Variations in the temporal evolution of seismicity pointed out by non-extensive statistical physics approach

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Statistics and pattern recognition applied to the spatio-temporal properties of seismicity

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Aim: to study a **'descriptor'** of the criticality of a seismogenic system

the Earth's crust behaves as a **complex, nonlinear dynamical system** chracterized by **long-range** correlations and **multifractal** and hierarchical properties

similar composite systems are studied in statistical physics through the Boltzmann-Gibbs (BG) entropy which is appropriate to study systems with local interactions in space or time

Tool: we use the **Tsallis entropy** that generalizes some properties of BG entropy such as additivity

Newness:

- detailed analysis of the entropic q index and of the entropy for both **sub-additive** and **super-additive** states
- **Bayesian** estimation of the magnitude distribution in both cases

Brief notes on non-extensive statistical physics[1]

Tsallis entropy of a probability density function $f(x)$ is given by

$$S_q[f] = \frac{1 - \int f^q(x)\,dx}{q-1},$$

where q is the entropic index.

S_q is not additive, e.g. $S_q(A+B) = S_q(A) + S_q(B) + (1-q)S_q(A)\,S_q(B)$ for any two independent systems; in particular, if $q < 1$ we speak of super-additivity and of sub-additivity when $q > 1$

We look for the distribution that optimizes the entropy S_q under the constraints

- $\int_0^{+\infty} f(x)\,dx = 1$ normalization condition
- $\int_0^{+\infty} x\,f_q(x)\,dx = X_q$ where $f_q(x) = \dfrac{f^q(x)}{\int_0^{+\infty} f^q(x)\,dx}$ is called **escort** distribution,

maximizing the functional

$$\phi(f, \lambda_0, \lambda_1) = S_q + \lambda_0 \left(\int_0^{+\infty} f(x)\,dx - 1 \right) + \lambda_1 \left(\int_0^{+\infty} x\,f_q(x)\,dx - \bar{x} \right)$$

being λ_0, λ_1 Lagrange multipliers.

[1] more details on Bayesian inference in the last slide

Q-exponential distributions

We obtain the **q-exponential** distributions

$$f(x) = \frac{1}{X_q}\left[1 - \frac{(1-q)}{(2-q)}\frac{x}{X_q}\right]^{1/(1-q)} \qquad 1 < q < 2 \qquad x \in \mathcal{R}^+$$

$$f(x) = \frac{1}{Z_q}\left[1 - (1-q)\lambda^{'}x\right]^{1/(1-q)} \qquad 0 < q < 1 \qquad x \in (0, x_{max})$$

where Z_q is the normalizing constant, x_{max} is the cutoff to be imposed whenever the argument of $f(\cdot)$ becomes negative, $c_q = \int_0^{x_{max}} f^q(x)\,dx$ and

$$\lambda^{'} = \frac{\lambda_1}{c_q + (1-q)\,\lambda_1\bar{x}}$$

Q-exponential distributions for the magnitude

Following to the fragment-asperity interaction model* [1], and applying the relationships $x \propto E^{2/3}$ and $\log_{10} M_0 = 1.5\, M_w + 9.1$, we transform the previous q-exponential distributions into

$\boxed{q \in (1,2)}$ $\qquad M \geq m_0$

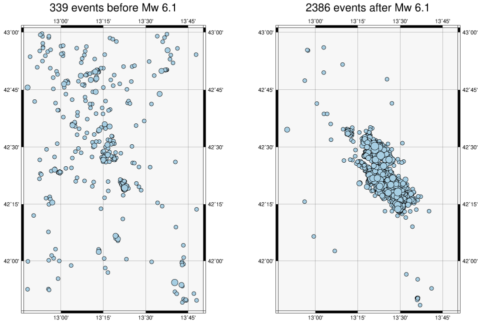
$$\frac{f(M)}{1-F(m_0)} = \frac{(\ln 10)\, \dfrac{10^M}{a^{2/3}\,\bar{x}} \left[1 - \dfrac{1-q}{2-q}\, \dfrac{10^M}{a^{2/3}\,\bar{x}}\right]^{1/1-q)}}{\left[1 - \dfrac{1-q}{2-q}\, \dfrac{10^{m_0}}{a^{2/3}\,\bar{x}}\right]^{(2-q)/(1-q)}}$$

