Graphical illustrations of LEM topography using plots of curvature versus the steepness index

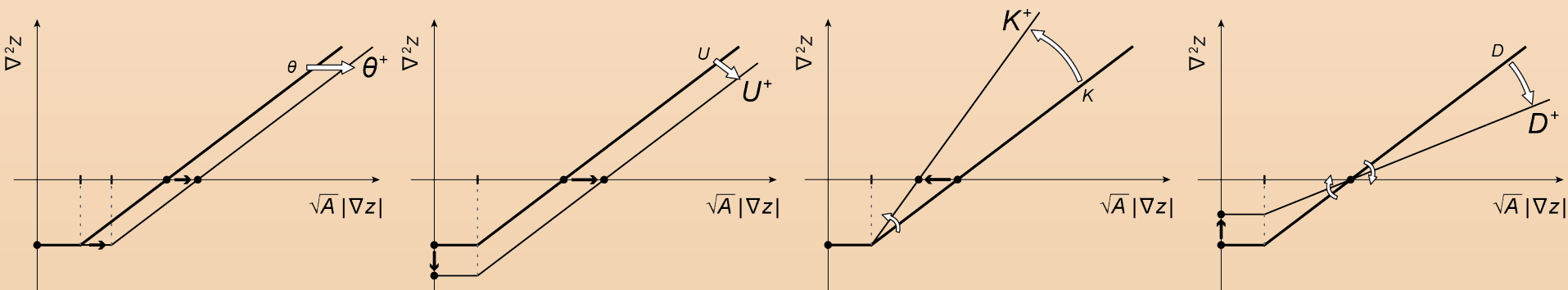## Nikos Theodoratos[1]* and James W. Kirchner[1,2]

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Executive summary

We examine stream-power incision and linear diffusion landscape evolution models (LEMs), with and without incision thresholds.

We present a steady-state relationship between curvature and the steepness index. This relationship plots as a straight line.

We view these lines as counterparts to the slope–area relationship for the case of landscapes with hillslope diffusion.

We illustrate the graphical explanatory power of curvature–steepness-index lines.

LEM without incision threshold
(e.g., Howard, 1994; Dietrich et al., 2003)

$$\partial z/\partial t = -K\sqrt{A}|\nabla z| + D\nabla^2 z + U$$

Uplift

Linear diffusion

Total rate of elevation change

Stream-power incision

Symbol	Description	Dimensions
z	elevation	H (height)
t	time	T (time)
A	drainage area	L^2 (L: length)
$|\nabla z|$	topographic slope	$H\,L^{-1}$
$\nabla^2 z$	Laplacian curvature	$H\,L^{-2}$
K	incision coefficient	T^{-1}
D	diffusion coefficient	$L^2\,T^{-1}$
U	uplift rate	$H\,T^{-1}$

In steady state ($\partial z/\partial t = 0$), we rearrange the equation as:
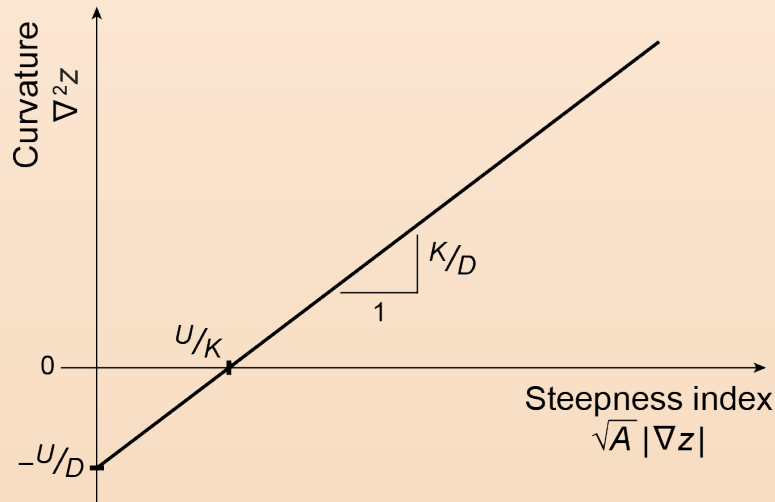
$$\nabla^2 z = (K/D)\sqrt{A}|\nabla z| - U/D$$

$\nabla^2 z$: Curvature
$\sqrt{A}|\nabla z|$: Steepness index (e.g., Whipple, 2001)

- The curvature–steepness-index relationship plots as a straight line with slope K/D.

