A Multiple 1D Earth Approach (M1DEA) to account for lateral viscosity variations in solutions of the sea level equation:

An application for glacial isostatic adjustment by Antarctic deglaciation

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All in brief

The problem:

Pseudo-spectral sea level equation:

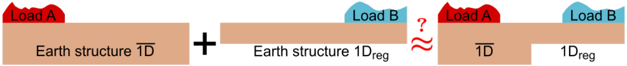
- Global 1D Earth structure ⇒ **No lateral variations!**
- **Computationally cheaper** than spatial solutions!

The goal:

A method that combines both, **lateral variations** in a **computationally cheap** way!

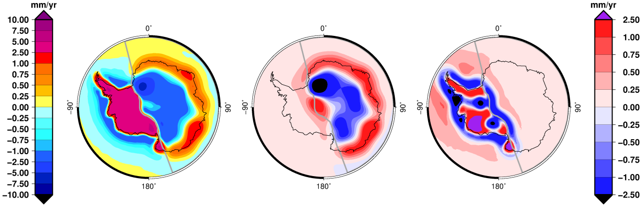
The idea: Multiple 1D Earth Approach (M1DEA) → Sec.2

Locally-appropriate 1D Earth structures for each separately computed load component.



Our tests: → Sec.4.1,4.2

Impact of viscosity variations beneath West or East Antarctica on the GIA rates outside the region they represent:



Our findings:

The **M1DEA can account for lateral variations** in Earth structure **if**...
→ Sec.5

All details in:

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1. Motivation

It is widely established to predict sea level change (SLC) and glacial isostatic adjustment (GIA) of melting events (Fig.1) by the pseudo-spectral form of the sea level equation (SLE) [1].

Problem: The pseudo-spectral SLE depends on two physical quantities:

- (Ice) load on the surface
- Global spherically-symmetric (1D) visco-elastic Earth structure
  ⇒ **No lateral variations in Earth structure!**

Idea: A **Multiple 1D Earth Approach** (M1DEA) that computes the SLC and GIA responses to deglaciation in each regional basin separately, and sums the contributions. Thereby, each contribution utilize its **own 1D Earth structure** that is **locally-appropriate** for the represented basin.

Our benchmark: We asses the M1DEA by analyzing the **impact of viscosity variations** on the GIA rates **outside the region they represent** using **East and West Antarctica** as **large-scale load components** that differ significantly in their lithospheric [2] and viscosity structures [3].

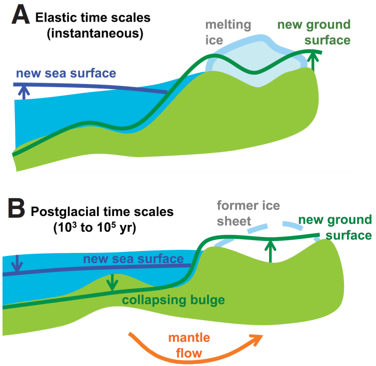

Figure 1: Basic contributions to SLC and GIA in the SLE [4, 5]:

$$\boxed{S = \text{eustatic} + \text{elastic} + \text{viscous}}$$

(A) The elastic (instantaneous) response, **(B)** the viscous response that crucially depends on the mantle viscosity below the load. From Conrad, 2013 [6].

2. Multiple 1D Earth Approach (M1DEA)

- Pseudo-spectral SLE: Total SLC or uplift rates ψ are the superposition of the rates from all regional ice components L_{reg} on the same global 1D Earth model $\overline{1\mathrm{D}}$ (Fig.2(a)):

$$\psi_{\overline{1\mathrm{D}}}\Big(\sum_{\mathrm{reg}} L_{\mathrm{reg}}\Big) = \sum_{\mathrm{reg}} \psi_{\overline{1\mathrm{D}}}(L_{\mathrm{reg}})$$

- M1DEA: locally-appropriate 1D Earth structure $1\mathrm{D}_{\mathrm{reg}}$ for each regional component (Fig.2(b)):

$$\psi_{\mathrm{M1DEA}}\Big(\sum_{\mathrm{reg}} L_{\mathrm{reg}}\Big) = \sum_{\mathrm{reg}} \psi_{1\mathrm{D}_{\mathrm{reg}}}(L_{\mathrm{reg}})$$

- Uplift rates in each region are dominated by the local loading (if present), the effect of distant loading is generally small (Fig.2(c)).