$\boxed{q \in (0,1)}$ $\qquad M \in \left(m_0, \log_{10} \dfrac{a^{2/3}}{(1-q)\,\lambda'}\right)$

$$\frac{f(M)}{1-F(m_0)} = \frac{(\ln 10)\,(2-q)\, \dfrac{\lambda'}{a^{2/3}} \times 10^M \left[1 - (1-q)\, \dfrac{\lambda'}{a^{2/3}}\, 10^M\right]^{1/1-q)}}{\left[1 - (1-q)\, \dfrac{\lambda'}{a^{2/3}}\, 10^{m_0}\right]^{(2-q)/(1-q)}}$$

L'Aquila earthquake, M_w 6.1 on April 6, 2009

Lat (41.8, 43.0) Long (12.8, 13.8) 2005/04/25 - 2009/07/31



$N = 2725$ earthquakes
$M_W \in (2.0, 6.1]$
drawn from the Italian Seismological Instrumental and Parametric Data-Base (ISIDe)

Figure 1. Spatial distribution of the seismicity, on the left, before and, on the right, after the main shock M_w 6.1

- We estimate the q-exponential distributions **on time windows of 100 events shifted at each event** of $\mathcal{M} = \left\{ M_w^{(i)} \right\}_{i=1}^{N}$
- we compare the two versions of the q-exponential distribution $f(M)$ on the basis of the posterior marginal likelihood $\mathcal{L}(f(\mathcal{M}))$

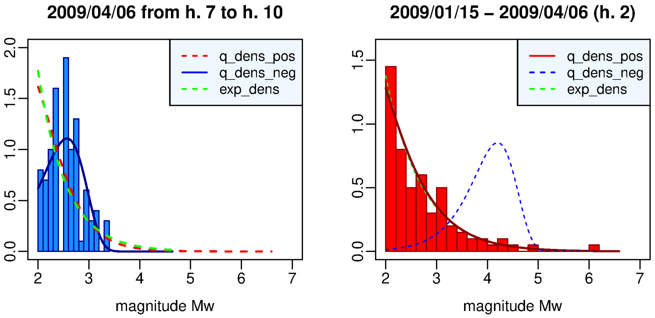


Figure 2. Time windows where (left) $q < 1$ and (right) $q > 1$ q-exponential distributions provide the "best" performance respectively

Which is the **best** q-exponential density between $q < 1$ and $q > 1$ over all the time windows?

- in 96% of intervals $\log \mathcal{L}(f_{q>1}(\mathcal{M})) > \log \mathcal{L}(f_{q<1}(\mathcal{M}))$, almost always showing **strong evidence** in favor of $f_{q>1}(M)$ according to the Jeffreys' scale, that is, $\Delta \log \mathcal{L}(f(\mathcal{M})) > 2.3026$
- in 70% of intervals $\log \mathcal{L}(f_{q>1}(\mathcal{M})) > \log \mathcal{L}(f_{q=1}(\mathcal{M}))$, of which 20% strongly and 46% substantially
- the 139 intervals in which $\log \mathcal{L}(f_{q<1}(\mathcal{M})) > \log \mathcal{L}(f_{q>1}(\mathcal{M}))$ are concentrated on the day of the mainshock

consequently, we show the values of the entropic index for the q-exponential distribution $f_{q>1}(M)$

Temporal variations of the Q entropic index[2] ($q > 1$)

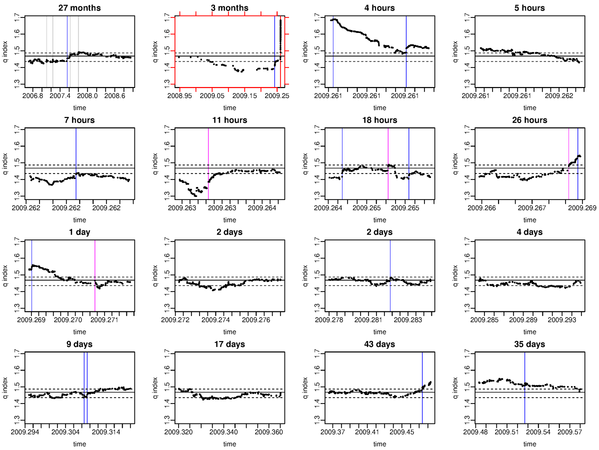


Figure 3. Vertical bars denote events of $M_w \in [4, 5)$, $M_w \in [5, 6)$, $M_w \geq 6$; horizontal, solid and dotted lines denote mean and quartiles of q values respectively.

