}$, $B_0 = \mathbf{15}$, $B_g = \mathbf{19}$; $G = B_0 - B_{\text{eff}} = \mathbf{6}$; $L = \mathbf{1}$

Further (clear-sky): $B_{\text{skin}} = B_0/2 = \mathbf{7.5} \Rightarrow \text{WIN} = B_{\text{eff}} - B_{\text{skin}} = \mathbf{2.5} \Rightarrow$

$B_g : B_0 : B_{\text{eff}} : B_{\text{skin}} : G : \text{WIN} : \text{LWCRE} =$

$\mathbf{20 : 15 : 10 : 7.5 : 5 : 2.5 : 1}$ (related to the spherical surface)

$\mathbf{80 : 60 : 40 : 30 : 20 : 10 : 4}$ (related to the intercepting disk)

Clear-sky: Costa-Shine (2012)

Name	CS12 (Wm ⁻²)	Round (Wm ⁻²)	Diff (Wm ⁻²)	Clear-Sky Units	All-Sky Units	Solar Units 1 = TSI / 51	N × UNIT (Wm ⁻²)	CERES (Wm ⁻²)
WIN	65	65	0	1	2.5	10 / 4	66.7	
G	127	130	3	2	5	20 / 4	133.4	132.4
ATM	194	195	1	3	7.5	30 / 4	200.1	
OLR	259	260	1	4	10	40 / 4	266.8	266.0
ULW	386	390	4	6	15	60 / 4	400.2	398.4
2OLR	518	520	2	8	20	80 / 4	533.6	532.0

$$B_g : B_0 : \text{OLR} : \text{ATM} : G : \text{WIN} = 2 : 3/2 : 1 : 3/4 : 1/2 : 1/4.$$

Equivalent to:

$$B_g : B_0 : \text{OLR} : \text{ATM} : G (= \text{SFC SW+LW Net}) : \text{WIN} = 80 : 60 : 40 : 30 : 20 : 10$$

Extended to TSI (based on the observation that both internal and external fluxes fit into the system):

$$\text{SFC SW+LW Gross} : \text{ULW} : \text{DLR} : \text{OLR} : \text{SFC SW Net} : \text{ATM} : G : \text{WIN} : \text{TOA SW Up} : \text{LWCRE}$$

$$= 80 : 60 : 48 : 40 : 32 : 30 : 20 : 10 : 8 : 4 ; \text{ after spherical weighting (divided by 4):}$$

$$= 20 : 15 : 12 : 10 : 8 : 7.5 : 5 : 2.5 : 2 : 1$$

$$\mathbf{1} = 26.68 \text{ Wm}^{-2}; \text{ TSI} = 1360.68 \text{ Wm}^{-2} = \mathbf{51}$$

The four equations + definitions in CERES notation system

- Eq. (1) SFC SW+LW net, clear-sky = OLR/2**
Eq. (2) SFC SW+LW net, all-sky = (OLR – LWCRE)/2
Eq. (3) SFC SW net + LW down, clear = 2OLR
Eq. (4) SFC SW net + LW down, all = 2OLR + LWCRE

- + SFC LW down clear = SFC LW down all – LWCRE
+ TOA LW clear = TOA LW all + LWCRE
+ LWCRE TOA = LWCRE SFC
+ SFC LW up all = SFC LW up clear

Accuracy of the equations and their integer solution

Accuracy in CERES EBAF Ed4.1, 19 years of data Wm^{-2}

Eq. (1) net, clear-sky:	Surface SW net + LW net	=	TOA LW / 2	-2.24
Eq. (2) net, all-sky:	Surface SW net + LW net	=	(TOA LW - LWCRE) / 2	+2.87
Eq. (3) gross, clear-sky:	Surface SW net + LW down	=	2 TOA LW	-2.86
Eq. (4) gross, all-sky:	Surface SW net + LW down	=	2 TOA LW + LWCRE	+2.46

Surface SW net, all-sky	=	6	Surface SW net, clear-sky	=	8
Surface LW net, all-sky	=	-2	Surface LW net, clear-sky	=	-3
Surface LW down, all-sky	=	13	Surface LW down, clear-sky	=	12
Surface LW up, all-sky	=	15	Surface LW up, clear-sky	=	15
TOA LW all-sky	=	9	TOA LW clear-sky	=	10
G greenhouse effect, all-sky	=	6	G greenhouse effect, clear-sky	=	5
LWCRE (surface, TOA)	=	1	SWCRE (surface)	=	-2

Eq. (1) (CERES EBAF 19 yrs)

- SFC SW net clear-sky = 211.75 **8**
- SFC LW down clear-sky = 317.40 **12**
- SFC LW up clear-sky = 398.38 **15**

SFC SW+LW net, clear-sky = 130.76 **5**

TOA LW /2, clear-sky = 133.00 **5**

$\Delta\text{Eq}(1)$
= - 2.24 Wm⁻²

Eq. (2) (CERES EBAF 19 yrs)

• SFC SW net all-sky	= 163.54	6
• SFC LW down all-sky	= 345.12	13
• SFC LW up all-sky	= 398.60	15
• TOA LW, all-sky	= 240.19	9
• LWCRE	= 25.82	1
SFC SW+LW net, all-sky	= 110.06	4
(TOA LW – LWCRE)/2	= 107.19	4
$\Delta E_{eq}(2)$	= 2.87	Wm^{-2}

Eq. (3) (CERES EBAF 19 yrs)

- SFC SW net clear-sky = 211.75 **8**
- SFC LW down clear-sky = 317.41 **12**

SFC SW net + LW down = 529.16 **20**

2TOA LW, clear-sky = 532.02 **20**

$\Delta E_q(3)$
= - 2.86 Wm⁻²

Eq. (4) (CERES EBAF 19 yrs)

• SFC SW net all-sky	= 163.54	6
• SFC LW down all-sky	= 345.12	13
• TOA LW, all-sky	= 240.19	9
• LWCRE	= 25.82	1

SFC SW net +LW down, all = 508.66 **19**

2TOA LW + LWCRE = 506.20 **19**

$\Delta E_{eq(4)}$ = 2.46 Wm^{-2}

$$\text{Eq.(1)} \quad \mathbf{8} + (\mathbf{12} - \mathbf{15}) = \mathbf{10/2}$$

- $211.75 = \mathbf{8} \times 26.68 - 1.69 \text{ Wm}^{-2}$
- $317.40 = \mathbf{12} \times 26.68 - 2.76 \text{ Wm}^{-2}$
- $398.38 = \mathbf{15} \times 26.68 - 1.82 \text{ Wm}^{-2}$

$$\mathbf{130.76} = \mathbf{5} \times 26.68 - 2.64 \text{ Wm}^{-2}$$

$$\mathbf{133.00} = \mathbf{5} \times 26.68 - 0.4 \text{ Wm}^{-2}$$

$$\Delta \text{Eq(1)} = -2.24 \text{ Wm}^{-2}$$

$$\text{Eq. (2)} \quad \mathbf{6} + (\mathbf{13} - \mathbf{15}) = (\mathbf{9} - \mathbf{1})/2$$

$$\bullet \quad 163.54 = \mathbf{6} \times 26.68 + 3.46 \text{ Wm}^{-2}$$

$$\bullet \quad 345.12 = \mathbf{13} \times 26.68 - 1.72 \text{ Wm}^{-2}$$

$$\bullet \quad 398.60 = \mathbf{15} \times 26.68 - 1.60 \text{ Wm}^{-2}$$

$$\bullet \quad 240.19 = \mathbf{9} \times 26.68 + 0.07 \text{ Wm}^{-2}$$

$$\bullet \quad 25.82 = \mathbf{1} \times 26.68 - 0.86 \text{ Wm}^{-2}$$

$$\mathbf{110.06} = \mathbf{4} \times 26.68 + \mathbf{3.34 \text{ Wm}^{-2}}$$

$$\mathbf{107.19} = \mathbf{4} \times 26.68 + \mathbf{0.47 \text{ Wm}^{-2}}$$

$$\Delta \text{Eq(2)} = \mathbf{2.87 \text{ Wm}^{-2}}$$

$$\text{Eq. (3)} \quad \mathbf{8} + \mathbf{12} = 2 \times \mathbf{10}$$

- $211.75 = \mathbf{8} \times 26.68 - 1.69 \text{ Wm}^{-2}$

- $317.41 = \mathbf{12} \times 26.68 - 2.75 \text{ Wm}^{-2}$

$$529.16 = \mathbf{20} \times 26.68 - 4.44 \text{ Wm}^{-2}$$

$$532.02 = \mathbf{20} \times 26.68 - 1.58 \text{ Wm}^{-2}$$

$$\Delta \text{Eq(3)} = -\mathbf{2.86 \text{ Wm}^{-2}}$$

$$\text{Eq. (4)} \quad \mathbf{6} + \mathbf{13} = 2 \times \mathbf{9} + \mathbf{1}$$

- $163.54 = \mathbf{6} \times 26.68 + 3.46 \text{ Wm}^{-2}$
- $345.12 = \mathbf{13} \times 26.68 - 1.72 \text{ Wm}^{-2}$
- $240.19 = \mathbf{9} \times 26.68 + 0.07 \text{ Wm}^{-2}$
- $25.82 = \mathbf{1} \times 26.68 - 0.86 \text{ Wm}^{-2}$

$$508.66 = \mathbf{19} \times 26.68 + 1.74 \text{ Wm}^{-2}$$

$$506.20 = \mathbf{19} \times 26.68 - 0.72 \text{ Wm}^{-2}$$

$$\Delta \text{Eq(4)} = \mathbf{2.46 \text{ Wm}^{-2}}$$

Accuracy of the equations

- The „gross” Eq. (3) and Eq. (4), contrary to the model differences, have the same accuracy of $< 3 \text{ Wm}^{-2}$ as the evident net equations.
- The Earth system seems to be able to ‘close the window’ and maintain an „effectively LW-opaque” atmosphere, with a prescribed global mean optical depth of $\tau = 2$.
- This is one of the most interesting results of our study.
- How? By LWCRE. — Why? Good question!
- Follow the simplest geometry! See in the Extras.

Ramanathan (1998, 2006)

- As we can see, the integer solution to the theoretical transfer equations, with LWCRE = **1**, prescribes $OLR(all) = \mathbf{9}$, $OLR(clear) = \mathbf{10}$, and $ULW = \mathbf{15}$.
- This means for the greenhouse effect $G(all) = ULW - OLR(all) = \mathbf{15} - \mathbf{9} = \mathbf{6}$ and $G(clear) = \mathbf{15} - \mathbf{10} = \mathbf{5}$.
- The normalized greenhouse factors are $g(theory, all) = \mathbf{6/15} = \mathbf{0.4}$ and $g(theory, clear) = \mathbf{5/15} = \mathbf{1/3}$.
- Ramanathan (1998) in his Volvo Prize Lecture gives the following description: “At a global average surface temperature of about 289 K, the globally averaged longwave emission by the surface is about $395 \pm 5 \text{ Wm}^{-2}$, whereas the OLR is only $237 \pm 3 \text{ Wm}^{-2}$. Thus, the intervening atmosphere and clouds cause a reduction of $158 \pm 7 \text{ Wm}^{-2}$ in the longwave emission to space, which is the magnitude of the total greenhouse effect (G)”. In that case $g(all) = 158/395 = \mathbf{0.4}$.
- Ramanathan and Inamdar (2006) found for clear-sky: “the normalized g_a is 0.33, i.e., the atmosphere reduces the energy escaping to space by 131 Wm^{-2} (or by a factor of 1/3).” Yes, $g(theory, clear) = \mathbf{1/3}$.

The Greenhouse Effect: Theory and Observation (CERES EBAF Ed4.1, 12 mo)

217	406.69	268.74	137.95	0.3392	407.47	243.29	164.18	0.402925
218	408.34	269.87	138.47	0.3391	408.66	244.31	164.35	0.402168
219	407.39	269.3	138.09	0.33896	407.8	243.9	163.9	0.401913
220	403.98	267.77	136.21	0.33717	404.46	242.74	161.72	0.399842
221	399.63	265.56	134.07	0.33549	400.14	240.21	159.93	0.399685
222	393.57	263.56	130.01	0.33034	393.8	237.71	156.09	0.396369
223	391.11	263.08	128.03	0.32735	391.1	237.04	154.06	0.393915
224	390.24	263.34	126.9	0.32518	389.92	237.46	152.46	0.391003
225	392.12	263.67	128.45	0.32758	391.56	238.29	153.27	0.391434
226	396.27	264.54	131.73	0.33242	395.85	238.86	156.99	0.39659
227	399.87	265.53	134.34	0.33596	400.31	239.43	160.88	0.401889
228	403.78	266.9	136.88	0.339	404.84	241.25	163.59	0.404086
Observed								
	399.42	265.99	133.43	0.3340	399.66	240.37	159.29	0.3985
	1360.68	400.20	266.80	133.40	400.20	240.12	160.08	0.4
Theory								
	51	15	10	5	15	9	6	2/5
	TSI	ULW_clr	OLR_clr	G_clr	g_clr	ULW_all	OLR_all	G_all
								g_all

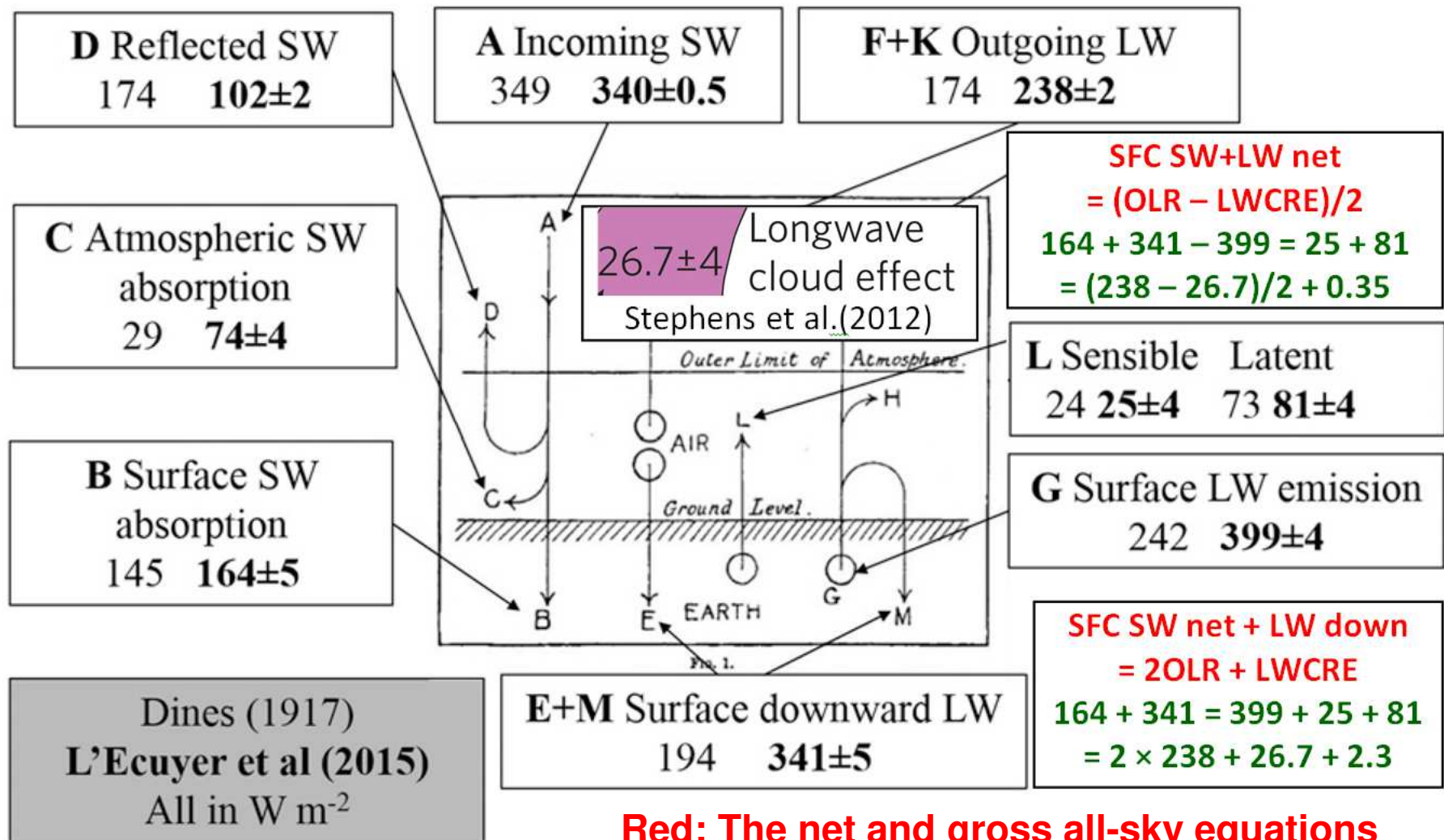
Sits exactly in its theoretically prescribed equilibrium position;
does not seem to show any deviation or enhancement.

Radiative forcing vs. Radiative equilibrium?

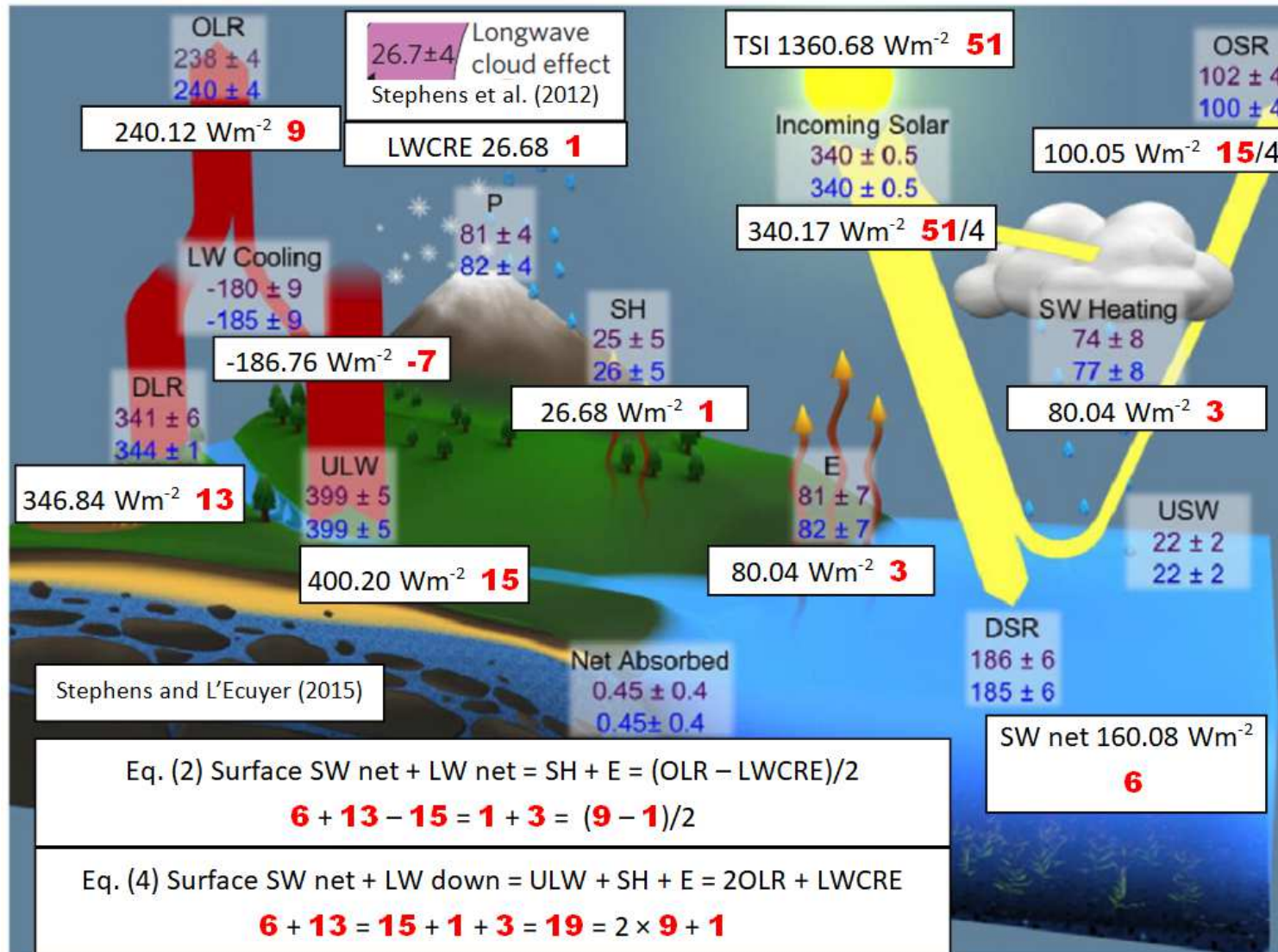
- Manabe and Strickler (1964), Manabe and Wetherald (1967, 1975, 1980), and their follow-ups (Ramanathan and Coakley 1978, Ramanathan et al. 1979) did not regard the surface net radiation (the size of the convective adjustment) theoretically constrained, and never equated to $OLR/2$.
- The Charney Report (1979) does not make any attempt to utilize these constraints. Their result, 3 ± 1.5 °C, equivalent to 16.7 ± 8.4 Wm^{-2} , falls far out any observed range of uncertainty of ± 0.5 °C (± 3 Wm^{-2}) of the examined relationships.
- The IPCC AR5 (2013) WGI report Chapter 2 mentions surface net radiation several times, but never declares its definite theoretical connection to the TOA fluxes.
- Ramaswamy et al. (2019), in their assessment of „The historical evaluation of the radiative forcing” concept, refer to the estimate of L’Ecuyer et al. (2015); but the all-sky net and gross equations (Eq. 2 and 4) are satisfied there within 0.35 Wm^{-2} and 2.3 Wm^{-2} , resp.