$\Rightarrow$ Earth structure variations in distant regions should have a minor effects on local rates (Fig.2(c)).

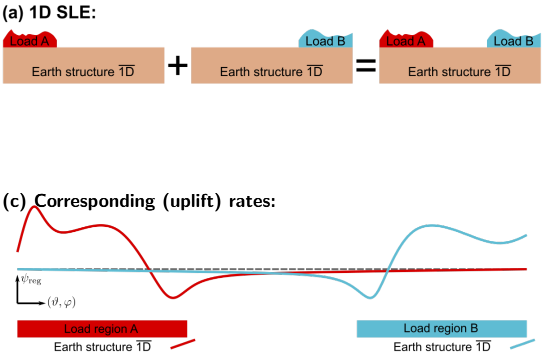

Figure 2: (a) Superposition of load components A (red) and B (blue) each on the same Earth structure $\overline{1D}$, **(b)** Superposition of load A (red) and B (blue) on different Earth structures $\overline{1D}$ and $1D_{\mathrm{reg}}$, **(c)** Schematic rates (e.g., uplift) induced by load A on the average Earth $\overline{1D}$ (solid red), load B on the average Earth $\overline{1D}$ (solid blue), and load B on the regional Earth $1D_{\mathrm{reg}}$ (dashed blue).

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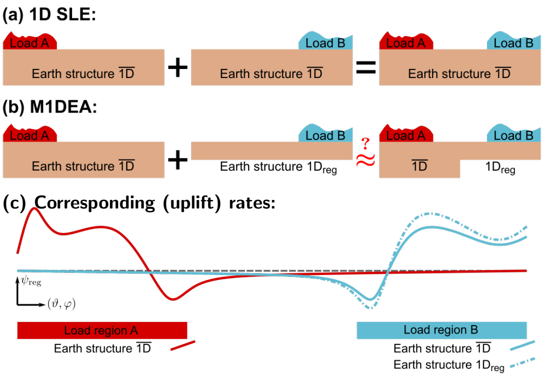

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3. Test cases - Ice load

Long-term loading scenario

- Modeling one glacial cycle using the W12 loading scenario [7, 8] (corrected for marine-grounded ice).
- Split into two large-scale components for East Antarctica $W12_{\mathrm{east}}$ and West Antarctica $W12_{\mathrm{west}}$

Loading scenario for recent ice loss

- Century of uniform ice loss ($165\ \frac{\mathrm{Gt}}{\mathrm{yr}}$) in West Antarctica

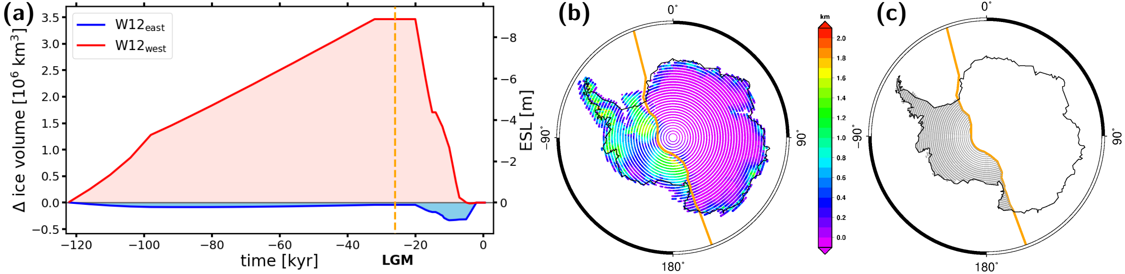


Figure 3: (a) Ice volume and eustatic sea level (ESL) of the loading scenario W12 divided into its East Antarctic (blue) and West Antarctic component (red). **(b)** Ice extent of the W12 loading scenario at LGM relative to the present state. The orange line shows the border between the East and West Antarctic components. **(c)** Ice extend of the recent loading scenario $WANT_{100}$, additionally applied in Sec.4.2.