- It is a testable prediction for landscapes that follow this LEM (e.g., Perron et al., 2009).

- We view it as a counterpart to the slope–area relationship, which is NOT a power law in landscapes with diffusion (e.g., Howard, 1994).

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Curvature–steepness-index line

$$\nabla^2 z = (K/D)\sqrt{A}|\nabla z| - U/D$$

The slope K/D, and the intercepts U/K and U/D of the curvature–steepness-index line can be expressed in terms of characteristic scales.

Given their dimensions ($[K]=T^{-1}$, $[D]=L^2\,T^{-1}$, $[U]=H\,T^{-1}$), parameters can be combined to define (Theodoratos et al., 2018):
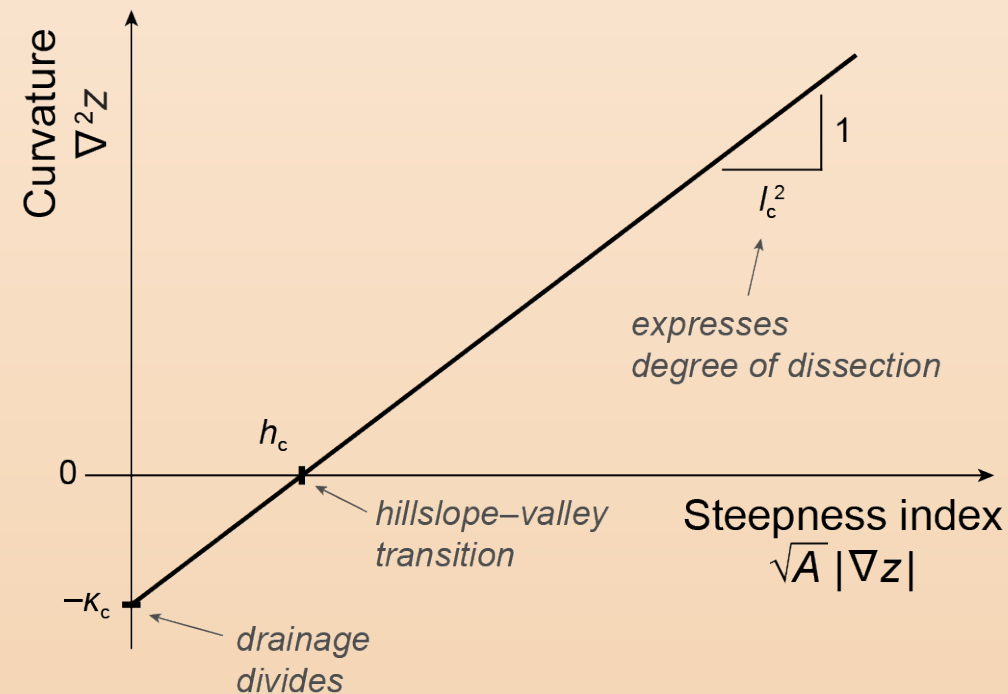
a characteristic length $\qquad l_c = \sqrt{D/K}$

a characteristic height $\qquad h_c = U/K$

a characteristic curvature $\quad \kappa_c = U/D = h_c/l_c^2$

Thus, the curvature–steepness-index relationship becomes:

$$\nabla^2 z = (1/l_c^2)\sqrt{A}|\nabla z| - \kappa_c$$
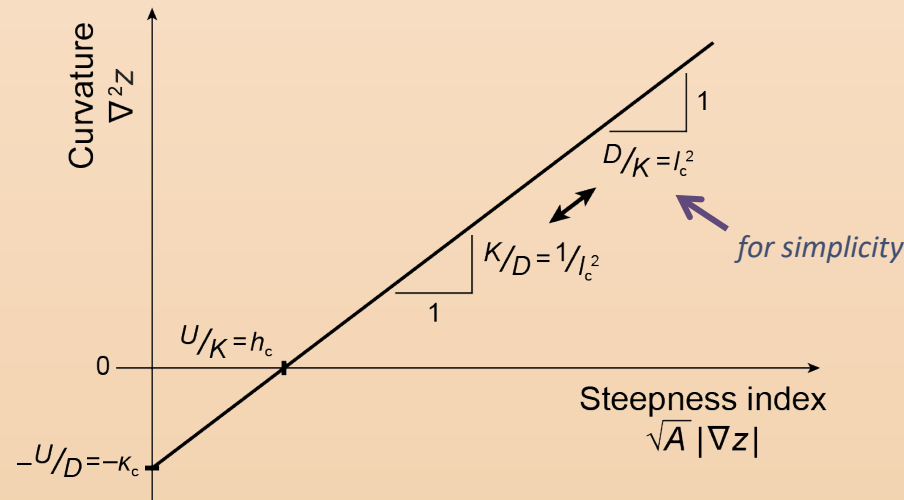
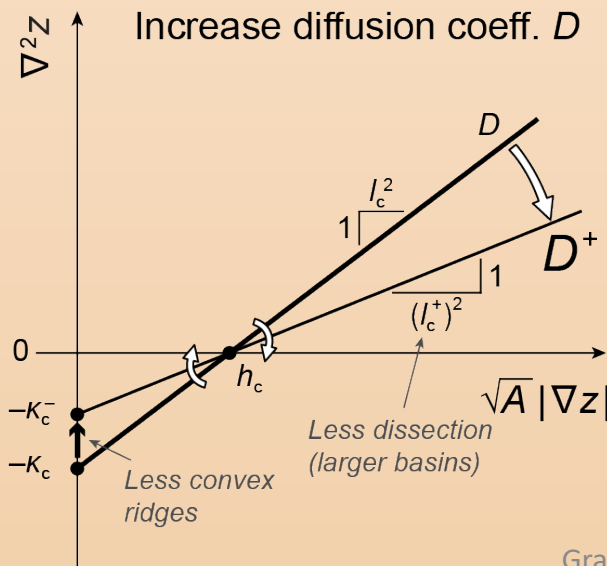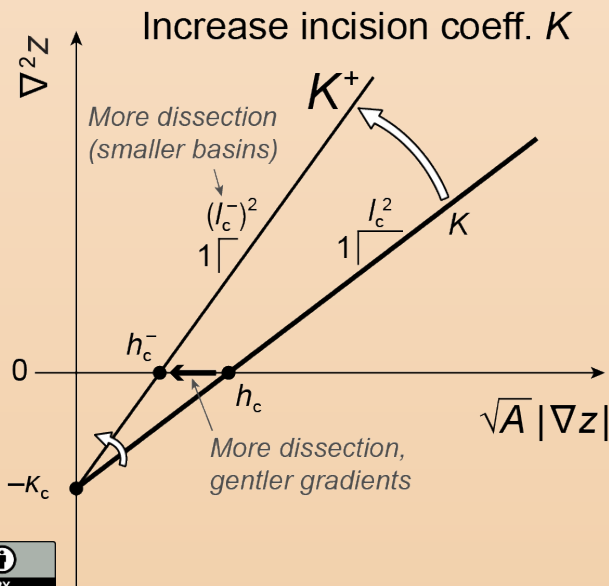# Landscape properties and points of special interest, visualized by the curvature–steepness-index line

- The slope of the curvature–steepness-index line depends on l_c, which characterizes the scales of landscape dissection (e.g., Perron et al, 2008).

- The intercept $\sqrt{A}|\nabla z| = h_c$ gives the steepness index at points with $\nabla^2 z = 0$, which define hillslope–valley transitions (e.g., Howard, 1994).