„Radiative forcing of climate”

Ramaswamy et al., Met Monographs (2019)



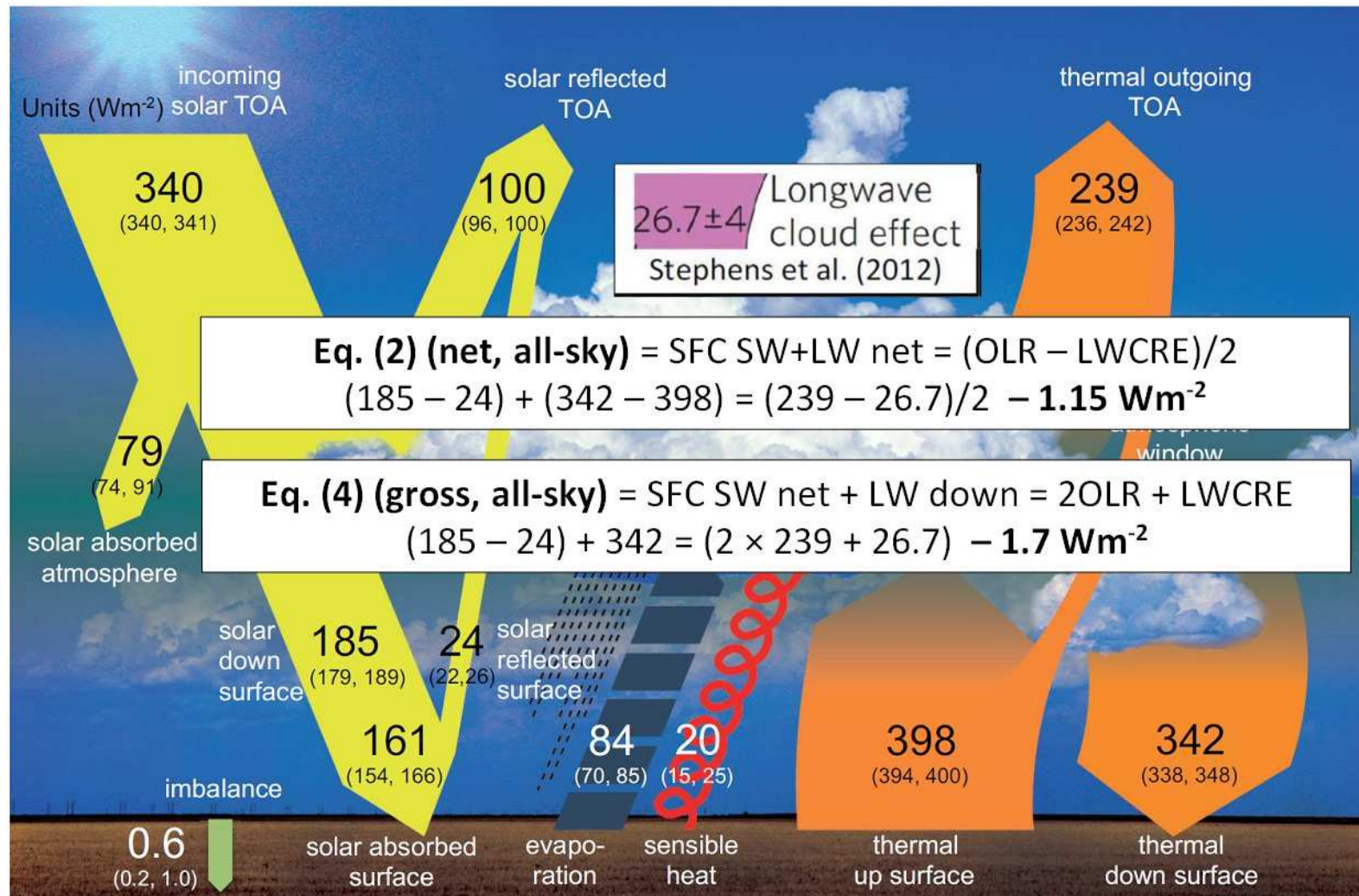
All-sky equations and integer structure in Stephens and L'Ecuyer (2015)



Eq. (2) (net, all-sky) $185 - 22 + 344 - 399 = (240 - 26.7)/2 + 1.35 \text{ Wm}^{-2}$

Eq. (4) (gross, all-sky) $185 - 22 + 344 = (2 \times 240 + 26.7) + 0.30 \text{ Wm}^{-2}$

All-sky equations in IPCC (2013)



The greenhouse effect $g = G/\text{ULW} = (\text{ULW} - \text{OLR})/\text{ULW}$
 g (theory) = $(15 - 9) / 15 = 0.4$; g (obs) = $(398 - 239) / 398 = 0.3995$

The differences might come from

- Observation uncertainty
- Measurement error
- Natural fluctuation around the N position
- Systematic deviation from the N position
- Theoretical accuracy of the equations
(Eddington two-stream approximation)
- Dynamical transition of the whole system

Conclusions

- Eq. (1) is a standard textbook formula; its validity in observations was an expectation.
- Eq. (2) is its evident all-sky extension.
- Eq. (3) and (4) belong to a specific „single-slab” (SW-transparent, LW-opaque) atmosphere, with $\tau = 2$.
- Their same accuracy as the net equations ($< 3 \text{ Wm}^{-2}$) is a remarkable fact and deserves attention.
- The extreme accuracy at TOA ($< 1 \text{ Wm}^{-2}$) requires further explanation.
- The internal integer structure is a consequence, but the external reference to TSI = **51** with LWCRE = **1** is a novum and points to new directions.

LWCRE = 1, TSI = 51

The complete global mean energy flow system follows from
TSI at 1 AU = $1360.9 \pm 0.5 \text{ Wm}^{-2}$.

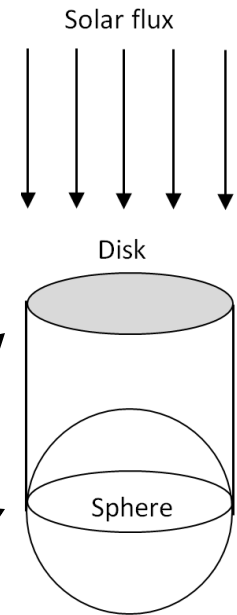
Let me use TSI = $1360.68 \text{ Wm}^{-2} = 51 \Rightarrow 1 = 26.68 \text{ Wm}^{-2}$

Disk, clear-sky: RSR = 8, ASR = 43, OLR = 40, IMB = 3

Disk, all-sky: RSR = 15, ASR = 36, OLR = 36

Each flux value is integer on the cross-section disk

Some quarters appear only after spherical weighting



All-sky: SFC SW+LW net (4) = SH (1) + LH (3). G : OLR : ULW = 6 : 9 : 15 = 2 : 3 : 5

All-sky	B _g	B ₀	DLR	TSI	ASR	B _{eff}	ATM	SAS	G	RSR	WIN	L
Disk	76	60	52	51	36	36	32	24	24	15	4	4
Sphere	19	15	13	12.75	9	9	8	6	6	3.75	1	1
Wm ⁻²	506.92	400.20	346.84	340.17	240.12	240.12	213.44	160.8	160.08	100.05	26.68	26.68

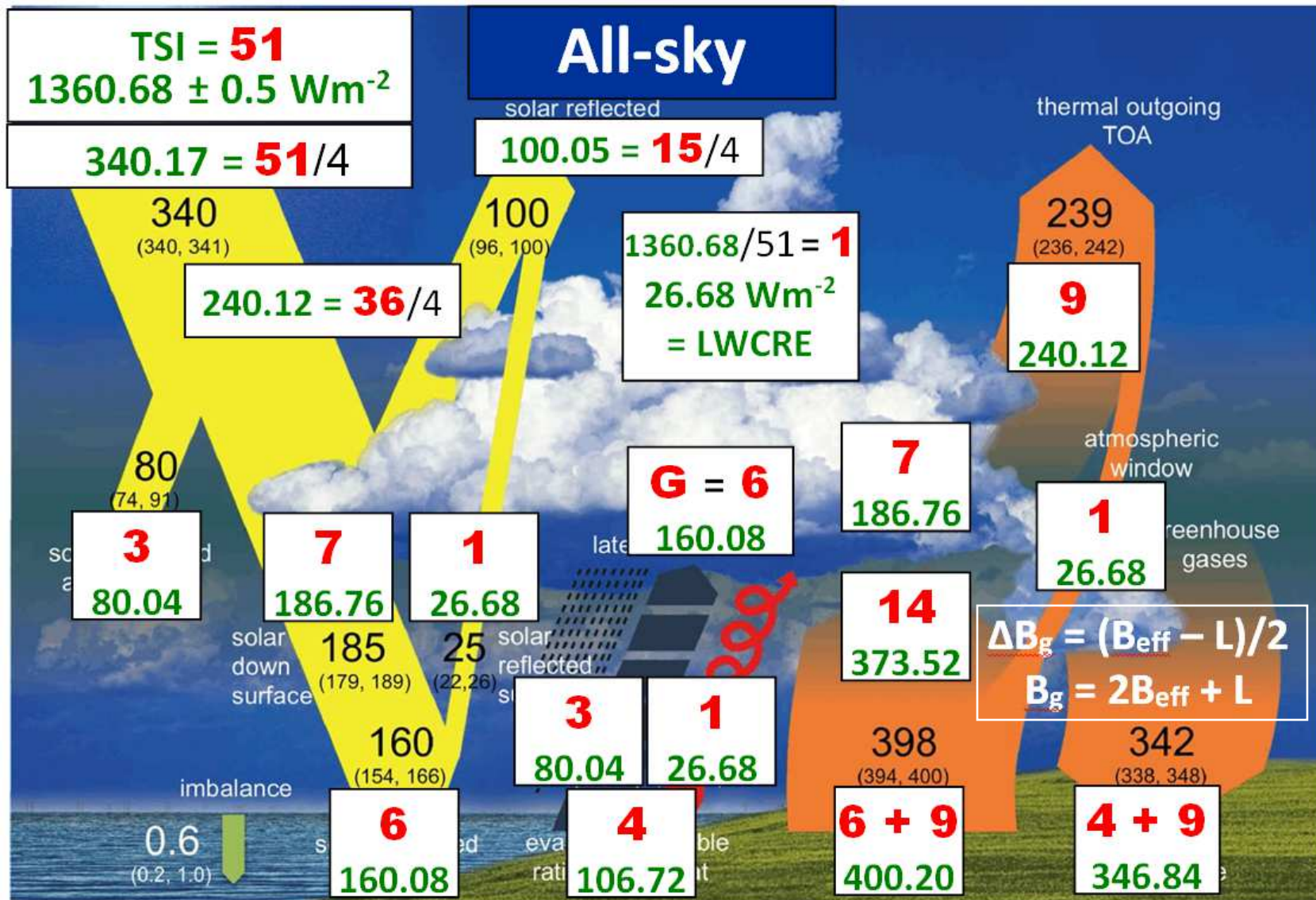
Clear-sky: G : OLR : ULW = 2 : 4 : 6 = 3 : 6 : 9 = 5 : 10 : 15 = 1 : 2 : 3

Clear-sky	B _g	B ₀	TSI	DLR	ASR	B _{eff}	SAS	ATM	G	WIN	RSR	IMB
Disk	80	60	51	48	43	40	32	30	20	10	8	3
Sphere	20	15	12.75	12	10.75	10	8	7.5	5	2.5	2	0.75
Wm ⁻²	533.60	400.20	340.17	320.16	286.81	266.80	213.44	200.10	133.40	66.70	53.36	20.01

All-sky	N integer	N×UNIT	Ed4.1 19yr	N×UNIT – 19yr
Total Solar Irradiance	51	1360.68	1361	–0.32
Unit	1	26.68	26.686	–0.006
TOA SW insolation	51 / 4	340.17	340.02	0.15
TOA LW up	36 / 4	240.12	240.19	–0.07
TOA SW up	15 / 4	100.05	99.07	0.98
SFC SW net	6	160.08	163.54	–3.46
SFC LW down	13	346.84	345.12	1.72
SFC LW up	15	400.20	398.60	1.60
SFC LW net	–2	–53.36	–53.48	0.12
SFC SW+LW net	4	106.72	110.06	–3.34
SFC SW+LW gross	19	506.92	508.66	–1.74
Greenhouse effect G	6	160.08	158.41	1.67
Clear-sky				
TOA SW insolation	51 / 4	340.17	340.02	0.15
TOA LW up	40 / 4	266.80	266.01	0.79
TOA SW up	8 / 4	53.36	53.76	–0.40
TOA net	3 / 4	20.01	20.25	–0.24
SFC SW net	8	213.44	211.75	1.69
SFC LW down	12	320.16	317.40	2.76
SFC LW up	15	400.20	398.38	1.82
SFC LW net	–3	–80.04	–80.98	0.94
SFC SW+LW net	5	133.40	130.77	2.63
SFC SW+LW gross	20	533.60	529.15	4.45

Recommendations for IPCC AR6 WGI Chapter 7

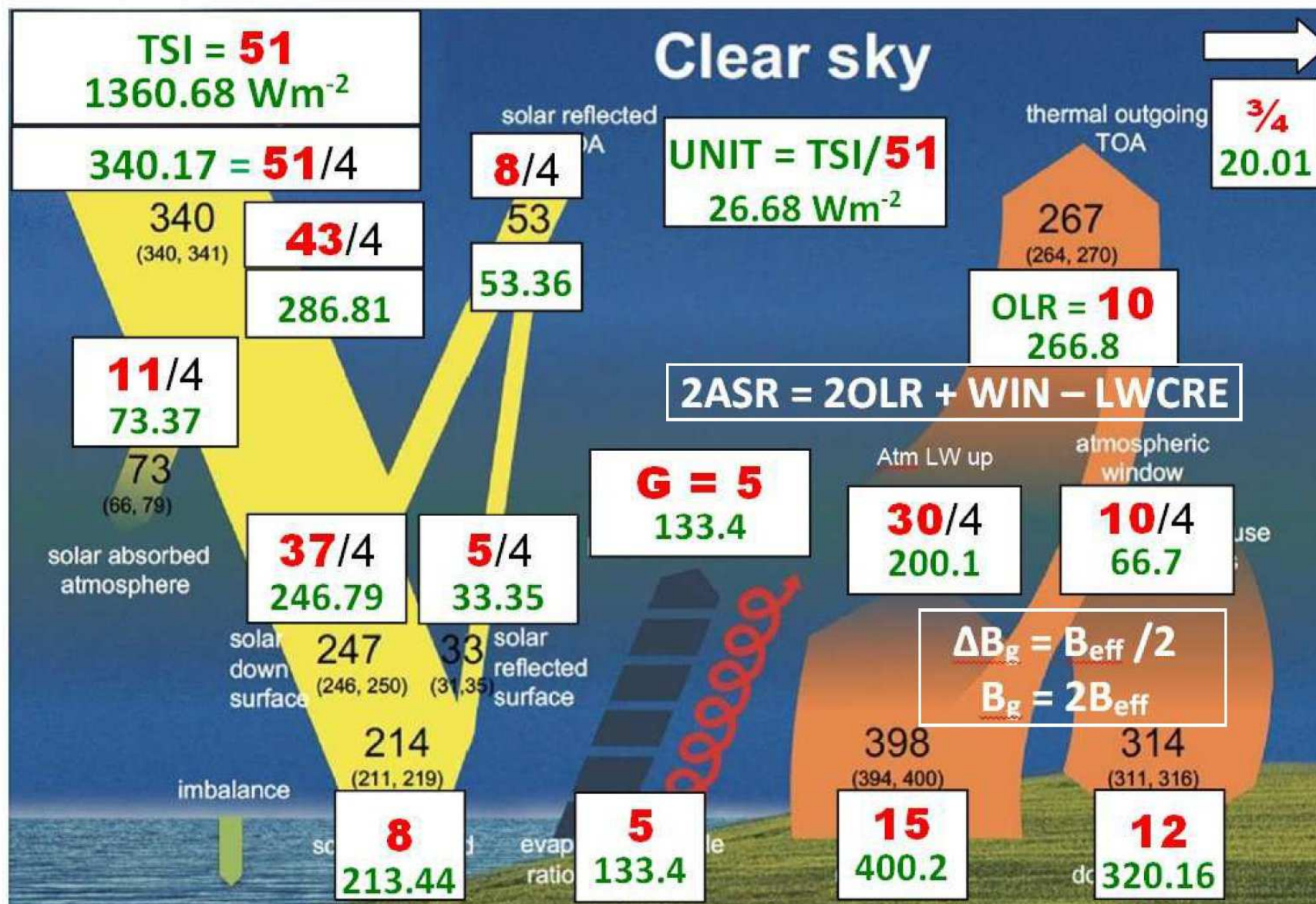
- The Earth's energy budget seems to be determined by theoretical transfer equations, satisfied by the Earth system with high accuracy.
- The all-sky theoretical greenhouse effect is $g_{\text{theory}} = (15 - 9)/15 = 0.4$, its observed value is $g_{\text{obs}} = (398 - 239)/398 = 0.3995$.
- The clear-sky theoretical greenhouse effect is $g_{\text{theory}} = (15 - 10)/15 = 1/3$, its observed value is $g_{\text{obs}} = (398 - 267)/398 = 0.329$.
- The essential information on the recent state of the energy budget is projected on the diagrams:



SFC SW+LW net = (OLR - LWCRE)/2; SFC SW net + LW down = 2OLR + LWCRE

$$6 + (13 - 15) = (9 - 1)/2$$

$$6 + 13 = 2 \times 9 + 1$$

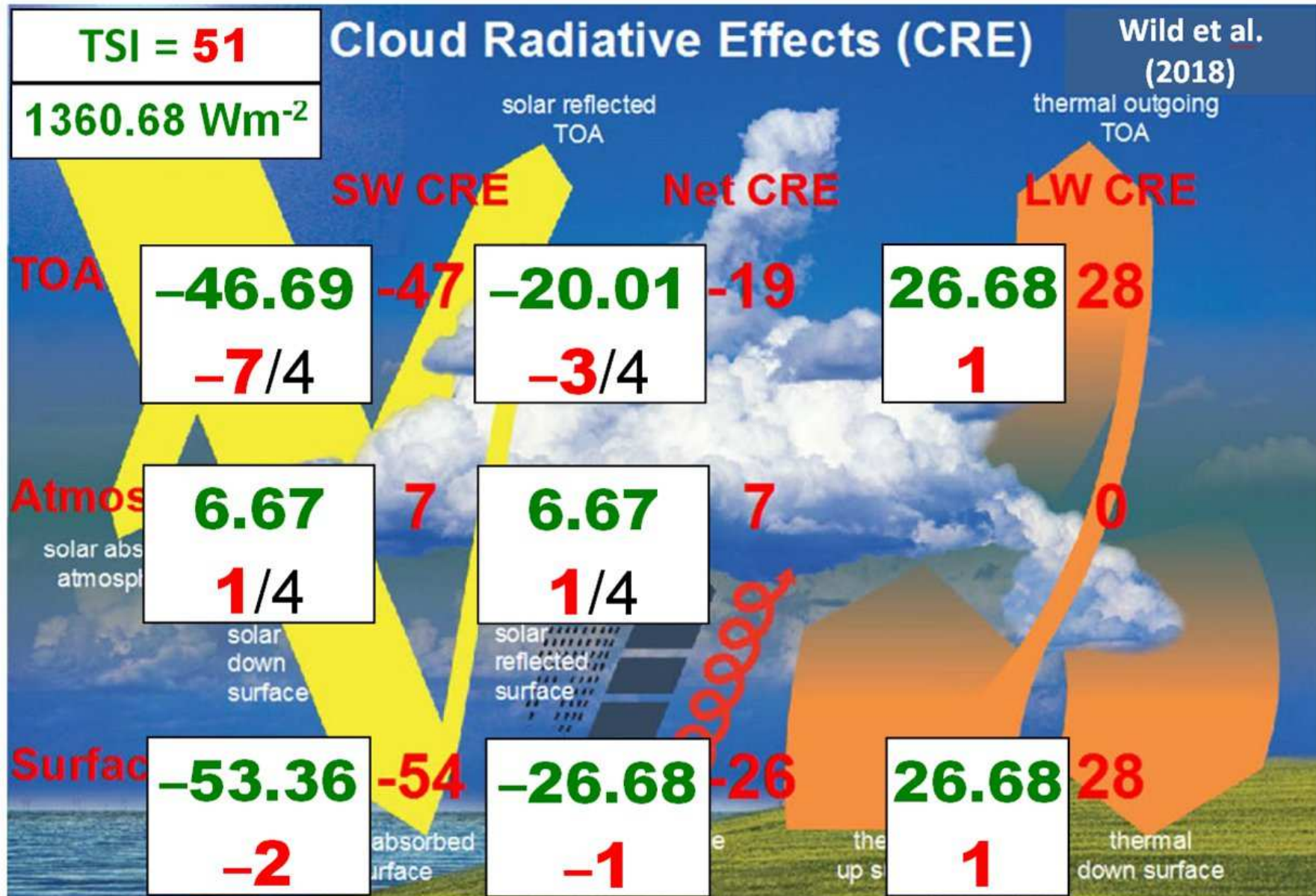


$$\text{SFC SW+LW net} = \text{OLR} / 2;$$

$$8 + (12 - 15) = 10 / 2$$

$$\text{SFC SW net} + \text{LW down} = 2\text{OLR}$$

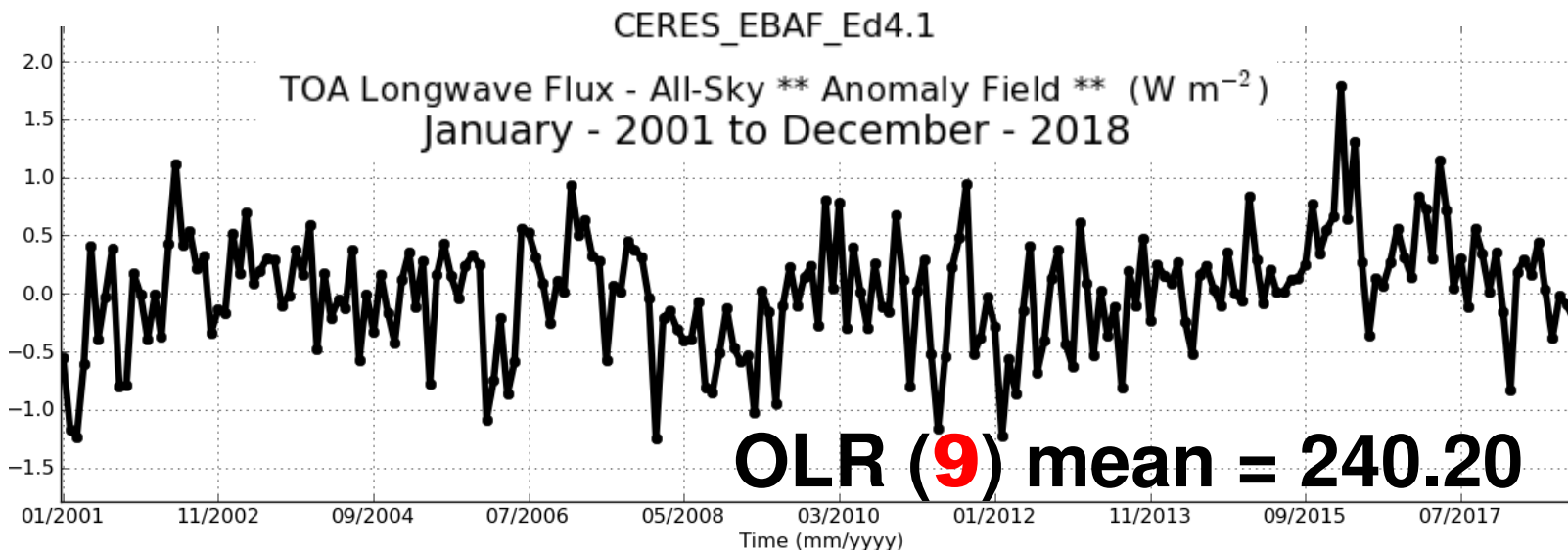
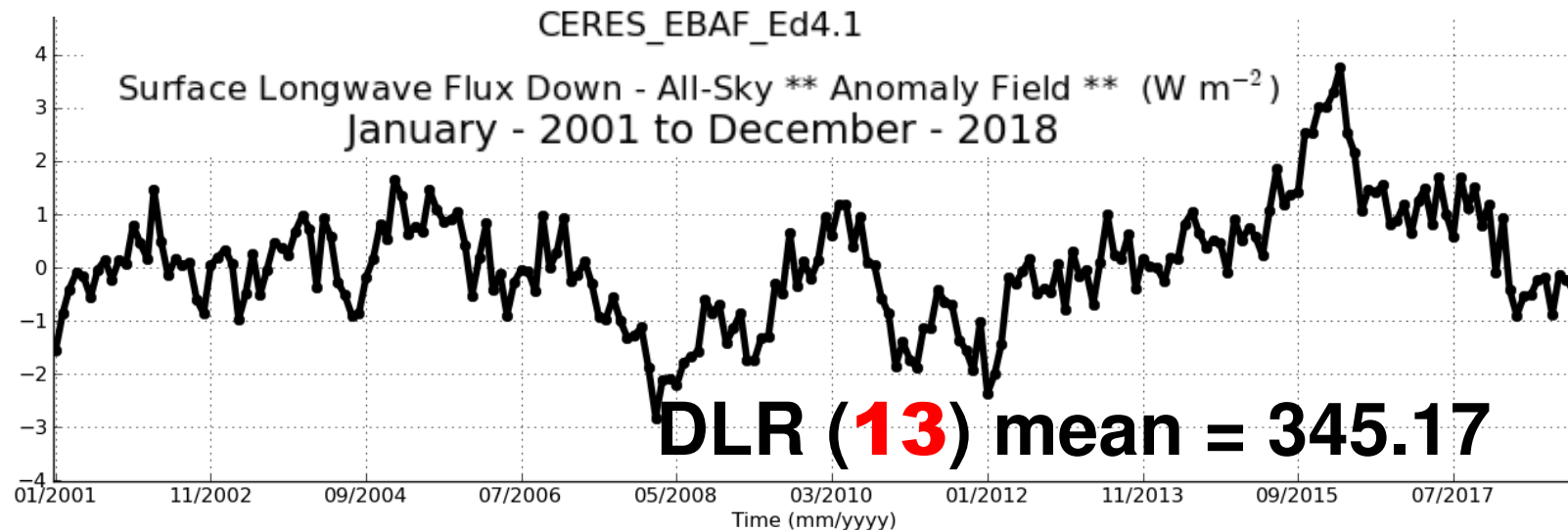
$$8 + 12 = 2 \times 10$$



Summary for this session

- **Earth radiation budget** seems to be more constrained than previously thought.
- **Radiative forcing** is only one half of the description; the other half are relevant stabilizing equilibrium relationships.
- **Climate change** might have other sources (e.g., shortwave perturbations) than the assumed enhanced greenhouse effect.
- **I predict** $\text{DLR}(\text{all}) = (13/9) \times \text{OLR}(\text{all})$.

$$\text{DLR} = (13/9)\text{OLR} - 1.8 \text{ Wm}^{-2}$$

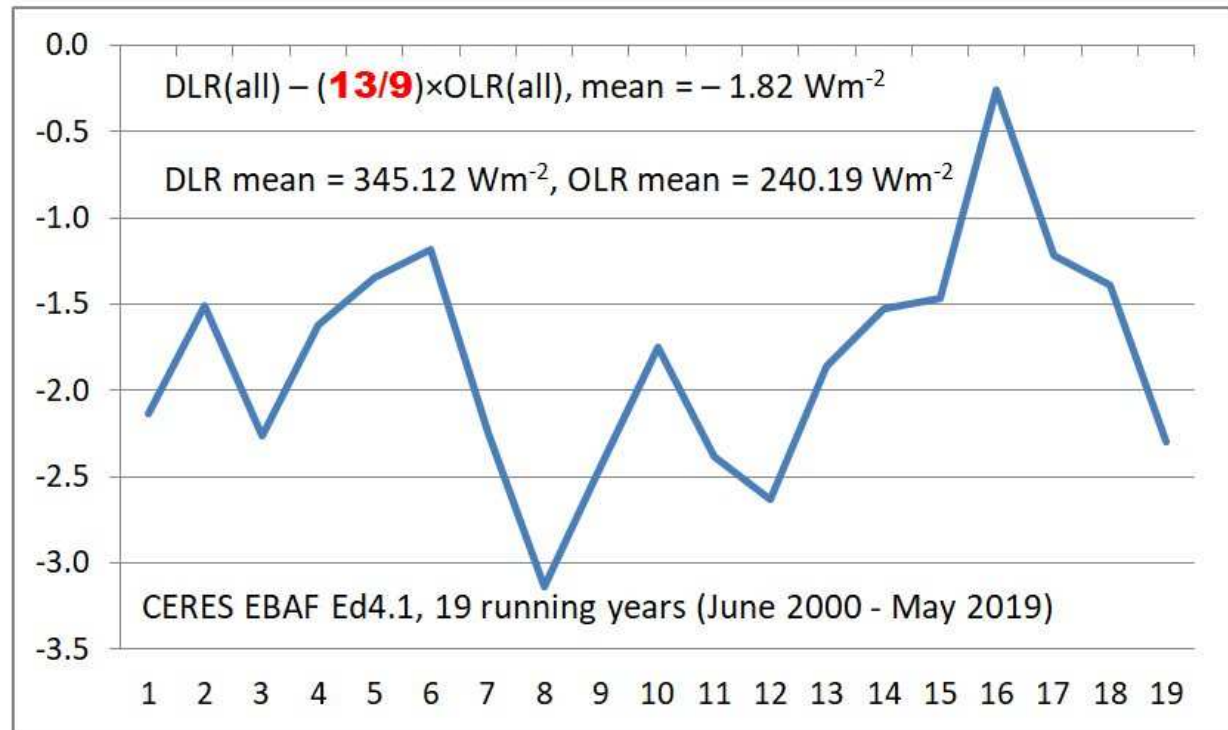


$$\text{TSI} = 1360.9 \pm 0.5 \text{ Wm}^{-2} = 51 \Rightarrow 9 = 240.16 \text{ Wm}^{-2}$$

$$\text{DLR} - (13/9) \text{ OLR} = -1.8 \text{ Wm}^{-2}$$

343.967	239.607	-2.132
345.348	240.133	-1.511
344.977	240.398	-2.265
345.433	240.268	-1.621
345.465	240.101	-1.347
345.526	240.027	-1.179
345.183	240.527	-2.244
343.598	240.049	-3.140
343.834	239.73	-2.443
345.303	240.268	-1.751
344.361	240.051	-2.379
343.96	239.949	-2.633
344.914	240.073	-1.857
345.347	240.143	-1.526
345.703	240.347	-1.464
347.507	240.76	-0.258
346.308	240.595	-1.218
345.692	240.289	-1.393
344.913	240.374	-2.295
345.123	240.194	-1.824

DLR OLR Diff

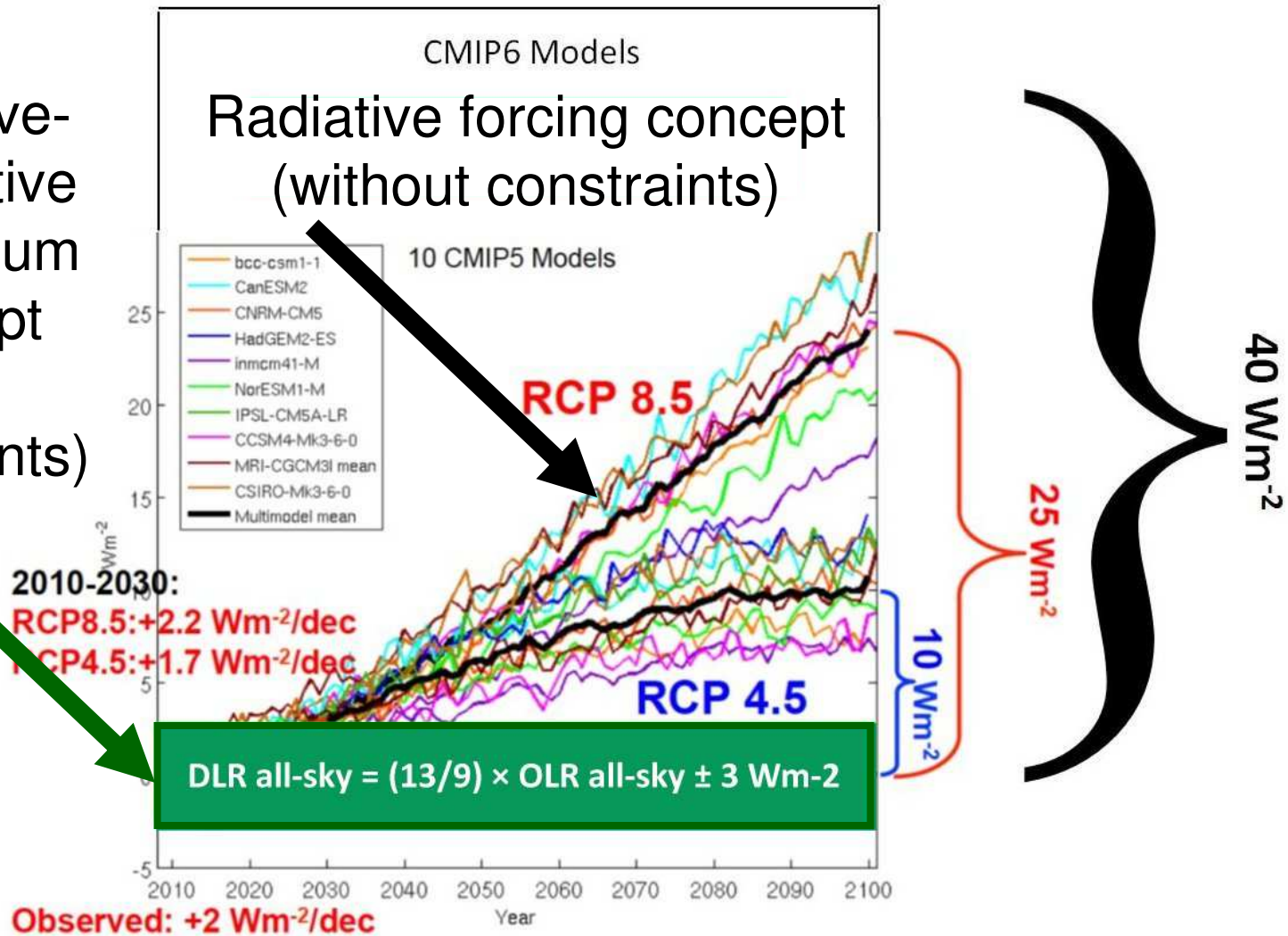


228 months of observations

DLR predictions 21st century

Radiative-convective equilibrium concept (with constraints)

Radiative forcing concept (without constraints)



$$\text{DLR all-sky} = (13/9) \text{ OLR all-sky} \pm 3 \text{ Wm}^{-2}$$

The Message

- These challenging times prove humankind needs the best science in every respect.
- Radiative forcing is not the best science.
- It is only one half of our understanding.
- The other half is equilibrium constraints.
- The equations are robust, proved to be valid in the past two decades.
- I expect them to remain valid in the forthcoming decades as well.
- How these constraints counteract additional CO₂ forcing (by reorganizing cloud / temperature / water vapor distributions?) is to be investigated.

Extras

Marshall and Plumb (2008)

The simplest greenhouse geometry

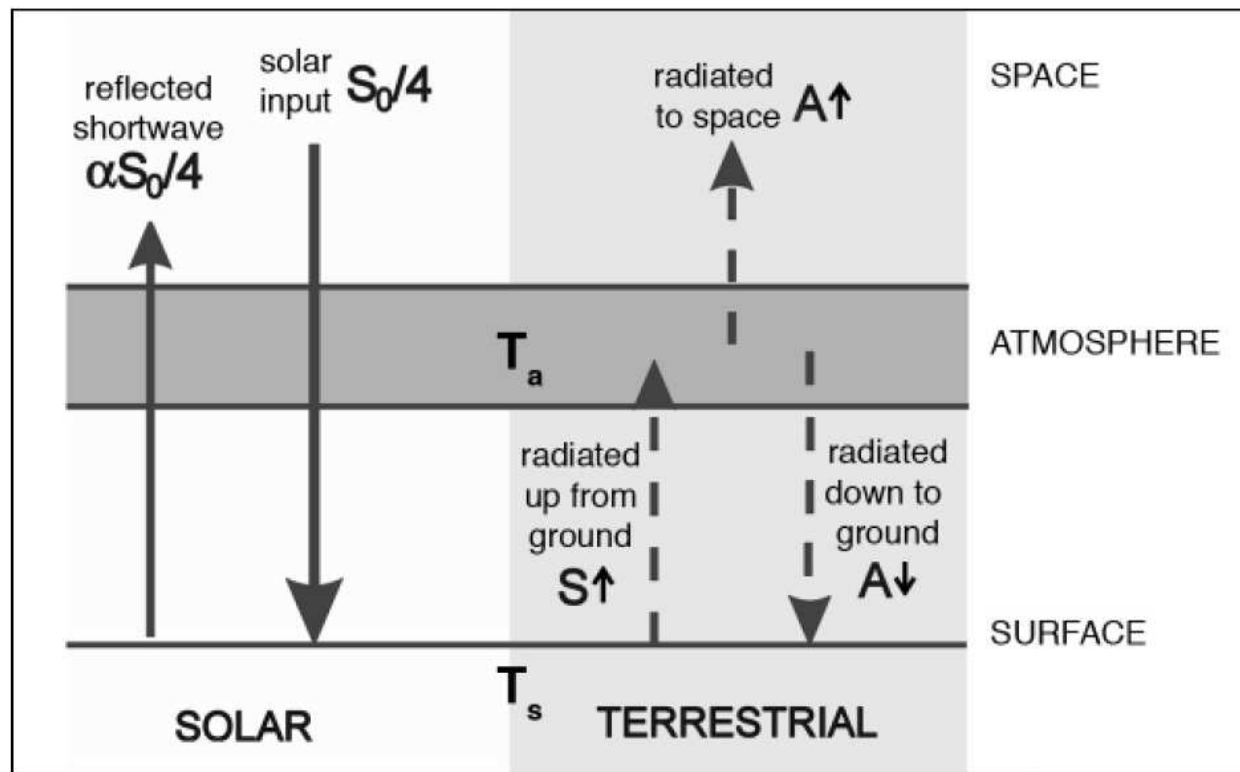


FIGURE 2.7. The simplest greenhouse model, comprising a surface at temperature T_s , and an atmospheric layer at temperature T_a , subject to incoming solar radiation $S_0/4$. The terrestrial radiation upwelling from the ground is assumed to be completely absorbed by the atmospheric layer.

SW-transparent, LW-opaque, non-turbulent

The two essential features of the simplest greenhouse model: $S = 2A = 2F$; $G = A = F$

$$F = A = S_0(1-\alpha)/4$$

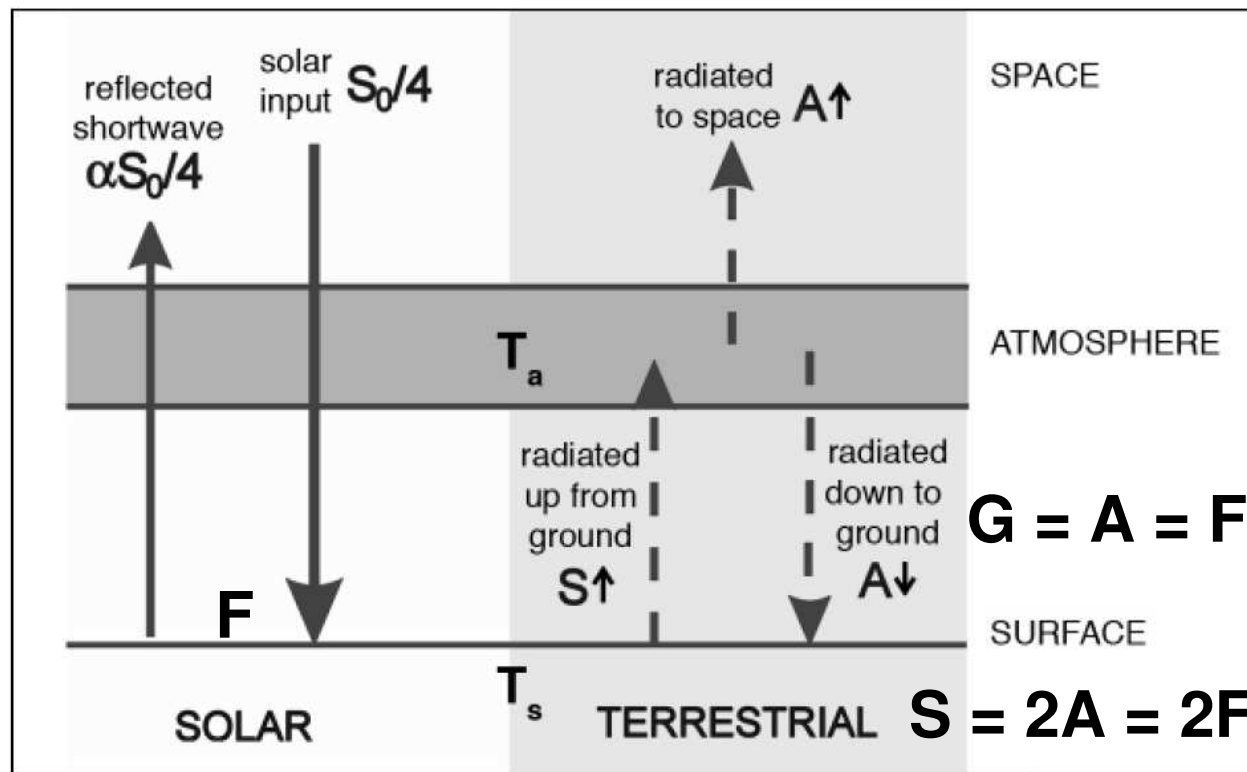


FIGURE 2.7. The simplest greenhouse model, comprising a surface at temperature T_s , and an atmospheric layer at temperature T_a , subject to incoming solar radiation $S_0/4$. The terrestrial radiation upwelling from the ground is assumed to be completely absorbed by the atmospheric layer.