3. Test cases - Earth structures

Sensitivity analysis (Sec.4.1)

Loading: W12 only
Earth structures: Test range (Fig.4)

- 6 layers: Core, LM, TZ, DUM, SUM, EL
- 4 variables: thickness of elastic lithosphere (EL), viscosities of upper mantle layers (SUM, DUM, TZ)

⇒ 192 possible combinations (gray dots, Fig.4)

M1DEA Tests (Sec.4.2)

Loading: W12 + recent ice loss $WANT_{100}$
Earth structures:

- $W12_{earth}$ (blue line, Fig.4): Rigid Earth structure for East Antarctica (based on [8])
- BAR_{earth} (red line, Fig.4): Low-viscous Earth structure for West Antarctica (based on [9])

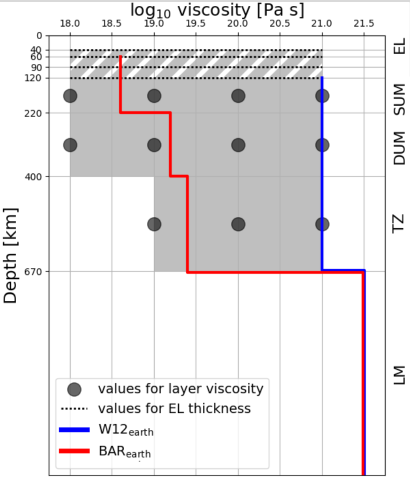


Figure 4: Applied viscosity profiles: Colored lines show the profiles for the M1DEA tests (Sec.4.2). Dark gray dots represent possible values for viscosities of the upper mantle layers in the sensitivity analysis (Sec.4.1): The gray and white shaded range can be viscous or elastic depending on the chosen EL thickness.

4.1 Sensitivity analysis: Definitions for analysis

Specific impact:

Assume ice **load** in region **B** with an **Earth structure j**, and ice **load** in region **A** (outside B) with an **Earth structure i**. The impact caused by a **variation of j** on the predicted uplift rates u is the difference Δu between the predicted M1DEA rates and the predictions using only Earth structure i:

$$\Delta u = u_{\mathrm{M1DEA}} - u_{\overline{\mathrm{1D}}} = (u_{\mathrm{A},i} + u_{\mathrm{B},j}) - (u_{\mathrm{A},i} + u_{\mathrm{B},i}) = u_{\mathrm{B},i} - u_{\mathrm{B},j} \tag{1}$$

Mean Impact:

Mean impact expected in region A due to any variation of the Earth structure in region B is simply the mean of all tested "specific impacts" (Eq.1):

$$\overline{\Delta u} = \frac{1}{N_{\mathrm{mod}}} \sum_{i,j} |u_{\mathrm{B},i} - u_{\mathrm{B},j}| \tag{2}$$

Interpretation:

Small differences Δu in region A (area of interest)
$\Rightarrow$ low impact of the Earth structure j in region B on the rates in region A
$\Rightarrow$ **good applicability for the M1DEA** in this specific combination i, j.

4.1 Sensitivity analysis: Impact of variations in Earth structure

Setup for analysis:

Load components: $\mathrm{W12}_{\mathrm{east}}$, $\mathrm{W12}_{\mathrm{west}}$

- EL variations (Fig.5(a),(b)):
  - $\mathrm{EL}_{\mathrm{EANT}} = 120$ km, $\mathrm{EL}_{\mathrm{WANT}} = 60$ km
  - 48 Viscosity settings: $N_{\mathrm{mod}} = 96$

$\Rightarrow$ **Neglectable** impacts in the connected neighboring region

- Viscosity variations (Fig.5(c),(d)):
  - EL$= 120$ km, fixed EANT Earth: $\mu_{\mathrm{SUM,DUM,TZ}} = 10^{21}$ Pa s
  - Viscosity combinations: $N_{\mathrm{mod}} = 47$

$\Rightarrow$ **Larger** impacts in the neighboring region
- especially in East Antarctica (Fig.5(d))!