- The intercept $\nabla^2 z = -\kappa_c$ gives the steady-state curvature of ridges and drainage divides (see also Roering et al., 2007; Perron et al., 2009).

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Response of topography to parameter changes, visualized by curvature–steepness-index lines

The slope and intercepts of the curvature–steepness-index line depend on the characteristic scales l_c, h_c, and κ_c.

These scales are defined in terms of the LEM parameters K, D, and U.

Therefore, curvature–steepness-index lines can readily visualize topographic changes due to parameter changes.

Increase uplift rate U

$\nabla^2 z$

U

U^+

$1 \quad l_c^2$

$\dfrac{1}{l_c^2} \quad 1$

No change in dissection

0

h_c

h_c^+

$\sqrt{A}\,|\nabla z|$

Steeper gradient

$-\kappa_c$

$-\kappa_c^+$

More convex ridges

Increase incision coeff. K

$\nabla^2 z$

K^+

More dissection (smaller basins)

$(l_c^-)^2$

1

l_c^2

1

K

h_c^-

0

h_c

$\sqrt{A}\,|\nabla z|$

More dissection, gentler gradients

$-\kappa_c$

Increase diffusion coeff. D

$\nabla^2 z$

D

l_c^2

1

D^+

1

$(l_c^+)^2$

0

h_c

$\sqrt{A}\,|\nabla z|$

$-\kappa_c^-$

Less dissection (larger basins)

$-\kappa_c$

Less convex ridges
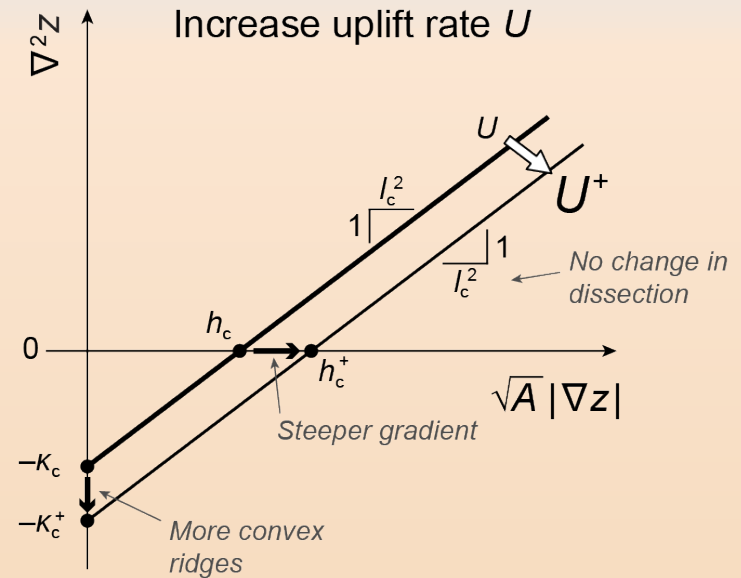
Adding an incision threshold to the LEM

The governing equation becomes (e.g., Perron et al., 2008):

$$\partial z/\partial t = \begin{cases} D\nabla^2 z + U \; , & \sqrt{A}|\nabla z| \leq \theta \\ -K(\sqrt{A}|\nabla z| - \theta) + D\nabla^2 z + U \; , & \sqrt{A}|\nabla z| > \theta \end{cases} \; ,$$

where θ is the incision threshold, which has dimensions of H.

The curvature–steepness-index relationship becomes:

$$\begin{cases} \nabla^2 z = -\kappa_c \; , & \sqrt{A}|\nabla z| \leq \theta \\ \nabla^2 z = (1/l_c^2)\sqrt{A}|\nabla z| - (1 + N_\theta)\kappa_c \; , & \sqrt{A}|\nabla z| > \theta \end{cases} \; ,$$
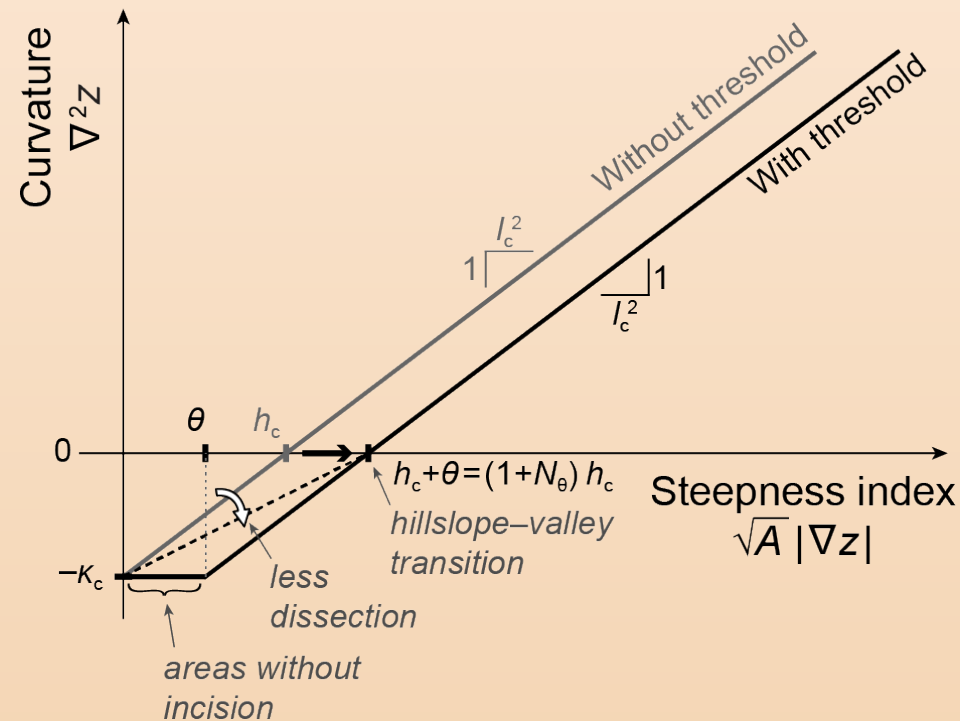
where N_θ is a dimensionless incision-threshold number that quantifies the relative importance of θ:

$$N_\theta = K\theta/U$$

The curvature–steepness-index relationship with incision threshold plots as a line with two segments, a horizontal and an inclined.