Eq. (3) – (4) (Gross): Single-Slab Geometry (same as in Marshall-Plumb)

380

CLIMATE MODELLING

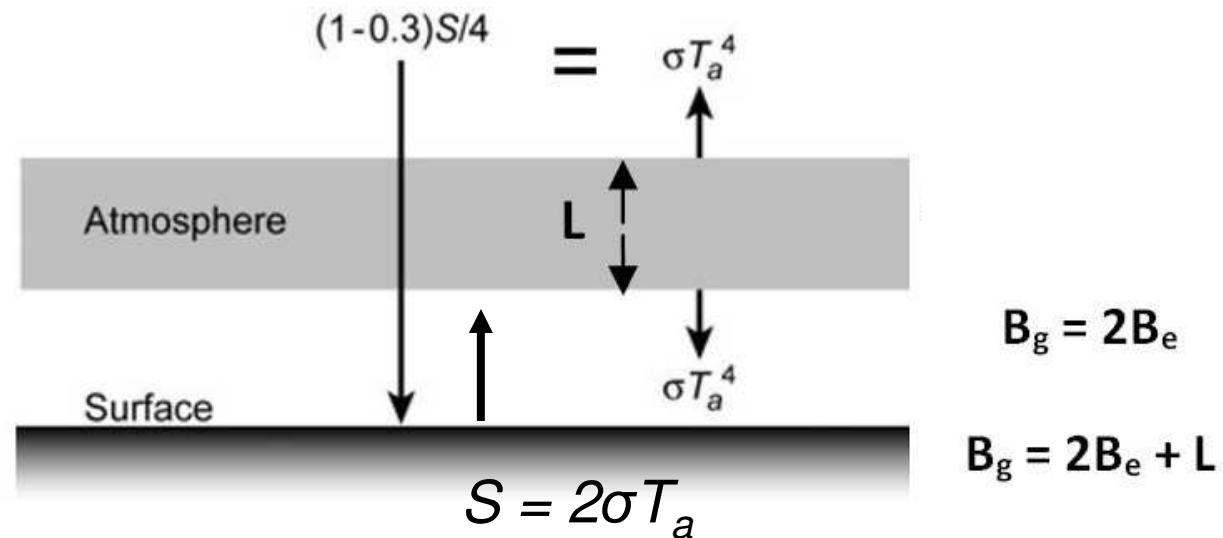


FIG. 11.1. A 'single-slab atmosphere' model of the greenhouse effect, in which the atmosphere is treated as a homogeneous layer of temperature T_a that is perfectly transparent to solar radiation and perfectly opaque in the thermal infra-red. The surface receives the equivalent of two solar constants, raising its mean temperature from 255 to 303 K.

Modified from Vardavas and Taylor (2006)

Same as Hartmann (1994, Fig. 2.3)

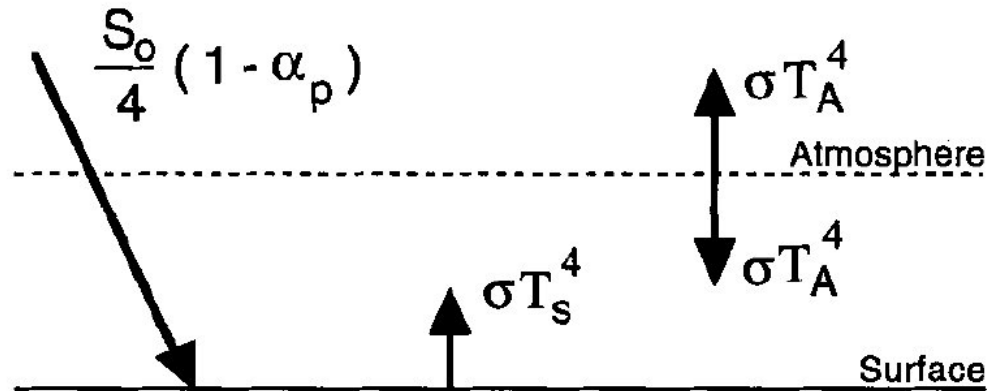


Fig. 2.3 Diagram of the energy fluxes for a planet with an atmosphere that is transparent for solar radiation but opaque to terrestrial radiation.

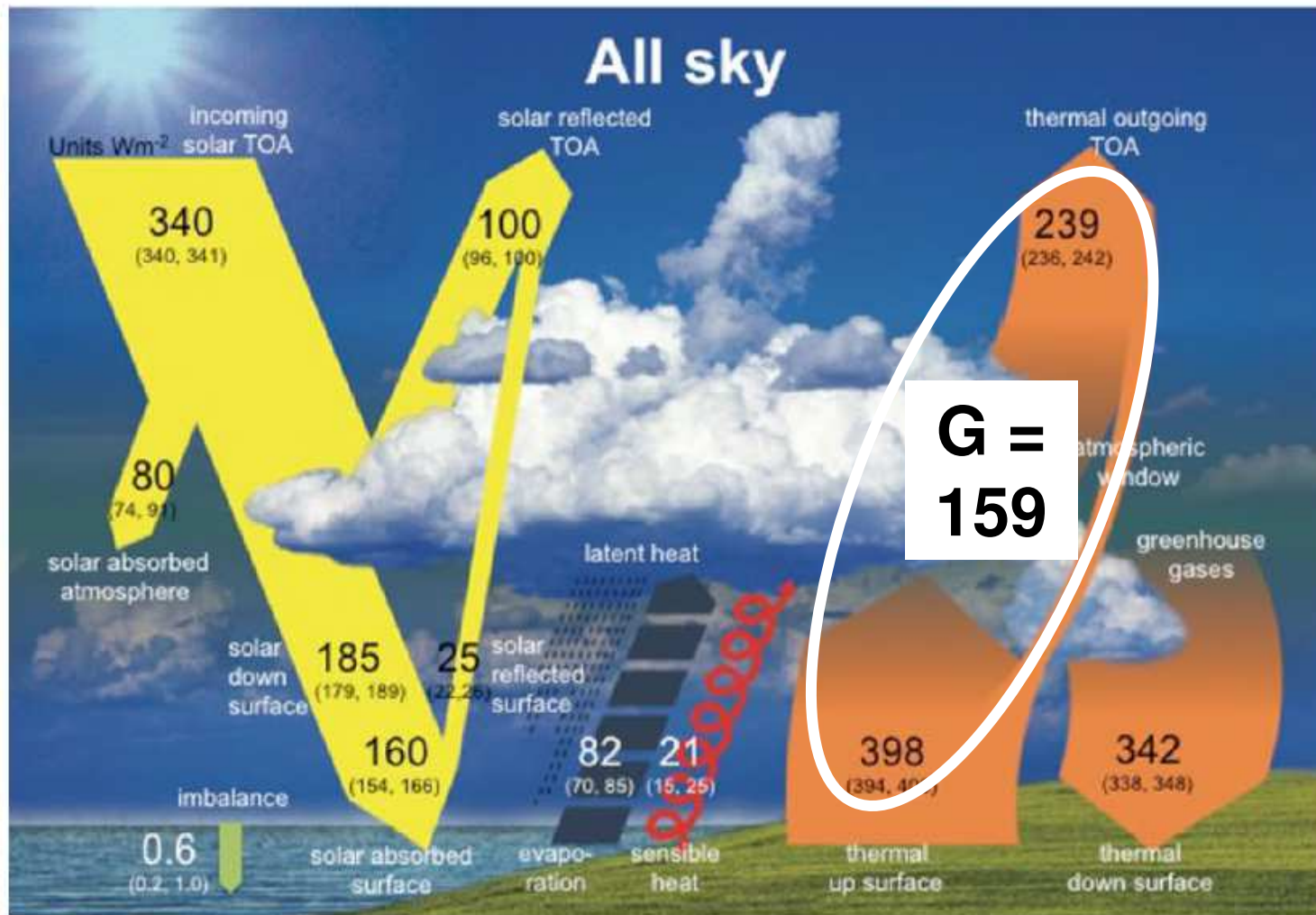
$$\text{Since } \sigma T_S^4 = 2\sigma T_A^4,$$

$$G = \sigma T_S^4 - \sigma T_A^4 = \sigma T_A^4 = S_0(1 - \alpha_p)/4$$

$$= \text{Surface Absorbed Solar}$$

But in our 'quasi LW-opaque' atmosphere,
G = Solar Absorbed Surface works **in the all-sky mean**:

Eq. (5) $G_{\text{all-sky}} = \text{SFC SW net}$



$\text{SFC SW net} = 160, G = 398 - 239 = 159$

G all = SFC SW net, if $\varepsilon = 1$ (Liou 1980)

$$Q(1 - \bar{r}) - \bar{\varepsilon}\sigma T_a^4 - (1 - \bar{\varepsilon})\sigma T^4 = 0, \quad \text{TOA} \quad (8.31)$$

$$Q(1 - \bar{r} - \bar{A}) + \bar{\varepsilon}\sigma T_a^4 - \sigma T^4 = 0, \quad \text{SFC} \quad (8.32)$$

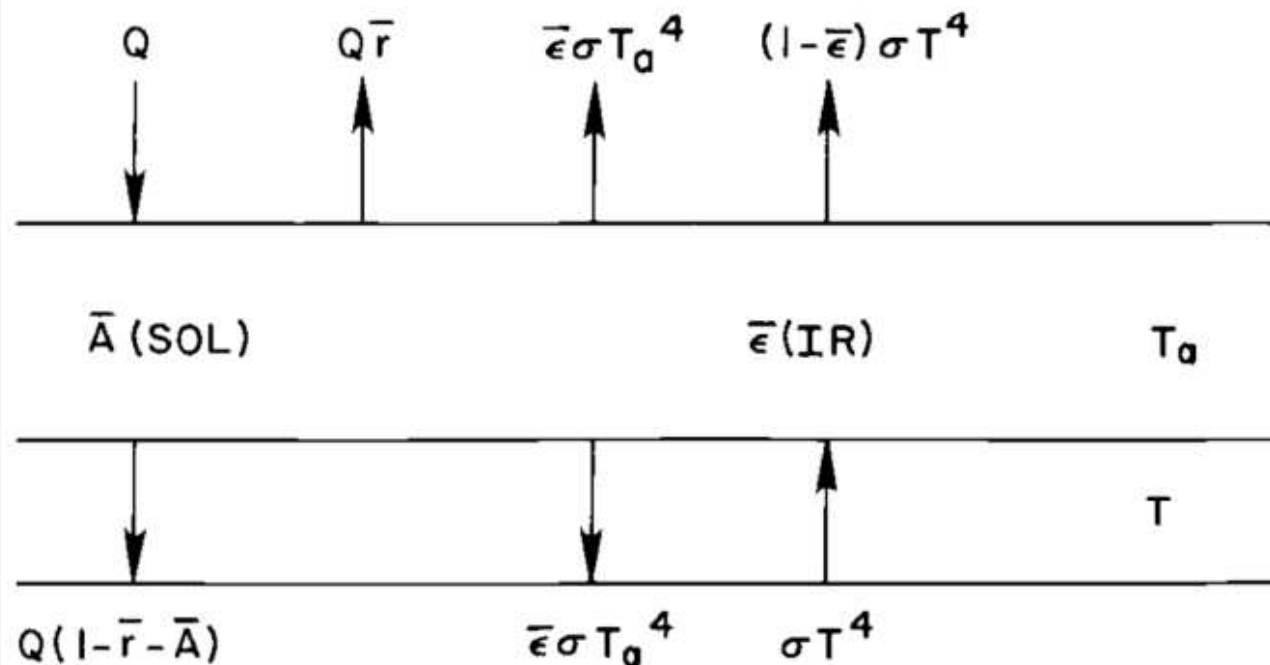


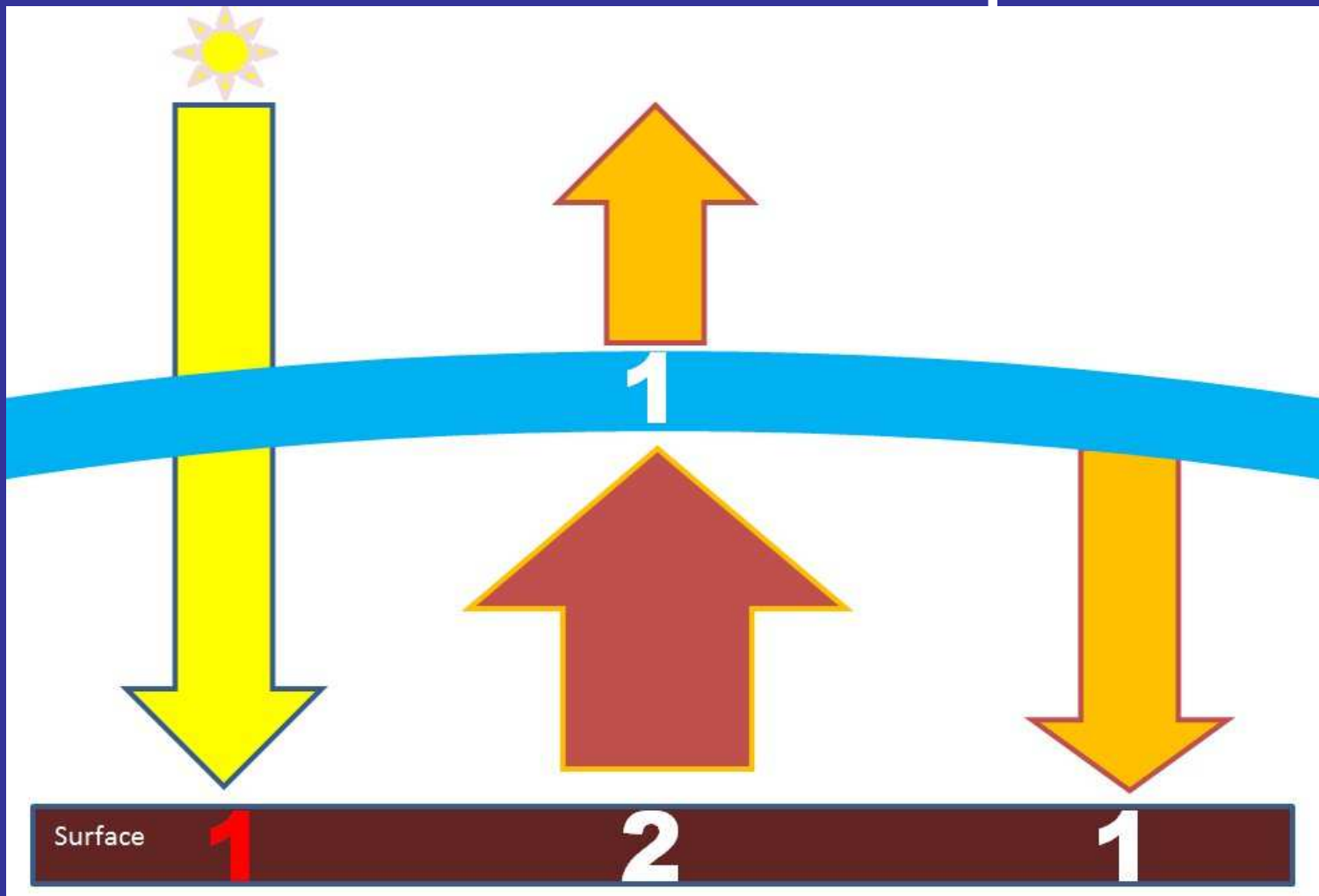
Fig. 8.20 Two-layer global radiative budget model.

The greenhouse effect of the system is $G = \text{SFC LW Up} - \text{TOA LW} = \sigma T^4 - (\bar{\varepsilon}\sigma T_a^4 + (1 - \bar{\varepsilon})\sigma T^4) = Q(1 - \bar{r} - \bar{A}) - (1 - \bar{\varepsilon})\sigma T^4$. In the infrared-opaque limit ($\bar{\varepsilon} = 1$), $G = Q(1 - \bar{r} - \bar{A})$ which is the solar radiation absorbed by the surface; and the equality stands with all SW atmospheric absorption \bar{A} . Notice also that in the case of $\bar{A} = 0$ and $\bar{\varepsilon} = 1$, it follows that $2\sigma T_a^4 = \sigma T^4$.

Eq. (5) $G_{\text{all-sky}} = \text{SFC SW net} \Rightarrow$

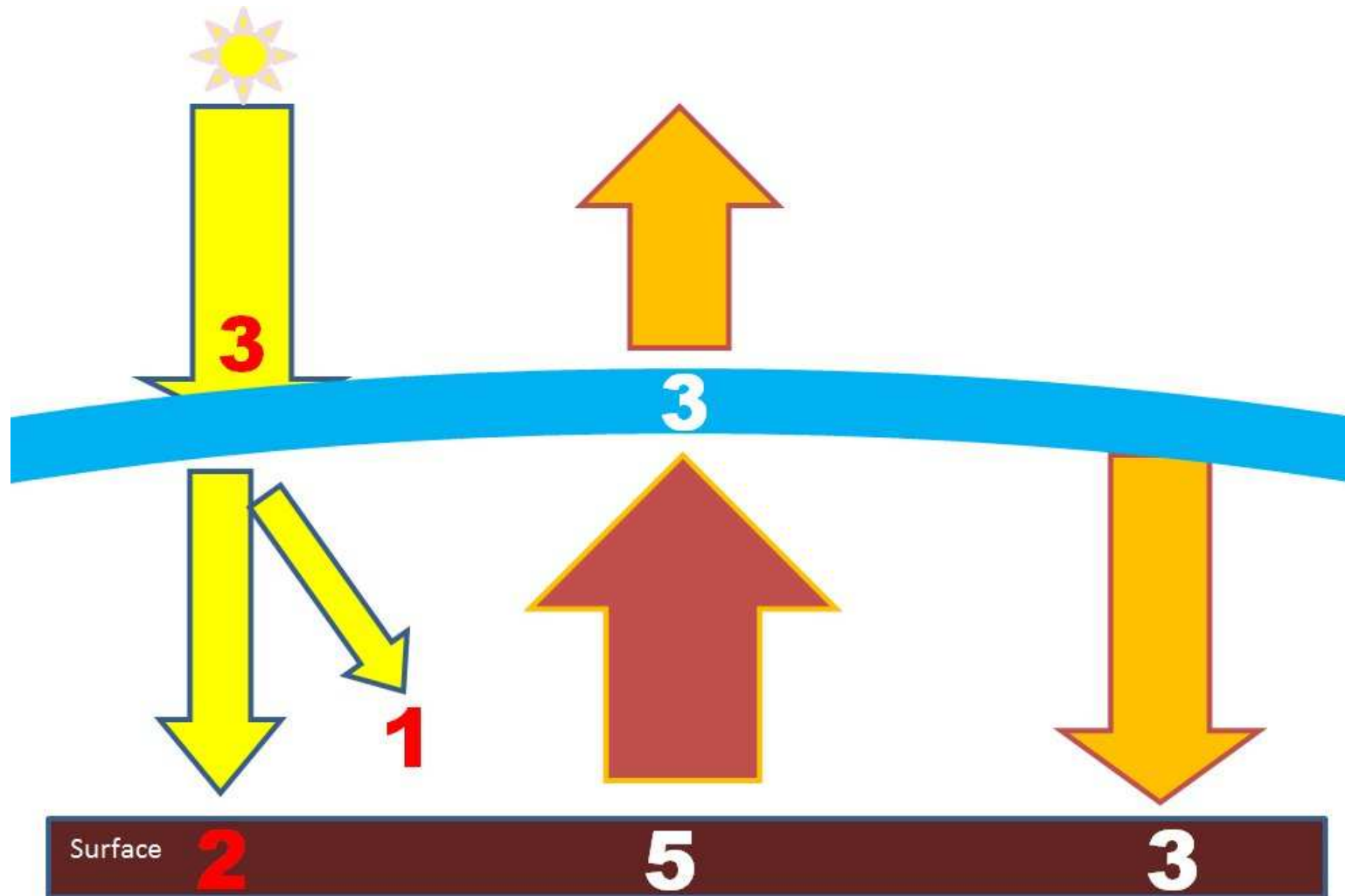
- $G_{\text{all}} = B_0 - B_{\text{eff}} = 15 - 9 = 6 = \text{SFC SW net}$
- $B_g(\text{all}) = \text{SFW SW net} + \text{LW down} = 19 \Rightarrow$
- $\text{LW down all} = 13 \Rightarrow \text{LW down clear} = 12$
- $B_g(\text{clear}) = 20 \Rightarrow$
- $\text{Clear-sky SFC SW net (clear)} = 8$
- $\text{SWCRE at surface} = -2.$

And now: How our system is able to create
an „effectively opaque” atmosphere?
A deduction in four steps