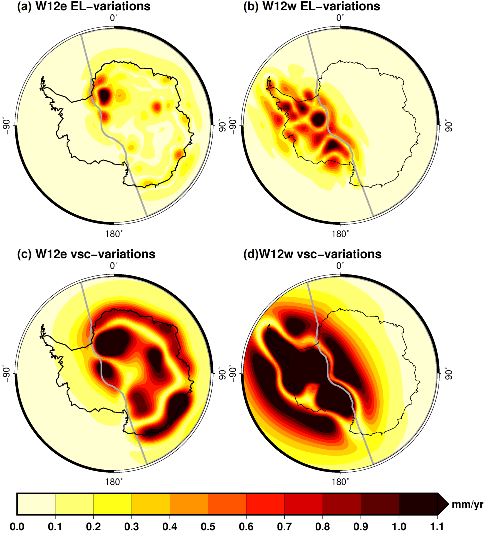


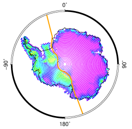
Figure 5: **(right)** Mean impact $\overline{\Delta u}$ (Eq.2) of Earth structure variations on present-day GIA uplift rates, for the components of the W12 loading scenario. **(a)** EL variations for $\mathrm{W12}_{\mathrm{east}}$, **(b)** EL variations for $\mathrm{W12}_{\mathrm{west}}$, **(c)** viscosity variations for $\mathrm{W12}_{\mathrm{east}}$, **(d)** viscosity variations for $\mathrm{W12}_{\mathrm{west}}$. The gray line marks the separation of the East and West Antarctic regions.

4.2 M1DEA test 1: Strong contrast & long-term load

Setup for test 1:

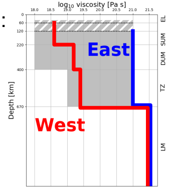
Load components: (Sec.3, Fig.3)

- Long-term loading: $\mathrm{W12}_{\mathrm{east,west}}$



Earth structures: (Sec.3, Fig.4)

- West Antarctica: low-viscous ($\mathrm{BAR}_{\mathrm{earth}}$)
- East Antarctica: rigid ($\mathrm{W12}_{\mathrm{earth}}$)



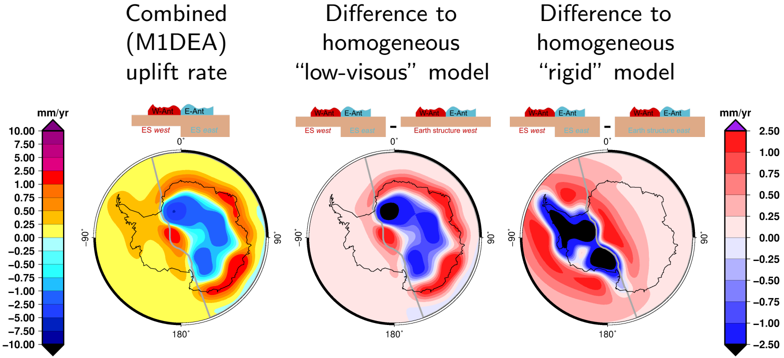


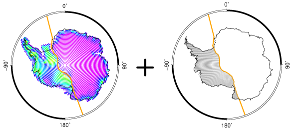
Figure 6: M1DEA applied for Antarctic deglaciation using a strong lateral contrast (rigid East Antarctic structure $\mathrm{W12}_{\mathrm{earth}}$, and low-viscosity West Antarctic structure $\mathrm{BAR}_{\mathrm{earth}}$) with **only long-term load** ($\mathrm{W12}_{\mathrm{east,west}}$) considered.

4.2 M1DEA test 2: Strong contrast & long-term + recent load

Setup for test 2:

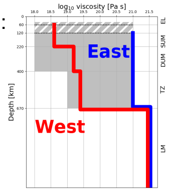
Load components: (Sec.3, Fig.3)

- Long-term loading: $W12_{east,west}$
- Recent loading: $WANT_{100}$



Earth structures: (Sec.3, Fig.4)

- West Antarctica: low-viscous (BAR_{earth})
- East Antarctica: rigid ($W12_{earth}$)



Combined (M1DEA) uplift rate | Difference to homogeneous "low-visous" model | Difference to homogeneous "rigid" model

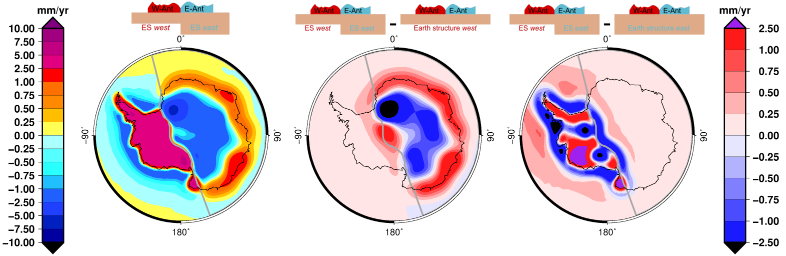


Figure 7: M1DEA applied for Antarctic deglaciation using a strong lateral contrast (rigid East Antarctic structure $W12_{earth}$, and low-viscosity West Antarctic structure BAR_{earth}) with **long-term load** ($W12_{east,west}$) **AND recent load** ($WANT_{100}$) considered.