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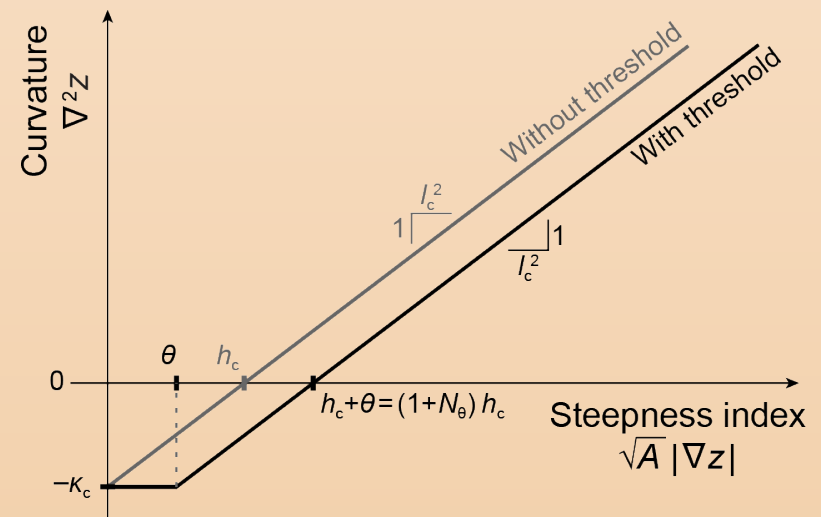
Effects of incision threshold on landscapes, visualized by the curvature–steepness-index line
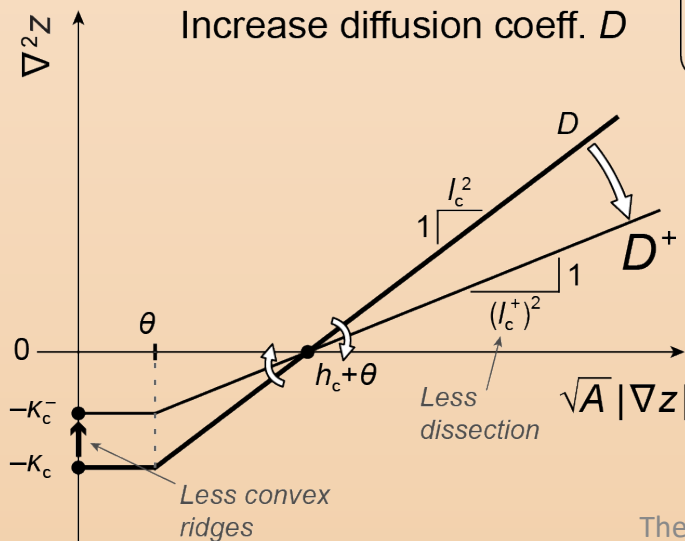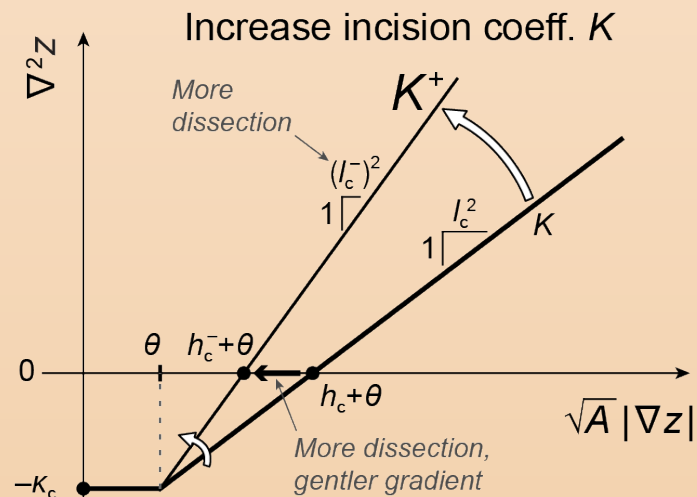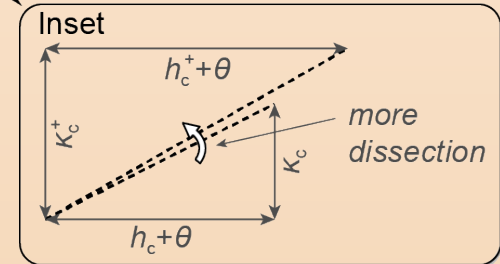

- The horizontal segment describes areas where incision is fully suppressed by the threshold. Their curvature is the same as that of drainage divides.
Therefore, hillslopes become more convex because bigger portions of them have the maximum convexity (e.g., Howard, 1994).

- Hillslope–valley transitions occur at larger drainage areas and/or steeper slopes (e.g., Montgomery and Dietrich, 1992; Perron et al., 2008).

- The slope of the black dashed line, which connects the intercepts $-\kappa_c$ and $(1 + N_\theta)h_c$, can be shown to characterize the scales of dissection. Therefore, the difference in the slopes of the black dashed line and the gray curvature–steepness-index line expresses graphically the effect of the incision threshold on landscape dissection.

Response of topography to parameter changes, visualized by curvature–steepness-index lines