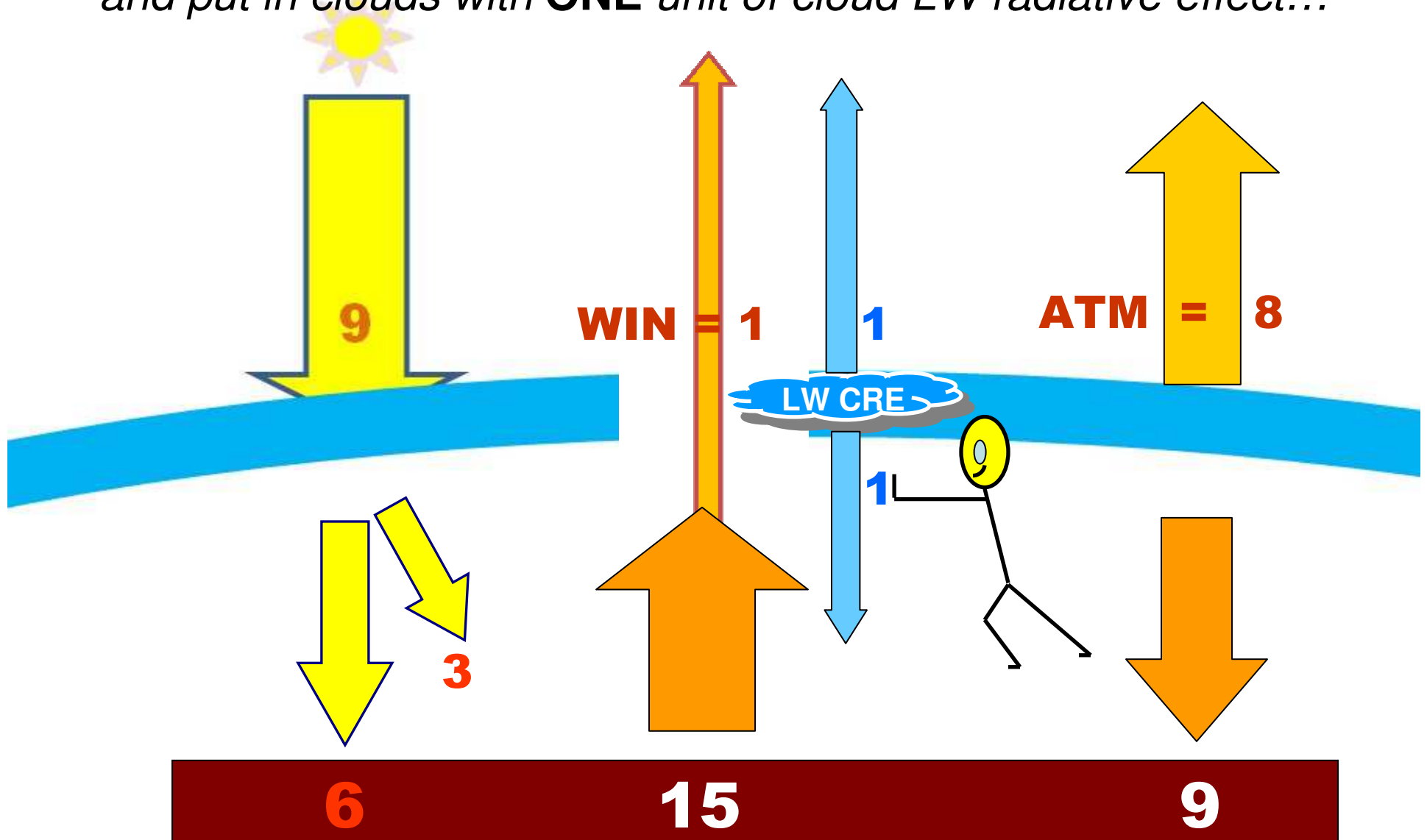
SW-transparent, LW-opaque, non-turbulent

Step 1. After *UNIT* change 1 \Rightarrow 3, introduce **ONE** unit of atmospheric SW-absorption:

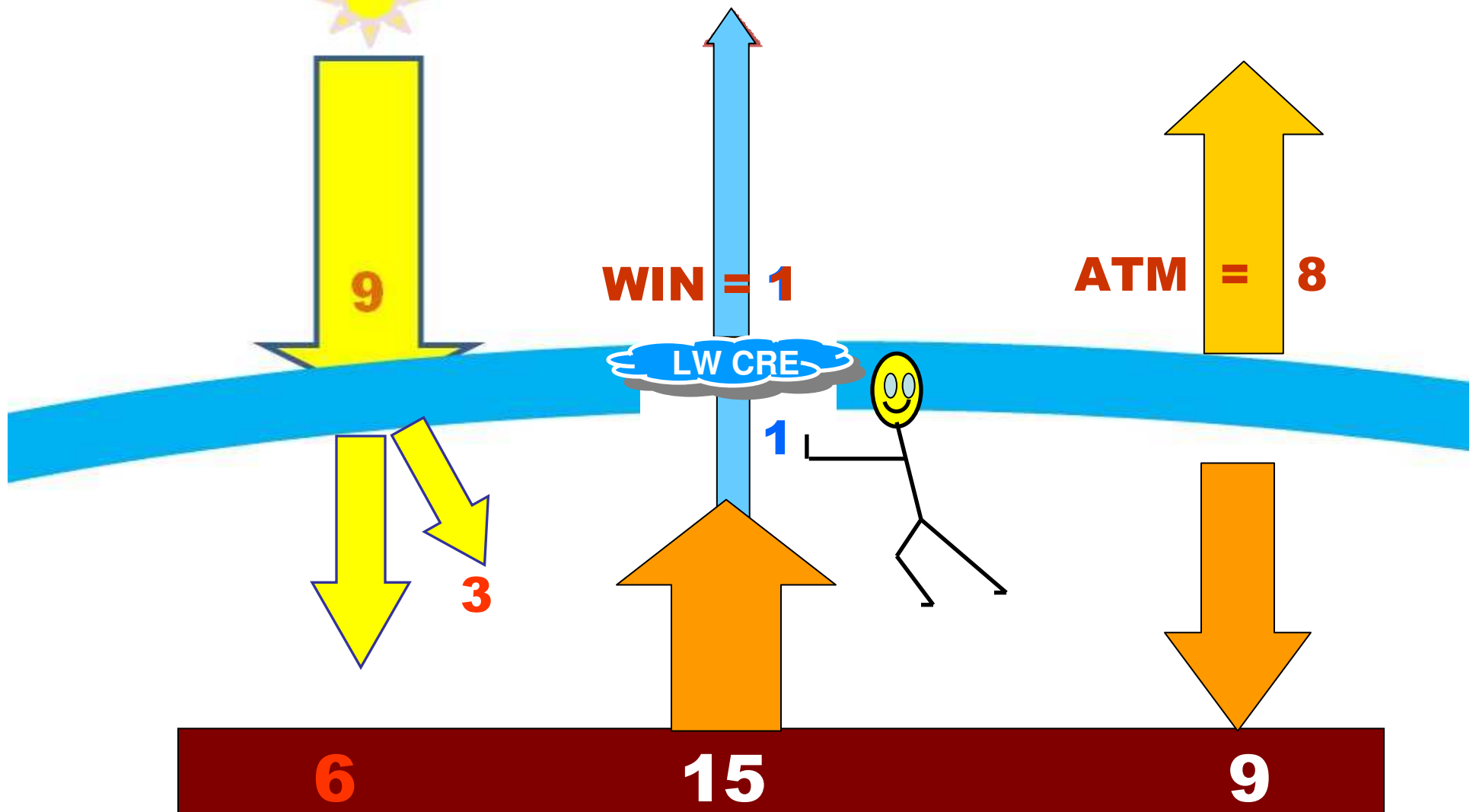


Solar Absorbed Atmosphere (SAA) = 1, Solar Absorbed Surface (SAS) = 2

Step 2. After unit change $3 \Rightarrow 9$, allow **ONE** unit for WIN and put in clouds with **ONE** unit of cloud LW radiative effect...

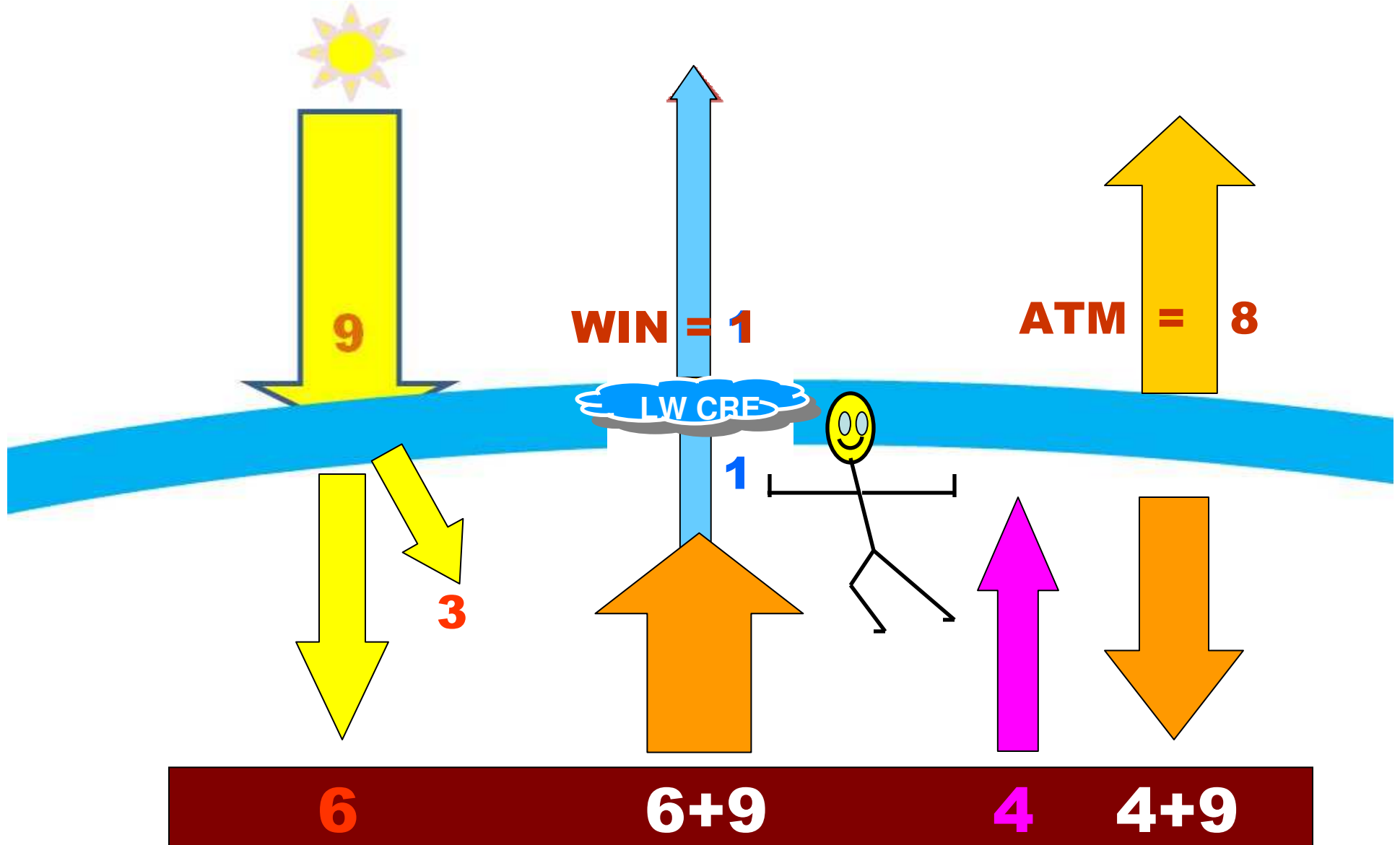


Step 3: ... and close the window with it !
The result is an effectively IR-opaque system.



From a surface perspective:
What is lost in the window is gained back by the LW effect of clouds

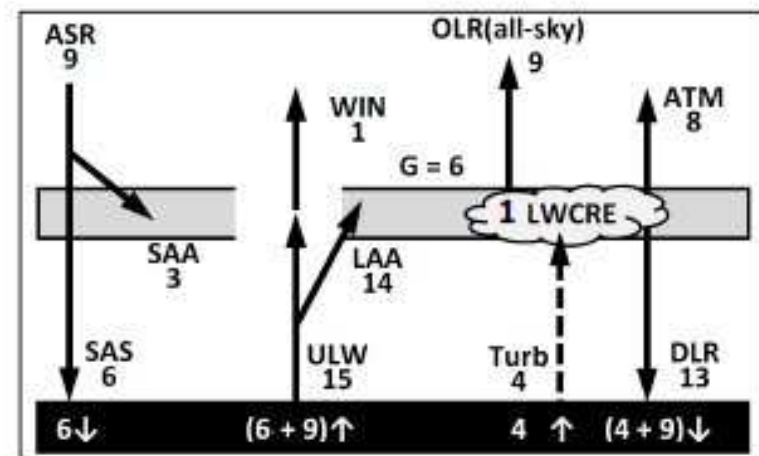
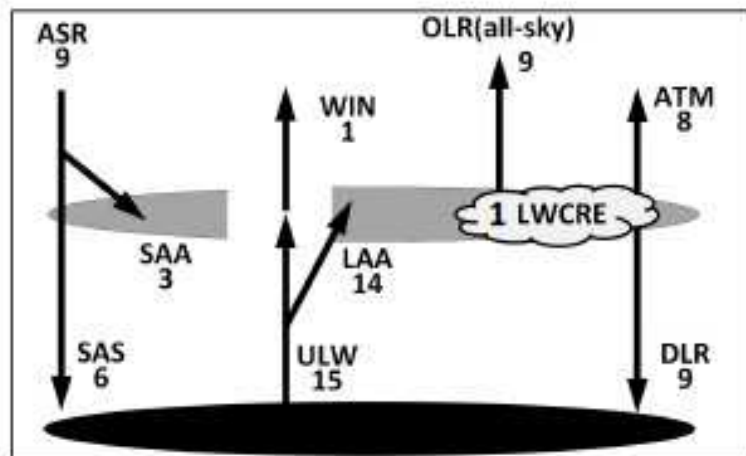
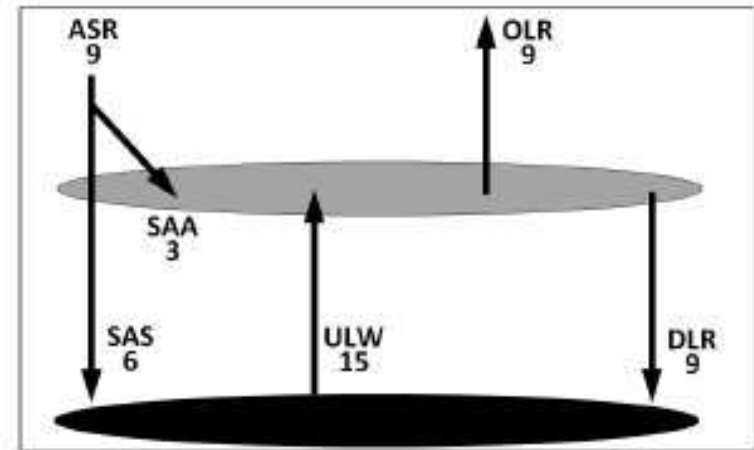
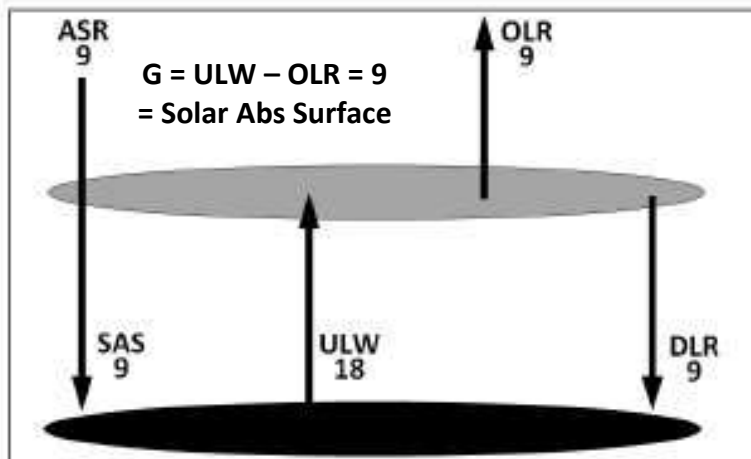
Step 4: Finally, by adding **4 UNITS** of convection, close the balance with turbulent fluxes.



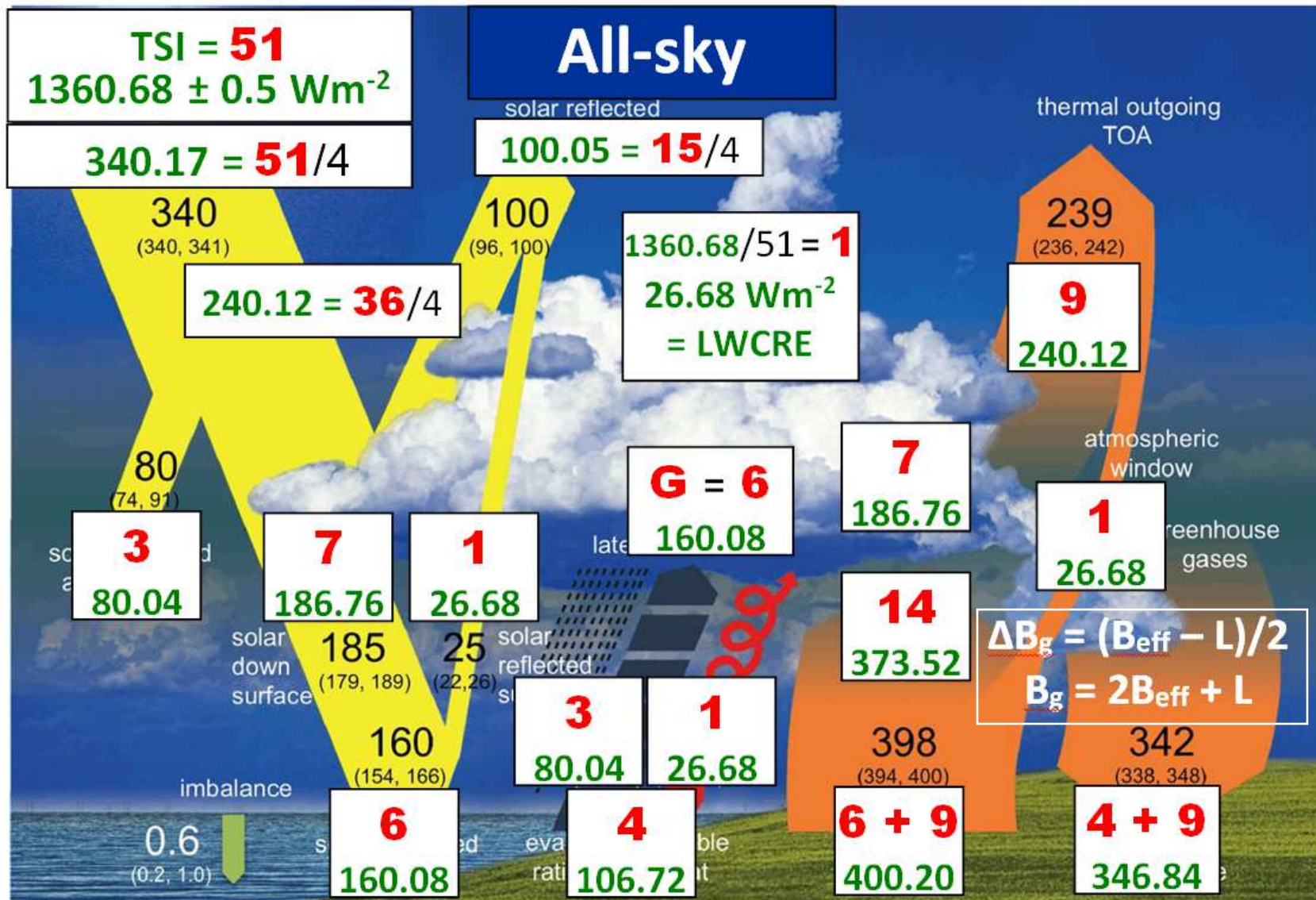
Deducing the **all-sky** system from the single-slab (description for the next slide)

- *Upper left* panel: SW-transparent, LW-opaque, non-turbulent atmosphere. Absorbed Solar Radiation ($ASR = 9$ units) is absorbed by the surface ($SAS = 9$). Upward emitted LW by the surface (ULW) is 18 units, absorbed completely by the atmospheric layer and re-emitted up (OLR) and down (DLR) equally as 9 units. $G = ULW - OLR = 9 = ASR = OLR = DLR = SAS$.
- *Upper right*: Allowing 3 units for partial Solar Atmospheric Absorption (SAA), SAS becomes 6 units, $ULW = 15$ units, atmospheric balance: $3 + 15 = 9 + 9$.
- *Lower left*: Allowing 1 unit for partial atmospheric LW transparency (WIN), Longwave Atmospheric Absorption (LAA) is 14 units, upward atmospheric LW emission becomes $ATM = 8$ units. Clouds are introduced by $LWCRE = 1$ unit, included here both in ATM and DLR . Energy balance: $3 + 14 = 8 + 9$.
- *Lower right*: To supply $LWCRE$ up and down, turbulence is allowed with 4 units, absorbed by the atmospheric layer. DLR becomes 13 units, balance: $SAA + LAA + Turb = 21 = ATM + DLR$. Surface energy budget and the two all-sky equations are also satisfied. Compare to Wild et al. (2015), next slide. The validity of $G = SAS$ of the initial geometry is reserved.