Observation:
The inclusion of **recent ice loss** in the **low-viscous region** drastically **reduces the impact** outside that region - even for a strong contrast!

5. Conclusions

The "Multiple 1D Earth Approach" (M1DEA) can account for

- lateral variations of the elastic lithosphere.
- lateral variations of the upper mantle viscosities, **if**
  - → each regional load component includes deglaciation associated with its individual time scale of relaxation.
  - → the load components maintain small viscosity contrasts between Earth structures of neighboring regions. (e.g., by adaptive subdivision of load components in areas of strong viscosity variations!)

⇒ Further tests of M1DEA against full 3D finite element models to compare **accuracy** and **computation time**.

Detailed description and discussion can be found in:

R. Hartmann, J. Ebbing, C.P. Conrad,
A Multiple 1D Earth Approach (M1DEA) to account for lateral viscosity variations in solutions of the sea level equation,
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Acknowledgements

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References

[1] J. X. Mitrovica and W. R. Peltier, “On postglacial geoid subsidence over the equatorial oceans,” *Journal of Geophysical Research: Solid Earth*, vol. 96, pp. 20053–20071, Nov. 1991.

[2] M. An, D. A. Wiens, Y. Zhao, M. Feng, A. A. Nyblade, M. Kanao, Y. Li, A. Maggi, and J.-J. Lévêque, “S-velocity model and inferred Moho topography beneath the Antarctic Plate from Rayleigh waves: Antarctic S-velocities and Moho,” *Journal of Geophysical Research: Solid Earth*, vol. 120, pp. 359–383, Jan. 2015.

[3] W. van der Wal, P. L. Whitehouse, and E. J. Schrama, “Effect of GIA models with 3d composite mantle viscosity on GRACE mass balance estimates for Antarctica,” *Earth and Planetary Science Letters*, vol. 414, pp. 134–143, Mar. 2015.

[4] W. E. Farrell and J. A. Clark, “On Postglacial Sea Level,” *Geophysical Journal of the Royal Astronomical Society*, vol. 46, no. 3, pp. 647–667, 1976.

[5] G. A. Milne and J. X. Mitrovica, “Postglacial sea-level change on a rotating Earth,” *Geophysical Journal International*, vol. 133, pp. 1–19, Apr. 1998.

[6] C. P. Conrad, “The solid Earth’s influence on sea level,” *Geological Society of America Bulletin*, vol. 125, pp. 1027–1052, July 2013.

[7] P. L. Whitehouse, M. J. Bentley, and A. M. Le Brocq, “A deglacial model for Antarctica: geological constraints and glaciological modelling as a basis for a new model of Antarctic glacial isostatic adjustment,” *Quaternary Science Reviews*, vol. 32, pp. 1–24, Jan. 2012.

[8] P. L. Whitehouse, M. J. Bentley, G. A. Milne, M. A. King, and I. D. Thomas, “A new glacial isostatic adjustment model for Antarctica: calibrated and tested using observations of relative sea-level change and present-day uplift rates: A new GIA model for Antarctica,” *Geophysical Journal International*, vol. 190, pp. 1464–1482, Sept. 2012.

[9] V. R. Barletta, M. Bevis, B. E. Smith, T. Wilson, A. Brown, A. Bordoni, M. Willis, S. A. Khan, M. Rovira-Navarro, I. Dalziel, R. Smalley, E. Kendrick, S. Konfal, D. J. Caccamise, R. C. Aster, A. Nyblade, and D. A. Wiens, “Observed rapid bedrock uplift in Amundsen Sea Embayment promotes ice-sheet stability,” *Science*, vol. 360, pp. 1335–1339, June 2018.