Increase inc. threshold θ

$\nabla^2 z$

$\sqrt{A}\,|\nabla z|$

$\theta \Rightarrow \theta^+$

$\frac{l_c^2}{1}$

$\frac{1}{l_c^2}$

$\theta \quad \theta^+ \quad h_c+\theta$

$h_c+\theta^+$

$-\kappa_c$

Less dissection, steeper gradient

Wider extent of max. convexity

Increase uplift rate U

$\nabla^2 z$

$\sqrt{A}\,|\nabla z|$

$U \Rightarrow U^+$

$\frac{l_c^2}{1}$

$\frac{1}{l_c^2}$

$\theta \quad h_c+\theta$

$h_c^++\theta$

$-\kappa_c$

$-\kappa_c^+$

Steeper gradient

More convex ridges

Inset

$h_c^++\theta$

κ_c^+

more dissection

κ_c

$h_c+\theta$

Increase incision coeff. K

$\nabla^2 z$

$\sqrt{A}\,|\nabla z|$

More dissection

K^+

$(l_c^-)^2$

$\frac{l_c^2}{1}$

K

$\theta \quad h_c^-+\theta$

$h_c+\theta$

$-\kappa_c$

More dissection, gentler gradient

Increase diffusion coeff. D

$\nabla^2 z$

$\sqrt{A}\,|\nabla z|$

$D \Rightarrow D^+$

$\frac{l_c^2}{1}$

$\frac{1}{(l_c^+)^2}$

$\theta \quad h_c+\theta$

$-\kappa_c^-$

$-\kappa_c$

Less dissection

Less convex ridges

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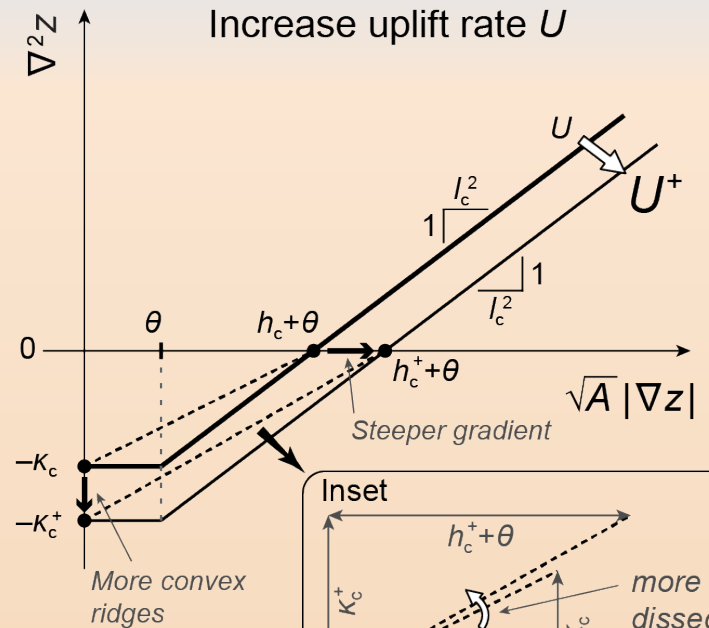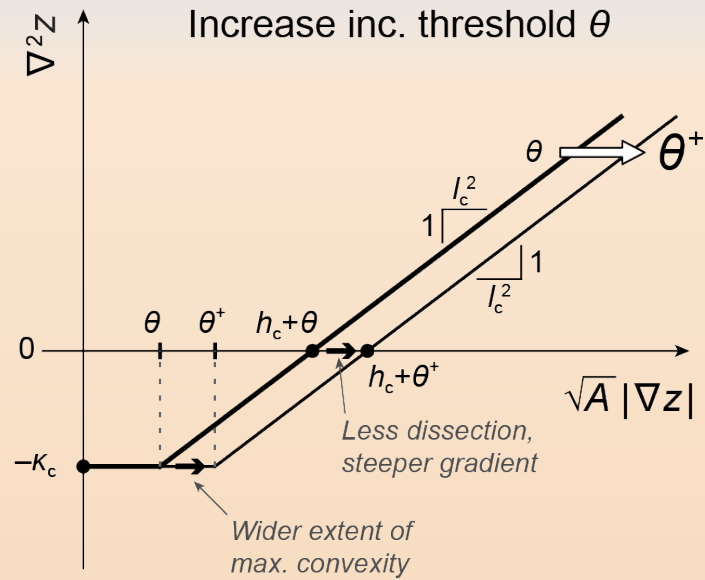
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Presentation based on:

Theodoratos, N., and Kirchner, J. W.: Graphically interpreting how incision thresholds influence topographic and scaling properties of modeled landscapes, *Manuscript in preparation*, 2020.