Deducing the **all-sky** system from the single-slab in four steps; **Eq. (5) $G = \text{SFC SW net}$**



Eq. (2), Eq. (4) and Eq. (5) valid



SFC SW+LW net = $(\text{OLR} - \text{LWCRE})/2$; SFC SW net + LW down = $2\text{OLR} + \text{LWCRE}$

$$6 + (13 - 15) = (9 - 1)/2$$

$$6 + 13 = 2 \times 9 + 1$$

Eq. (6) G clear-sky = SFC SW+LW net (verified by Costa and Shine 2012)

Name	CS12 (Wm ⁻²)	Round (Wm ⁻²)	Diff (Wm ⁻²)	Clear-Sky Units	All-Sky Units	Solar Units 1 = TSI / 51	N × UNIT (Wm ⁻²)	CERES (Wm ⁻²)
WIN	65	65	0	1	2.5	10 / 4	66.7	
G	127	130	3	2	5	20 / 4	133.4	132.4
ATM	194	195	1	3	7.5	30 / 4	200.1	
OLR	259	260	1	4	10	40 / 4	266.8	266.0
ULW	386	390	4	6	15	60 / 4	400.2	398.4
2OLR	518	520	2	8	20	80 / 4	533.6	532.0

$$B_g : B_0 : OLR : ATM : G : WIN = 2 : 3/2 : 1 : 3/4 : 1/2 : 1/4$$

Equivalent to:

$$B_g : B_0 : OLR : ATM : G (= \text{SFC SW+LW Net}) : WIN = 80 : 60 : 40 : 30 : 20 : 10$$

$$\begin{aligned} \text{SFC SW+LW Gross} : \text{ULW} : \text{DLR} : \text{OLR} : \text{SFC SW Net} : \text{ATM} : \text{G} : \text{WIN} : \text{TOA SW Up} : \text{LWCRE} \\ = 80 : 60 : 48 : 40 : 32 : 30 : 20 : 10 : 8 : 4 ; \text{ after spherical weighting (divided by 4):} \\ = \mathbf{20 : 15 : 12 : 10 : 8 : 7.5 : 5 : 2.5 : 2 : 1} \end{aligned}$$

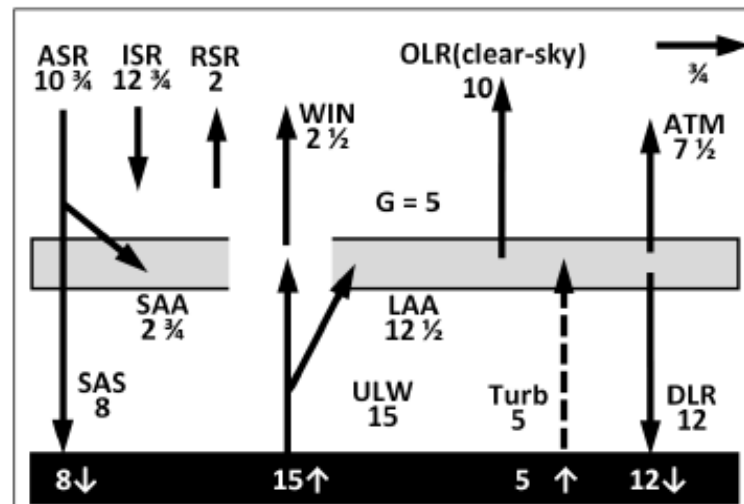
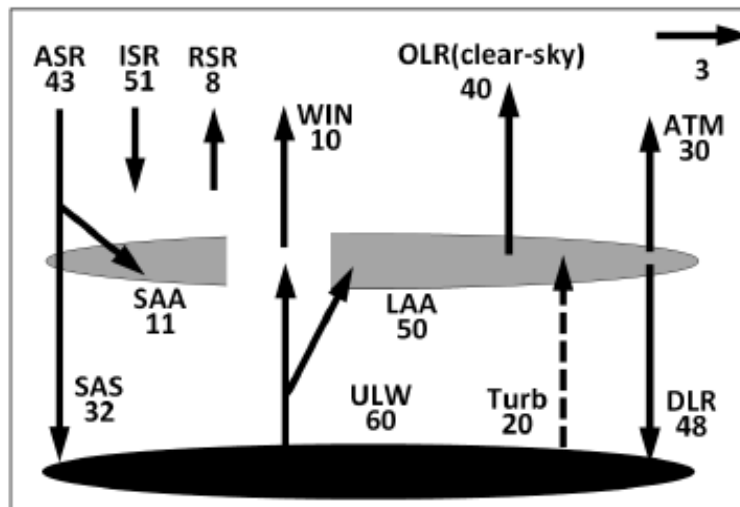
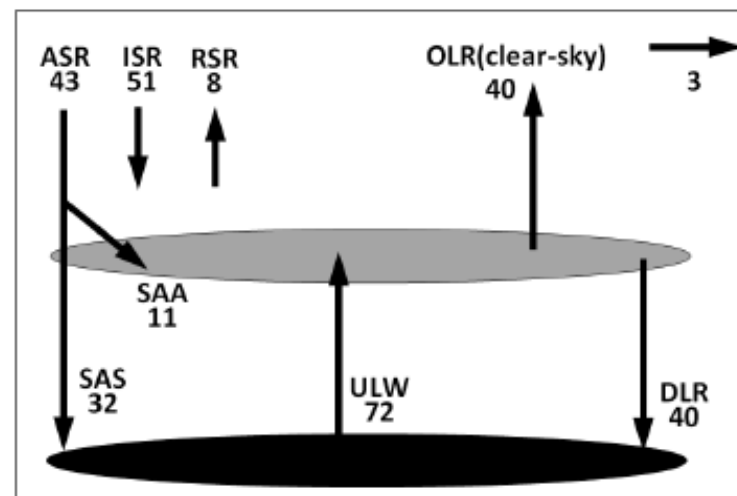
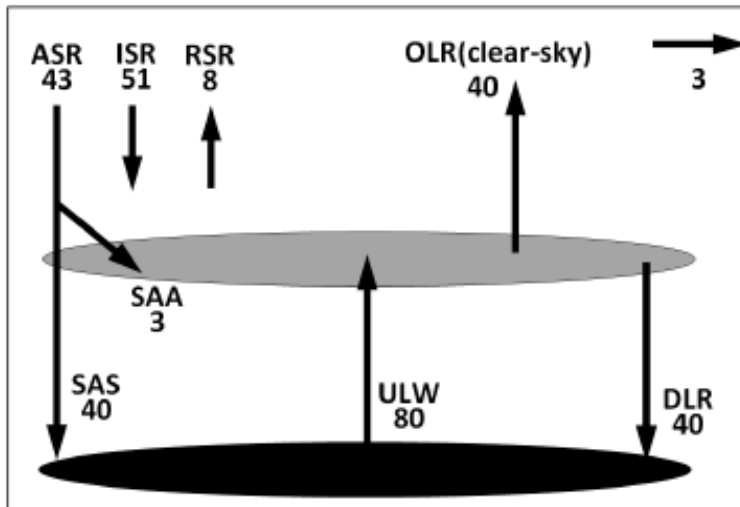
$$\mathbf{1} = 26.68 \text{ Wm}^{-2}; \text{ TSI} = 1360.68 \text{ Wm}^{-2} = \mathbf{51}$$

Deducing the **clear-sky** system

(description to the next slide)

- *Upper left:* On the intercepting cross-section disk, incoming solar radiation is 51 units, from which 43 absorbed, 8 reflected, 40 emitted as LW, 3 units are transferred to the cloudy part of the atmosphere, supplied by 3 units of solar atmospheric absorption (SAA). Hence $SAS = DLR = 40$, $ULW = 80$ units.
- *Upper right:* Allowing 8 units of more solar atmospheric absorption, $SAA = 11$, SAS becomes 32, ULW decreases to 72 units.
- *Lower left:* Allowing 10 units for the window, atmospheric upward emission (ATM) must decrease to 30 units. Introducing 20 units of turbulence, and constraining ULW to its all-sky value (15 units on the sphere, 60 units at the disk), LAA is 50 and DLR must become 48 units ($11 + 50 + 20 = 30 + 48 + 3$). The two clear-sky equations are valid ($Turb = OLR/2$ and $ULW + Turb = SAS + DLR = 2OLR$). Eq. (6) also satisfied: $G = Turb$; and a new equation reveals itself: Eq. (7) $2ASR = 2OLR + WIN - LWCRE$. Each value is integer.
- *Lower right:* After spherical weighting (divide by 4), the result can be compared to the clear-sky energy budget of Wild et al. (2018). In some fluxes, quarters appear.

Deducing the **clear-sky** system, Eq. (6), and a new
Eq. (7) $2ASR = 2OLR + WIN - LWCRE$



Eqs. (1), (3), (6) and (7) valid

More about the single-slab

Eq. (7) $2ASR = 2OLR + WIN - LWCRE$

$$B_g = \frac{\phi}{2\pi}(\chi_0^* + 2) \quad \text{Houghton (2002)} \quad (2.15)$$

where χ_0^* is the optical depth at the bottom of the atmosphere. If $\chi_0^* = 0$, $B_g = \phi/\pi$ and the surface temperature is in equilibrium with the incoming and the outgoing radiation, which are both equal to ϕ .

- Because Earth's atmosphere is not a closed but a 'leaky' single-slab, at $\chi_0^* = 2$, in clear-sky, Φ *cannot* be equal to the incoming radiation (which is given), only to the outgoing, which is set to a lower value: **OLR (clear) < ASR (clear)**.
- That's why in the clear-sky we have $B_g = 2\Phi/\pi = 2B_{\text{eff}} = \mathbf{2OLR}$, instead of $B_g = 2ASR$.
- In the all-sky, 'leak' (WIN) is closed by LWCRE:
- $ASR(\text{all}) = OLR(\text{all}); WIN = LWCRE$.

Eq. (7) $2ASR = 2OLR + WIN - LWCRE$

All-sky: $ASR = OLR$; $WIN_{all} = LWCRE = 1$

Clear-sky $OLR = 10$; $WIN_{clear} = 2.5 \Rightarrow$

Clear-sky $ASR = 10.75$; Clear TOA net = $3/4$

This was for the spherical surface of Earth \Rightarrow

For the intercepting cross-section disk:

$TSI = 51$, $ASR = 43$, $OLR = 40$, $WIN = 10$,

Reflected = 8 , Clear-sky TOA net = 3 .

Substitute $TSI = 51 = 1360.68 \text{ Wm}^{-2}$ and divide by 4 \Rightarrow

$RSR = 2 = 53.36 \text{ Wm}^{-2}$, $ASR = 10.75 = 286.81 \text{ Wm}^{-2}$,

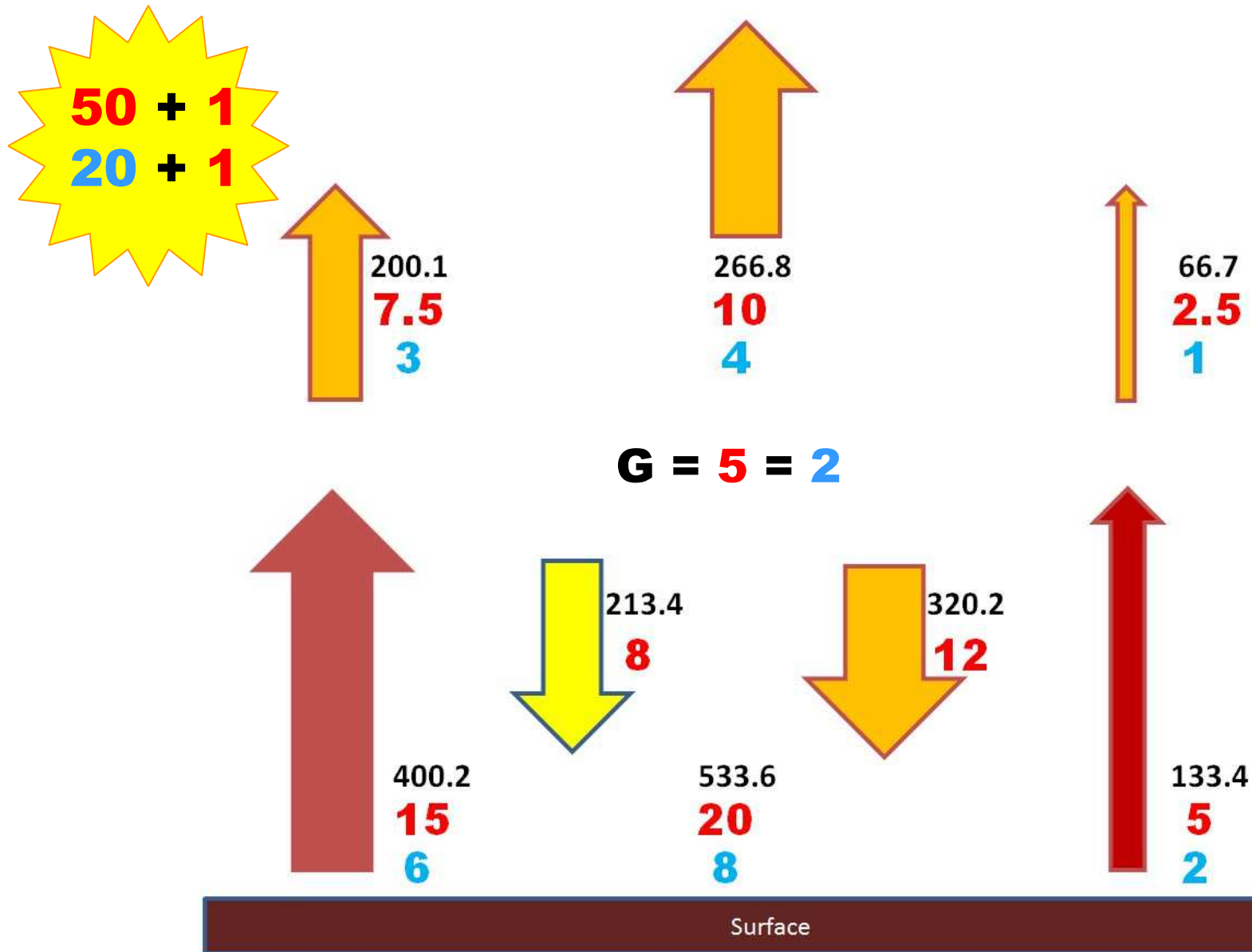
$OLR = 10 = 266.80 \text{ Wm}^{-2}$; Clear-sky TOA net = 20.01 Wm^{-2}

It seems that the energetic role of clouds in the LW is to close the open atmospheric window. The radiative energy being lost in the window is gained back by the greenhouse effect of clouds. This interplay is expressed by Eq. (7).

Clear-sky basics in their own units

- Costa and Shine (2012) computed WIN (clear) = 65 Wm^{-2} for their model atmosphere with OLR (clear) = 259 Wm^{-2} .
- Proportionally, with our theoretical OLR = 266.8 Wm^{-2} WIN would be 66.96 Wm^{-2} . Our theoretical WIN (clear) value is 66.7 Wm^{-2} .
- Notice that both TSI and the basic Earth fluxes (OLR, ATM, ULW and G) can also be expressed as integers in clear-sky unit of WIN (clear):
- TSI = **50** + **1** all-sky units = **20** clear-sky units + **1** all-sky unit
- **1** = LWCRE = WIN (all) = 26.68 Wm^{-2}
- **1** = WIN (clear) = 66.7 Wm^{-2}
- OLR = **4** = **10** = 266.8 Wm^{-2} ; ULW = **6** = **15** = 400.20 Wm^{-2} ; WIN = **1** = **2.5** = 66.7 Wm^{-2} ; ATM = **3** = **7.5** = 200.10 Wm^{-2} ; G = **2** = **5** = 133.40 Wm^{-2} ; TSI = 1360.68 Wm^{-2} .

Clear-sky basics: $g = (\text{ULW} - \text{OLR}) / \text{ULW} = \mathbf{5/15} = \mathbf{2/6} = 1/3$



All-sky basics related to TSI



Eq. (8) TSI = **51**, LWCRE = **1**

TSI = $1360.68 \pm 0.5 \text{ Wm}^{-2}$; Clear: **1** = 66.7 Wm^{-2} ; Cloudy: **1** = 44.67 Wm^{-2} ; All = **1** = 26.68 Wm^{-2} .

On the Earth's cross-section disk intercepting incoming solar flux:

Clear:	TSI = 20 + 1 = 16 + 11	RSR = 8	ASR = 16 + 3	OLR = 16
Cloudy:	TSI = 30 + 1 = 24 + 11	RSR = 4 + 13	ASR = 20 - 2	OLR = 20
All:	TSI = 50 + 1 = 40 + 11	RSR = 15	ASR = 36	OLR = 36

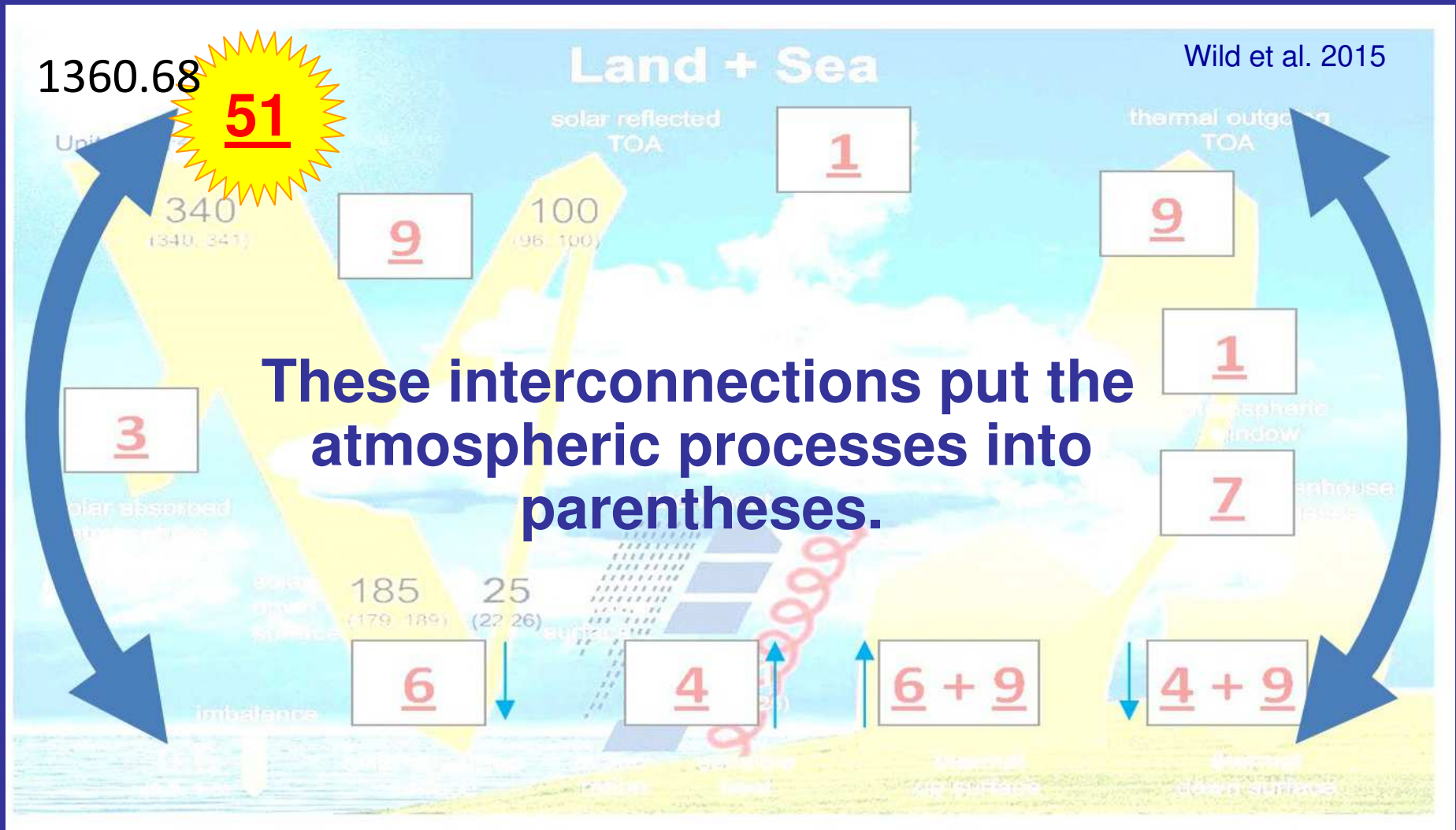
On the surface of the globe (after division by 4):

Clear:	TOA ISR = 4 + 11/4	RSR = 2	ASR = 4 + $\frac{3}{4}$	OLR = 4
Cloudy:	TOA ISR = 6 + 11/4	RSR = 1 + $3\frac{1}{4}$	ASR = 5 - $\frac{1}{2}$	OLR = 5
All:	TOA ISR = 10 + 11/4	RSR = 15/4	ASR = 9	OLR = 9

Eq. (1)-(4):

OLR(all-sky) = 9	OLR(clear-sky) = 10	ULW = 15
SFC SW net (all-sky) = 6	SFC SW net (clear-sky) = 8	SWCRE = - 2
SFC LW down (all-sky) = 13	SFC LW down (clear-sky) = 12	LWCRE = 1

The equations represent **direct surface-TOA** energetic relationships



Endnotes

- The first who realized that in radiative equilibrium there is a temperature discontinuity at the surface was Robert Emden (married Klara Schwarzschild, sister of Karl) in 1913. He calculated from the Schwarzschild-equations that the 'jump' is 20°C but, in the same sentence, he noted that this 'Temperatursprung' was greatly diminished by conduction of heat and evaporation.

$$89) \quad T_{\text{Erde}} = 254^{\circ} \sqrt[4]{2,2} = 309^{\circ} = + 36^{\circ}.$$

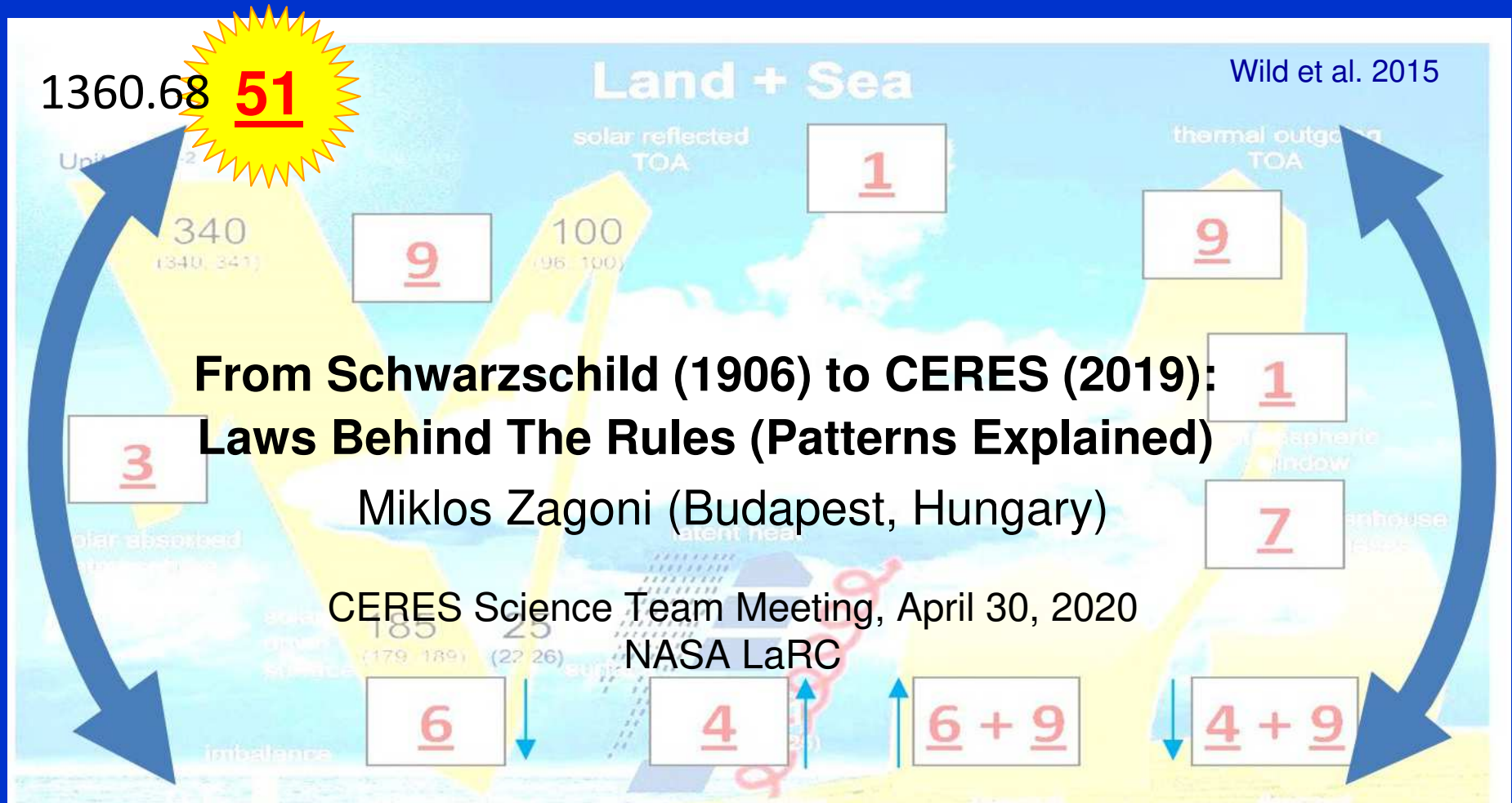
An der Berührungsfläche Atmosphäre und Erde ergibt sich somit ein Temperatursprung von 20° C, der in Wirklichkeit durch äußere Wärmeleitung stark herabgesetzt wird, namentlich auf Wasser, wo der Wasserdampf mit der Temperatur der Oberfläche in die Atmosphäre übertritt. Auch diese Strahlungstemperatur der Erdoberfläche hat einen durchaus annehmbaren Wert.