[2] **in red the window covering the mainshock** $Mw\ 6.1$

Temporal variations of the Tsallis entropy for the $q > 1$ q-exponential distribution

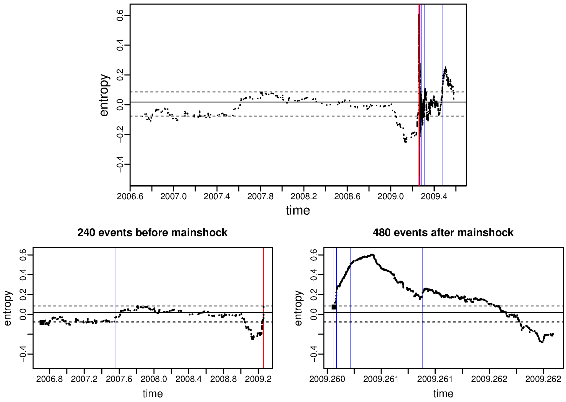


Figure 4. Solid and dotted lines denote mean and quartiles of q values respectively; (bottom) zoom around the date of L'Aquila shock.

Comments on the graphical results

- about **q-index**
  - background phase: - q is about constant for a long period (2005 - July 2007), - **slight increase since** $M_w\,4.1$ **on December 7, 2007** up to the end of 2008, - **gradual decrease** until March 28, 2009, then q **increases monotonically since March 30, 2009 (**$M_w\,4.1$**) up to** $q = 1.46$ **on the mainshock day**
  - aftershock sequence: q reaches the maximum ($q_{max} = 1.689$) within the 100 days following the mainshock
  - caution must be paid to the different time scales: the longer the trend is, the more reliable the indication which it provides
  - during the aftershock sequence the "average" decreasing trend is significant; looking at in detail the behavior - decrease pre-shock, increase after shock - repeats during the sequence
- about the **entropy**
  - analogously, the entropy shows a slight increase after $M_w\,4.1$ on December 7, 2007, it remains essentially constant for one year and a half, **it starts to decrease since January 19, 2009 up to March 30, 2009, from here it increases (energy dispersion) rapidly up to the maximum 0.605 three hours after the mainshock**
  - further significant increase after the pair $M_w\,4.2 - 4.4$ within June 22, 2009 - July 12. 2009 suggests that the sequence is in progress

Central Italy ... between L'Aquila and Amatrice (M_w 6 on 2016/08/24) earthquakes

Lat (42.3, 43.2) Long (12.7, 13.5) 2009/01/01 - 2016/10/30

$N = 5673$ earthquakes of $M_W \in (2.0, 6.5]$ drawn from the Italian Seismological Instrumental and Parametric Data-Base (ISIDe) considered as complete for $M_w \geq 2.0$

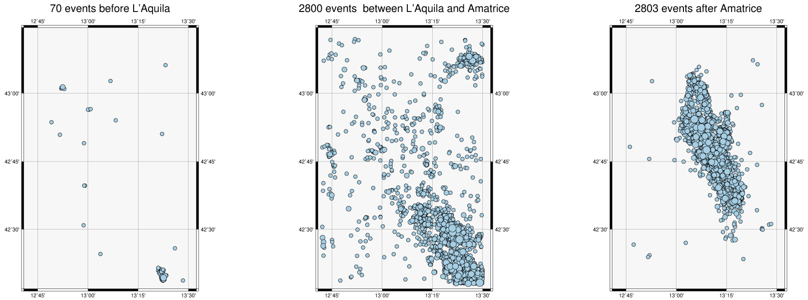


Figure 5. Spatial distribution of the seismicity, on the left, before L'Aquila shock, on the middle, between L'Aquila and Amatrice earthquakes, and, on the right, after Amatrice shock