- My equations do not separate the surface net flux into sensible heat and evaporation; neither surface net solar radiation into its downward and upward components; their integer values therefore are only tentative.
- Open questions: Climate transitions, ice ages.

Extension:

Presentation delivered at
CERES 33rd (online) Science Team Meeting

Patterns in the CERES Global Mean Data, Part 4.



“Equation (2.17) is known as the equation of transfer, and was first given in this form by Schwarzschild. While it sets the pattern of the formalism used in transfer problems, its physical content is very slight.” — Goody and Yung (1989)

“The Eddington approximation will generally be employed; while it is not precise it omits no essential physical principles, provided that the medium is stratified.” — Goody (1964)

Ueber das Gleichgewicht der Sonnenatmosphäre

Von **K. Schwarzschild.**

Vorgelegt in der Sitzung vom 13. Januar 1906.

Consider now, at some point in the solar atmosphere, the radiative energy A which is transmitted outward, and the radiative energy B , which (due to the radiation of outer layers) is transmitted inward.

Treat first the inward energy B . When traveling inward through an infinitesimally thin layer dh , the fraction $aBdh$ of B will be lost; on the other hand, the contribution $aEdh$ due to the lateral radiation of the layer itself will be added to B . All in all,

$$\frac{dB}{dh} = a(E - B). \quad (7)$$

In the case of the outward energy A , we proceed analogously and obtain

$$\frac{dA}{dh} = -a(E - A). \quad (8)$$

Given the absorption coefficient a as a function of depth h , define the “average optical depth”* of the atmosphere lying above the depth h by

$$\bar{\tau} = \int^h adh. \quad (9)$$

The differential equations then become

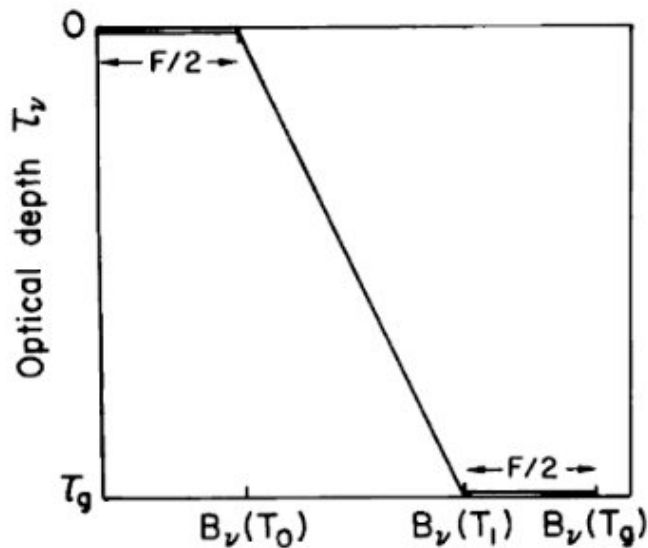
$$\frac{dB}{d\bar{\tau}} = E - B, \quad \frac{dA}{d\bar{\tau}} = A - E. \quad (10)$$

This leads to the final result

$$E = \frac{A_0}{2} (1 + \bar{\tau}), \quad A = \frac{A_0}{2} (2 + \bar{\tau}), \quad B = \frac{A_0}{2} \bar{\tau}. \quad (11)$$

$$E = \frac{A_0}{2} (1 + \bar{\tau}), \quad A = \frac{A_0}{2} (2 + \bar{\tau}), \quad B = \frac{A_0}{2} \bar{\tau}. \quad (11)$$

$$A - E = \Delta A = A_0/2 \quad \text{independent of } \tau$$



Chamberlain (1978)

Theory of Planetary Atmospheres,
Academic Press

Fig. 1.4 The MRE solution for $T(\tau)$, presented as $B_v(T)$ vs. τ_v . Note the discontinuity at the ground and the finite skin temperature at $\tau = 0$.

$$B_g - B_0 = \frac{\phi}{2\pi}$$

Houghton (2002, Eq. 2.13)

The Physics of Atmospheres,
Cambridge Univ Press

$$\Delta B_g = B_g - B_0 = B_{\text{eff}}/2$$

ATMOSPHERES IN RADIATIVE EQUILIBRIUM

9.1. Introduction

In this chapter we discuss *radiative equilibrium models* of the earth's atmosphere and the closely related *radiative-convective models*, for which small-scale convection is included in a highly parameterized form. In both cases, heat transports by planetary-scale motions are neglected.

$$B(\tau) = \frac{\sigma\theta(\tau)^4}{\pi} = \frac{-F_s(1 + 3\tau/2)}{2\pi} \quad \text{There are discontinuities,}$$

$$\Delta B = \frac{F_s}{2\pi} \quad (9.5)$$

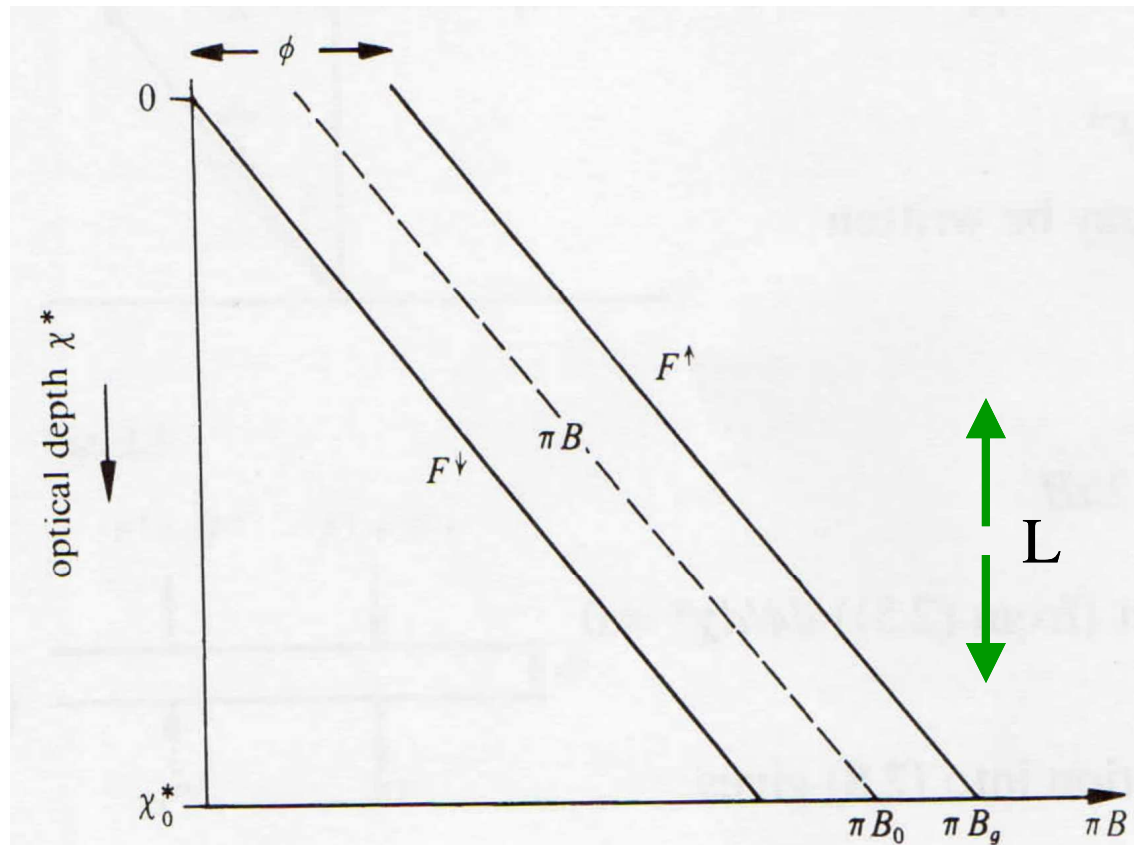
$$B^*(\tau_1) = \frac{\sigma\theta_g^4}{\pi} = \frac{-F_s(2 + 3\tau_1/2)}{2\pi}$$

My Eq. (1) $\Delta B = B_{\text{eff}}/2$

The solution, (9.5), although based upon many simplifications, has features that are instructive for planetary atmospheres.

Houghton (2002, Fig. 2.4)

The Physics of Atmospheres, Cambridge Univ Press



Eq. (1) (clear-sky)

$$\Delta B = B_{\text{eff}}/2$$

My Eq. (2) (all-sky)

$$\Delta B = (B_{\text{eff}} - L)/2$$

Separating atmospheric radiation from longwave cloud effect (L):

$$\text{Eq. (2): } \Delta B_g = (B_{\text{eff}} - L)/2 \quad (\text{surface net, all-sky})$$

Hartmann (1994)

Global Physical Climatology

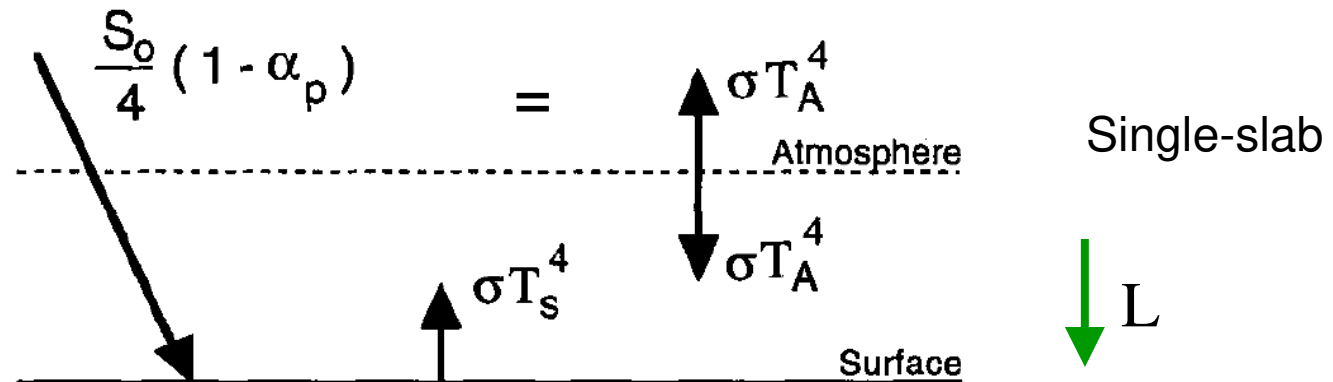


Fig. 2.3 Diagram of the energy fluxes for a planet with an atmosphere that is transparent for solar radiation but opaque to terrestrial radiation.

atmosphere and the surface. The atmospheric energy balance gives

$$\sigma T_s^4 = 2\sigma T_A^4 \Rightarrow \sigma T_s^4 = 2\sigma T_e^4 \quad (2.12)$$

and the surface energy balance is consistent:

$$\frac{S_0}{4} (1 - \alpha_p) + \sigma T_A^4 = \sigma T_s^4 \Rightarrow \sigma T_s^4 = 2\sigma T_e^4 \quad (2.13)$$

Surface total (gross) SW + LW energy income:	$B_g = 2B_{\text{eff}}$
Adding cloud effect, the surface absorption is:	$B_g = 2B_{\text{eff}} + L$

Houghton (2002)

$$B = \frac{\phi}{2\pi}(\chi^* + 1) \quad (2.12)$$

At the bottom of the atmosphere where $\chi^* = \chi_0^*$, $F^\uparrow = \pi B_g$, B_g being the black-body function at the temperature of the ground. It is easy to show that there must be a temperature discontinuity at the lower boundary, the black-body function for the air close to the ground being B_0 , and

$$B_g - B_0 = \frac{\phi}{2\pi} \quad (2.13)$$

2.5 The greenhouse effect

Combining (2.12) and (2.13) we find that for the radiative equilibrium atmosphere:

$$B_g = \frac{\phi}{2\pi}(\chi_0^* + 2) \quad (2.15)$$

With optical depth $\chi_0^* = 2$,

My Eq. (3) Surface total, clear-sky: $\pi B_g = 2\phi$

My Eq. (4) With cloud effect, all-sky: $\pi B_g = 2\phi + L$

My four equations

Eq. (1) Schwarzschild (1906, Eq. 11), net, clear-sky

$$\mathbf{A - E = \Delta A = A_0 / 2}$$

Eq. (2) Schwarzschild (1906, Eq. 11), incl LWCRE, net, all-sky

$$\mathbf{A - E = \Delta A = (A_0 - L) / 2}$$

Eq. (3) Schwarzschild (1906, Eq. 11), at $\tau = 2$, gross, clear-sky

$$\mathbf{A = 2A_0}$$

Eq. (4) Schwarzschild (1906, Eq. 11), at $\tau = 2$, incl LWCRE, gross, all-sky

$$\mathbf{A = 2A_0 + L}$$

My four equations

Eq. (1): Houghton Eq. (2.13)

Eq. (2): Houghton Eq. (2.13) incl LWCRE

Eq. (3): Houghton Eq. (2.15) at $\chi^*_0 = 2$

Eq. (4): Houghton Eq. (2.15) at $\chi^*_0 = 2$, incl LWCRE

Eq. (1) Surface net, clear-sky: $\Delta B_g = B_g - B_0 = B_{\text{eff}}/2$

Eq. (2) Surface net, all-sky: $\Delta B_g = B_g - B_0 = (B_{\text{eff}} - L)/2$

Eq. (3) Surface gross, clear-sky: $B_g = 2B_{\text{eff}}$

Eq. (4) Surface gross, all-sky: $B_g = 2B_{\text{eff}} + L$

The four equations in CERES notation

Eq. (1) $\Delta B_g = \text{SFC SW net} + \text{LW net, clear-sky} = \text{OLR}/2$

Eq. (2) $\Delta B_g = \text{SFC SW net} + \text{LW net, all-sky} = (\text{OLR} - \text{LWCRE})/2$

Eq. (3) $B_g = \text{SFC SW net} + \text{LW down, clear-sky} = 2\text{OLR}$

Eq. (4) $B_g = \text{SFC SW net} + \text{LW down, all-sky} = 2\text{OLR} + \text{LWCRE}$

Surface LW up (ULW) = LW down + LW net (both for clear and all)

LWCRE at the TOA = LWCRE at the surface

Accuracy of the equations in CERES EBAF Ed4.1, annual global means for 19 running years, 12/2000 – 11/2019

2001	340.166	239.788	266.178	344.674	316.613	397.756	397.695	186.831	23.612	163.218	211.672
2002	340.177	240.337	266.436	345.294	317.346	398.645	398.410	186.273	23.315	162.958	211.883
2003	340.068	240.401	266.273	345.169	317.282	398.513	398.125	186.742	23.375	163.368	211.359
2004	340.013	240.138	265.963	345.212	316.898	398.103	397.862	186.952	23.478	163.474	211.991
2005	339.966	240.251	266.187	346.071	317.737	398.873	398.543	186.302	23.229	163.072	211.479
2006	339.943	240.033	266.044	345.053	317.309	398.400	398.218	186.719	23.154	163.565	211.846
2007	339.914	240.468	266.152	344.670	317.161	398.448	398.228	186.355	23.095	163.260	211.585
2008	339.908	239.765	265.631	343.500	316.110	397.466	397.345	186.920	23.400	163.521	211.567
2009	339.912	239.915	265.778	344.295	317.067	398.124	398.023	186.886	23.372	163.515	211.476
2010	339.968	240.345	266.129	345.444	318.108	398.578	398.428	185.628	23.013	162.614	211.400
2011	340.027	240.038	265.628	343.808	316.484	397.723	397.646	186.399	23.061	163.338	211.577
2012	340.091	239.880	265.623	344.411	316.866	398.165	398.066	186.643	23.105	163.538	211.546
2013	340.083	240.075	265.804	345.163	317.223	398.360	398.238	186.700	23.398	163.302	211.773
2014	340.052	240.248	265.853	345.442	317.603	398.717	398.567	186.961	23.363	163.598	211.676
2015	340.138	240.424	265.925	346.364	318.692	399.428	399.287	186.902	23.114	163.787	211.405
2016	340.038	240.708	266.364	347.201	319.471	400.291	399.944	186.899	22.684	164.217	212.232
2017	339.953	240.610	266.193	346.265	318.313	399.740	399.363	187.320	22.838	164.483	212.387
2018	339.944	240.170	265.812	344.956	317.767	399.339	398.996	187.227	22.975	164.252	212.049
2019	339.942	240.576	266.166	344.940	318.131	400.007	399.733	187.623	22.840	164.783	211.965
	340.02	240.22	266.01	345.15	317.48	398.67	398.46	186.75	23.18	163.57	211.73
	isr	olr_a	olr_c	dlr_a	dlr_c	ulw_a	ulw_c	sw_d_a	sw_u_a	swnet_a	swnet_c
		ΔEq1		ΔEq2		ΔEq3		ΔEq4			
		-2.25		2.84		-2.80		2.50			

Accuracy of the equations, EBAF Ed4.1, 19 years

Eq. (1) Clear-sky, net

SFC SW net clear-sky	= 211.73
SFC LW down clear-sky	= 317.48
SFC LW up clear-sky	= 398.46
SFC SW+LW net, clear-sky	= 130.75
TOA LW /2, clear-sky	= 133.00

$$\Delta\text{Eq}(1) = -2.25 \text{ Wm}^{-2}$$

Eq. (2) All-sky, net

SFC SW net all-sky	= 163.57
SFC LW down all-sky	= 345.15
SFC LW up all-sky	= 398.67
TOA LW, all-sky	= 240.22
LWCRE	= 25.79
SFC SW+LW net, all-sky	= 110.05
(TOA LW – LWCRE)/2	= 107.21

$$\Delta\text{Eq}(2) = 2.84 \text{ Wm}^{-2}$$

Eq. (3) Clear-sky, gross

SFC SW net clear-sky	= 211.73
SFC LW down clear-sky	= 317.48
SFC SW net + LW down	= 529.21
2TOA LW, clear-sky	= 532.02

$$\Delta\text{Eq}(3) = -2.80 \text{ Wm}^{-2}$$

Eq. (4) All-sky, gross

SFC SW net all-sky	= 163.57
SFC LW down all-sky	= 345.15
TOA LW, all-sky	= 240.22
LWCRE	= 25.79
SFC SW net +LW down, all	= 508.72
2TOA LW + LWCRE	= 506.22

$$\Delta\text{Eq}(4) = 2.50 \text{ Wm}^{-2}$$

Definitions and integer solution

SFC LW down clear-sky = SFC LW down all – LWCRE
TOA LW clear-sky = TOA LW all + LWCRE
LWCRE TOA = LWCRE SFC
SFC LW up all-sky = SFC LW up clear-sky

Surface LW up, all-sky	=	15	Surface LW up, clear-sky	=	15
Surface SW net, all-sky	=	6	Surface SW net, clear-sky	=	8
Surface LW net, all-sky	=	-2	Surface LW net, clear-sky	=	-3
Surface SW+LW net, all-sky	=	4	Surface SW+LW net, clr-sky	=	5
Surface SW+LW gross, all	=	19	Surface SW+LW gross, clear	=	20
Surface LW down, all-sky	=	13	Surface LW down, clear-sky	=	12
TOA LW all-sky	=	9	TOA LW clear-sky	=	10
G greenhouse effect, all-sky	=	6	G greenhouse effect, clear-sky	=	5
LWCRE (surface, TOA)	=	1	SWCRE (surface)	=	-2

Accuracy of the **N** positions, EBAF Ed4.1, 19 years

Eq. (1) **8** + (**12** – **15**) = **10**/2

Eq. (3) **8** + **12** = 2 × **10**

Eq. (2) **6** + (**13** – **15**) = (**9** – **1**)/2

Eq. (4) **6** + **13** = 2 × **9** + **1**

<p>Clear: SW+LW net = OLR/2</p> <p>211.73 = 8 × 26.68 – 1.71</p> <p>317.48 = 12 × 26.68 – 2.68</p> <p>398.46 = 15 × 26.68 – 1.74</p> <p>130.75 = 5 × 26.68 – 2.65</p> <p>133.00 = 5 × 26.68 – 0.40</p> <p>$\Delta\text{Eq}(1) = -2.25 \text{ Wm}^{-2}$</p>	<p>Clear: SW net + LW down = 2OLR</p> <p>211.73 = 8 × 26.68 – 1.71</p> <p>317.48 = 12 × 26.68 – 2.68</p> <p>529.21 = 20 × 26.68 – 4.39</p> <p>532.02 = 20 × 26.68 – 1.58</p> <p>$\Delta\text{Eq}(3) = -2.80 \text{ Wm}^{-2}$</p>
<p>All: SW+LW net = (OLR-LWCRE)/2</p> <p>163.57 = 6 × 26.68 + 3.47</p> <p>345.15 = 13 × 26.68 – 1.69</p> <p>398.64 = 15 × 26.68 – 1.56</p> <p>240.22 = 9 × 26.68 + 0.10</p> <p>25.79 = 1 × 26.68 – 0.89</p> <p>110.05 = 4 × 26.68 + 3.33</p> <p>107.21 = 4 × 26.68 + 0.47</p> <p>$\Delta\text{Eq}(2) = 2.84 \text{ Wm}^{-2}$</p>	<p>All: SW net + LW down = 2OLR + LWCRE</p> <p>163.57 = 6 × 26.68 + 3.45</p> <p>345.15 = 13 × 26.68 – 1.69</p> <p>240.22 = 9 × 26.68 + 0.10</p> <p>25.79 = 1 × 26.68 – 0.89</p> <p>508.72 = 19 × 26.68 + 1.80</p> <p>506.23 = 19 × 26.68 – 0.69</p> <p>$\Delta\text{Eq}(4) = 2.50 \text{ Wm}^{-2}$</p>

Accuracy of the Greenhouse Effect: Theory and Observation

CERES EBAF Ed4.1, last 12 months

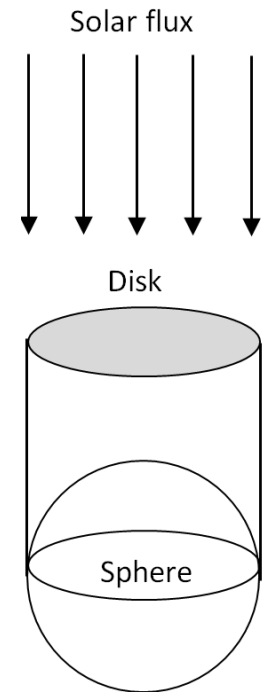
217	406.69	268.74	137.95	0.339202	407.47	243.29	164.18	0.402925
218	408.34	269.87	138.47	0.339105	408.66	244.31	164.35	0.402168
219	407.39	269.3	138.09	0.338963	407.8	243.9	163.9	0.401913
220	403.98	267.77	136.21	0.33717	404.46	242.74	161.72	0.399842
221	399.63	265.56	134.07	0.335485	400.14	240.21	159.93	0.399685
222	393.57	263.56	130.01	0.330335	393.8	237.71	156.09	0.396369
223	391.11	263.08	128.03	0.32735	391.1	237.04	154.06	0.393915
224	390.24	263.34	126.9	0.325185	389.92	237.46	152.46	0.391003
225	392.12	263.67	128.45	0.327578	391.56	238.29	153.27	0.391434
226	396.27	264.54	131.73	0.332425	395.85	238.86	156.99	0.39659
227	399.87	265.53	134.34	0.335359	400.31	239.43	160.88	0.401889
228	403.78	266.9	136.88	0.338996	404.84	241.25	163.59	0.404086
Observed	399.42	265.99	133.43	0.3340	399.66	240.37	159.29	0.3985
1360.68	400.20	266.80	133.40	0.3333	400.20	240.12	160.08	0.4
Theory	51	15	5	1/3	15	9	6	2/5
TSI	ULW_clr	OLR_clr	G_clr	g_clr	ULW_all	OLR_all	G_all	g_all

ULW = **15**, OLR clr = **10** => G (clr) = **5** = 133.40 Wm⁻² , G (all) = **6** = 160.08 Wm⁻²

Accuracy of the TOA fluxes

(clear-sky for total area, EBAF Ed4.1, 12/2000 – 11/2019)

TSI = 1360.68	51	N × UNIT	CERES	Diff
LW all-sky	36 / 4	240.12	240.22	-0.10
SW all-sky	15 / 4	100.05	99.06	0.99
LW clear-sky	40 / 4	266.80	266.01	0.79
SW clear-sky	8 / 4	53.36	53.74	-0.38
TOA LW CRE	4 / 4	26.68	25.79	0.89
TOA SW CRE	-7 / 4	-46.69	-45.30	-1.39
TOA Net CRE	-3 / 4	-20.01	-19.51	-0.50



Each flux is an **integer** on the intercepting cross-section disk

Eq. (5) TSI = **51** = 1360.68 Wm⁻² => LWCRE = **1** = 26.68 Wm⁻²

Clear-sky: RSR = **8** ASR = **43** OLR = **40** IMB = **3**

All-sky: RSR = **15** ASR = **36** OLR = **36**

Accuracy of the surface fluxes

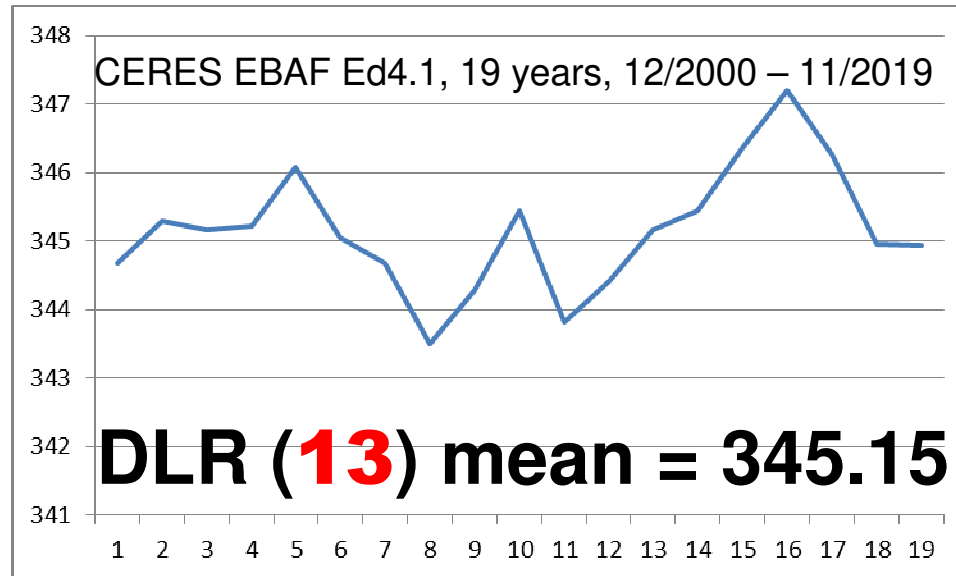
(clear-sky for total area, EBAF Ed4.1, 12/2000 – 11/2019)

	N	N × UNIT	CERES	Diff Wm ⁻²
Clear-sky				
LW down	12	320.16	317.48	2.68
LW up	15	400.20	398.46	1.74
SW net	8	213.44	211.73	1.71
All-sky				
LW down	13	346.84	345.15	1.69
LW up	15	400.20	398.67	1.53
SW Net	6	160.08	163.57	-3.49

SFC SW net is not resolved into
downward and upward components

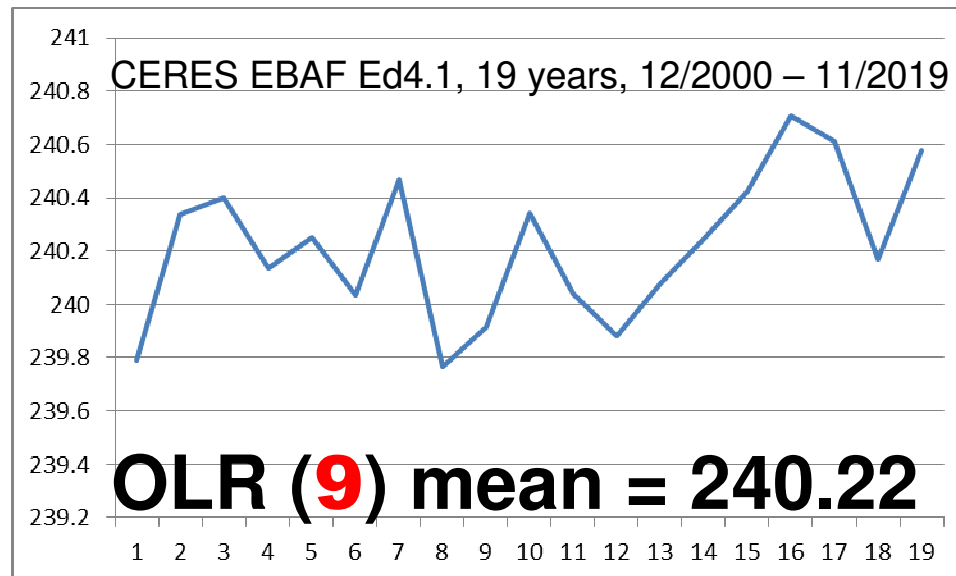
$$\text{DLR}(\text{all-sky}) = (13/9)\text{OLR}(\text{all-sky}) - 1.8 \text{ Wm}^{-2}$$

CO₂
increased by
40 ppm
during these
two decades



Radiative
forcing
balanced by
transfer
constraints

ULW =
(15/9) OLR
according to
transfer
equations



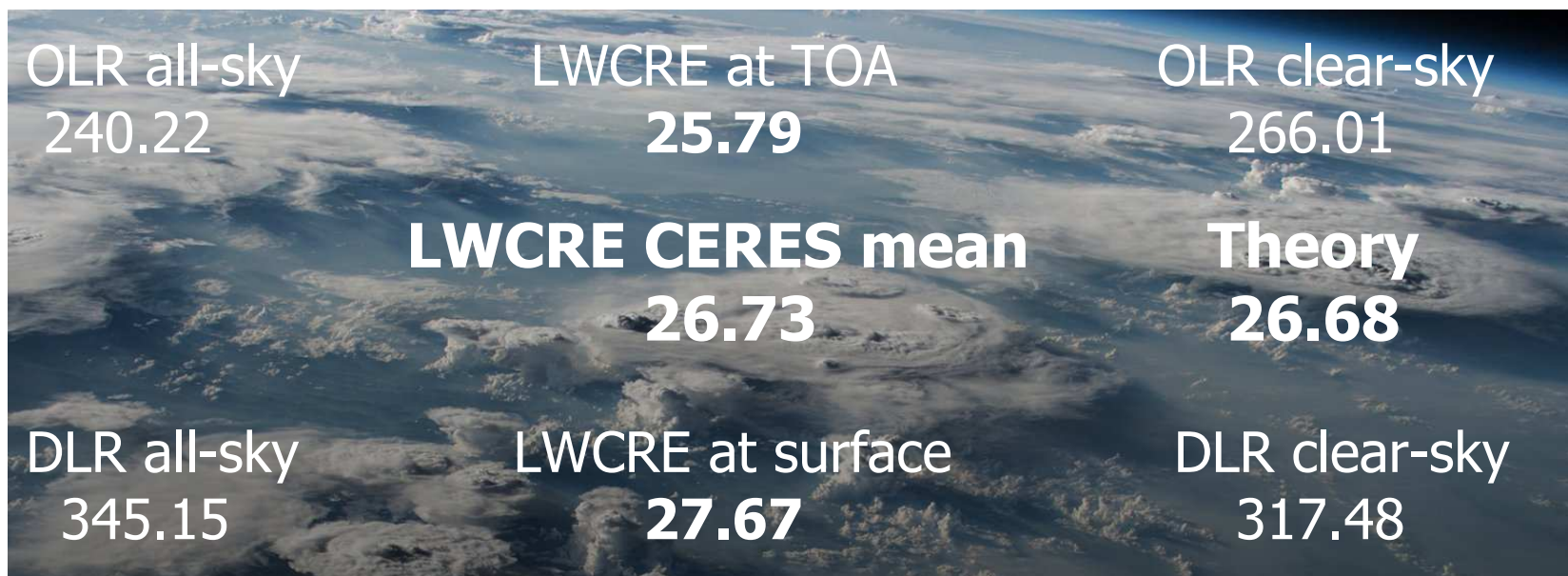
ULW –
(15/9) OLR
= -1.70 Wm^{-2}
according to
observation

$$\text{TSI} = 1360.9 \text{ Wm}^{-2} = 51 \Rightarrow 9 = 240.16 \text{ Wm}^{-2}$$

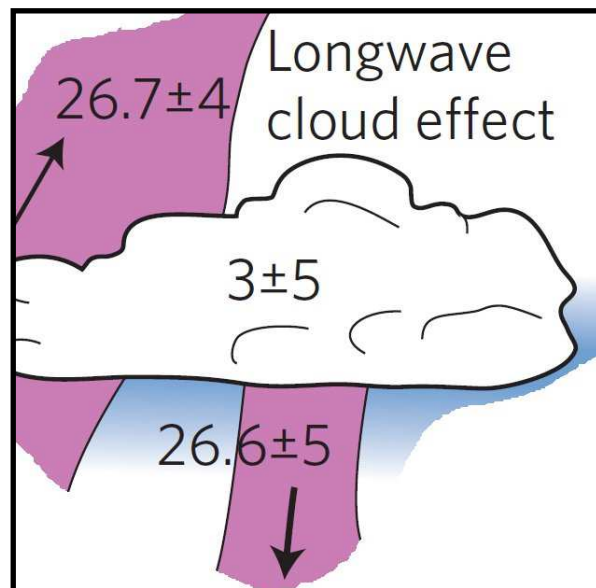
Accuracy of the new clear-sky parameter: no adjustment and with Δ^C adjustment

	TSI 1360.882 = 51 (disk)	N integer	Theory Wm ⁻²	no adj Wm ⁻²	theory – no adj	with Δ^C adjustment	theory – Δ^C adj
ISR	1360.882/4	51 /4	340.22	340.0	0.22	340.0	0.22
Clear-Sky	LW	40 /4	266.84	268.1	-1.26	266.0	0.84
	SW	8 /4	53.37	53.3	0.07	53.8	-0.43
	Net	3 /4	20.01	18.6	1.41	20.3	-0.29
CRE	LW	4 /4	26.68	27.9	-1.22	25.8	0.78
	SW	-7 /4	-46.70	-45.8	-0.90	-45.3	-1.40
	Net	-3 /4	-20.01	-17.9	-2.11	-19.6	-0.41
			Surface				
Clear-Sky	LW down	12	320.21	313.9	6.31	317.5	2.71
	LW up	15	400.26	397.6	2.66	398.5	1.76
	LW Net	-3	-80.05	-83.7	3.65	-81.0	0.95
	SW Net	8	213.47	213.5	-0.03	211.7	1.77
	SW+LW Net	5	133.42	129.8	3.62	130.7	2.72

Accuracy of mean CERES LWCRE = **0.05** Wm^{-2}



Stephens
et al. (2012)

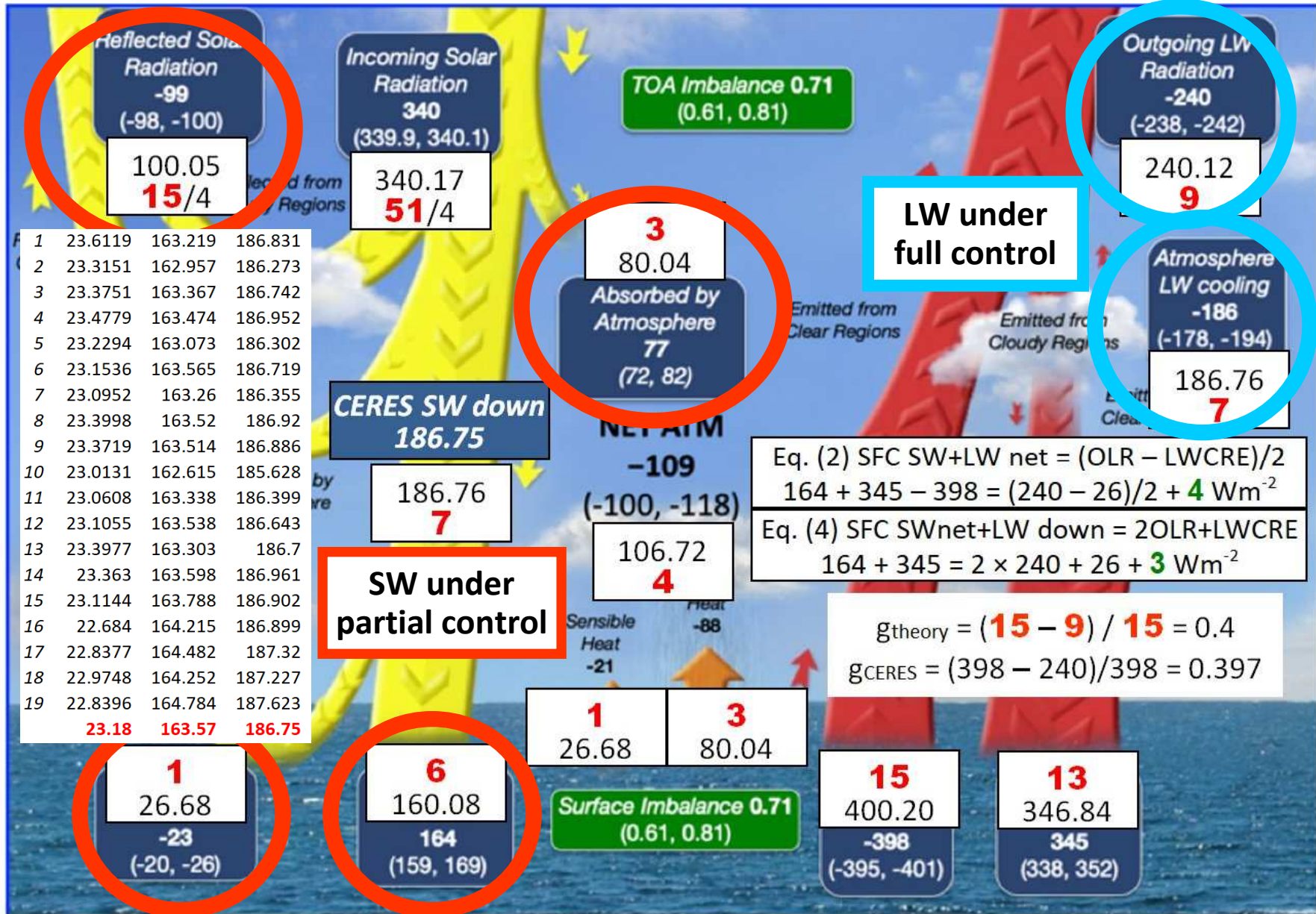


LWCRE Theory

$$\begin{aligned} \mathbf{1} &= \text{TSI} / 51 \\ &= 1360.68 / 51 \\ &= \mathbf{26.68 \text{ Wm}^{-2}} \end{aligned}$$

$$\begin{aligned} \text{CERES} - \text{Theory:} \\ \mathbf{0.05 \text{ Wm}^{-2}} \end{aligned}$$

The Bluehouse Effect, detected by CERES



Eq. (1) – (5): A theoretical steady state for our Aquaplanet

Summary and Conclusions

- Earth's global energy budget is controlled by radiation transfer equations originated in Schwarzschild's theory. Eq. (1) and (2) may be derived from first principles.
- Each of the four equations is satisfied by two decades of CERES observations within $\pm 3 \text{ Wm}^{-2}$. Forcing and feedbacks are expected to act within these limits.
- The fundamental individual fluxes (both SW and LW) are within $\pm 1 \text{ Wm}^{-2}$.
- The accuracy of CERES data (fit to theory) is much better than indicated in DQS.
- There are other constraints: the extension of the **N** system to total solar irradiance is unexpected, but extremely precise:
- Eq. (5) $\text{LWCRE} = \mathbf{1} = \text{TSI} / 51 \pm 0.01 \text{ Wm}^{-2}$.
 $\text{LWCRE} = 26.68 \text{ Wm}^{-2}$ (SORCE TSI) or 26.69 Wm^{-2} (TSIS1) .
- Eq. (6) $2\text{ASR} = 2\text{OLR} + \text{WIN} - \text{LWCRE}$ is a valid equation as well (not detailed here, see EGU2020 display).
- I expect $\pm 3 \text{ Wm}^{-2}$ fluctuations around, but not systematic deviation from, the equilibrium positions in the forthcoming decades.
- Open questions: limits, tipping points, shifts, ice ages (albedo?)

Thank you CERES Science Team for the excellent work!

Gupta, Kratz, Stackhouse, Wilber:

On Continuation of the Use of Daily TSI for CERES Processing

CERES Science Team Meeting, 29 April 2020