Temporal variations of the Q entropic index[3] ($q > 1$)

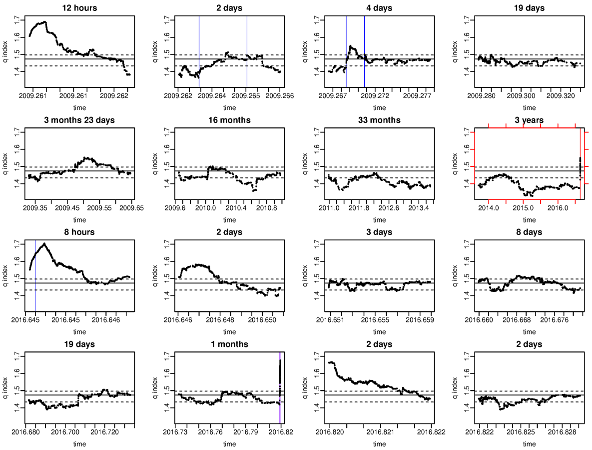


Figure 6. Vertical bars denote events of $M_w \in [4, 5)$, $M_w \in [5, 6)$, $M_w \geq 6$; horizontal, solid and dotted lines denote mean and quartiles of q values respectively.

[3] **in red the window covering the mainshock** *Mw* 6.0

Temporal variations of the Tsallis entropy for the $q > 1$ q-exponential distribution

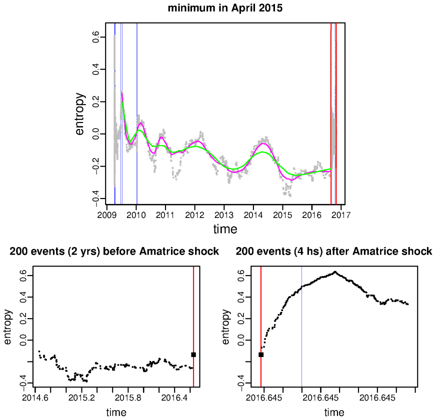


Figure 7. Smoothing of the entropy sequence, made through a locally weighted polynomial regression, highlights the **clear decrease** before Amatrice earthquake;
(bottom) zoom around the date of Amatrice shock.

Comments on the graphical results

- about **q-index**
  - since the beginning of 2015, the value of q is constantly and significantly less than the mean $q_{mean} = 1.474$
  - $q_{min} = 1.328$ on 2015/04/07
- about the **entropy**
  - the global trend of the entropy is decreasing since the L'Aquila sequence, and a further decrease starts since August 2014

Bayesian estimation: McMC methods

Let $\boxed{\boldsymbol{\eta} = (\boldsymbol{\theta}, \boldsymbol{\beta})}$ denote the parameter vector to estimate

in the Bayesian framework, quantities of interest (mean, variance, quantiles, ...) are obtained from the posterior distribution $\boxed{p(\boldsymbol{\eta}|\mathbf{data})}$ whose computation often involves intractable numerical integrations

to avoid these computational difficulties we resort to an iterative sampling Markov chain Monte Carlo method (**McMC**): the Metropolis-Hastings algorithm

Metropolis-Hastings algorithm

- *step 1* : select η_0 from $\pi_0(\eta)$ and set $i = 1$,
- *step 2* : draw a candidate $\tilde{\eta}$ from the proposal distribution $q(\eta \mid \eta_{i-1})$,
- *step 3* : compute the acceptance probability

$$\alpha(\tilde{\eta} \mid \eta_{i-1}) = min\left(1, \ \frac{\pi_0(\tilde{\eta})\, \mathcal{L}(data \mid \tilde{\eta})\, q(\eta_{i-1} \mid \tilde{\eta})}{\pi_0(\eta_{i-1})\, \mathcal{L}(data \mid \eta_{i-1})\, q(\tilde{\eta} \mid \eta_{i-1})}\right) \ ,$$

- *step 4* : accept $\tilde{\eta}$ as η_i with probability $\alpha(\tilde{\eta} \mid \eta_{i-1})$, set $\eta_i = \eta_{i-1}$ otherwise,
- *step 5* : repeat steps 2-4 a number R of times to get R draws from the posterior distribution, with optional burn-in and/or thinning.

References

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