Assessment of Tscherning-Rapp covariance in Earth gravity modeling using gravity gradient and GPS/leveling observations

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Introduction

Newly invented resources of data mainly help to achieve the more accurate evaluation of unknown parameters using information fusion techniques. As for Earth's gravity field matters, it is plausible to use such techniques for a better modeling since different geodetic data have been provided by different spaceborne missions like CHAMP, GRACE, and GOCE along with ground-based ones. Least Squares Collocation (LSC) is one of these methods which makes it possible to combine these kinds of data via covariance (COV) function to model the Earth's gravity field and more practically the geoid in unpresented accuracy. The precondition for getting a proper result in using LSC is the normalization of the input data. To do this, it is common to remove the topography portion and the long-wavelength info of the gravity field from the observations within the Remove-Compute-Restore (RCR) procedure (Sansò and Sideris, 2013). So, in this study, we analyze the COV (as a key stage in LSC) improvement and information assimilation technique on residual gravity data to improve geoid modeling.

Methodology and Data

Another prerequisite of using LSC is finding a functional relation between unknown parameters and anomalous potential (T). Therefore, Tscherning-Rapp (TR1974) analytical COV model for T between two points namely P and Q has been considered as

$$K(r_P, r_Q', \psi) = \alpha \sum_{n=2}^{N} \left(\frac{R_E^2}{r_P r_Q'}\right)^{n+1} k_n^{\sigma} P_n(\cos\psi) + \sum_{n=N+1}^{\infty} \left(\frac{R_B^2}{r_P r_Q'}\right)^{n+1} \frac{A}{(n-1)(n-2)(n+B)} P_n(\cos\psi),$$

where this COV model is a function of the spherical distance between two points (ψ). r_P and r_Q' are the radii of the Earth in the points, R_E is the mean radius of the Earth equal to 6371 km, N is the maximum degree and order (D/O) of the global gravity model (GGM) that is removed from the data, P_n is the Legendre polynomial, R_B is the Bjerhammer sphere radius, A denotes the scale factor of the residual signal variance at higher degrees, α represents the scale factor of the GGM global error variance and k_n^{σ} is the error degree variance of the reference GGM coefficients. On the other hand, based on Ramouz et al. (2020), the accuracy of LSC is directly related to the ability to localize the COV function which itself depends on the data distribution and topography. Here, we have analyzed these factors, on GOCE gradient (T_{zz}), gravity anomaly (Δg), and GPS/Leveling (N^G) data by considering the various case studies with different data arrangements which are depicted in the following along with their statistics (Table 1). The statistics of removing GGM (EIGEN6C4 for Δg and N^G up to D/O 360, GOCE_TIM_RL05 for T_{zz} up to D/O 25) and RTM (SRTM1" for Δg and N^G, GEBCO for T_{zz}) on used data are reported in Table 2.

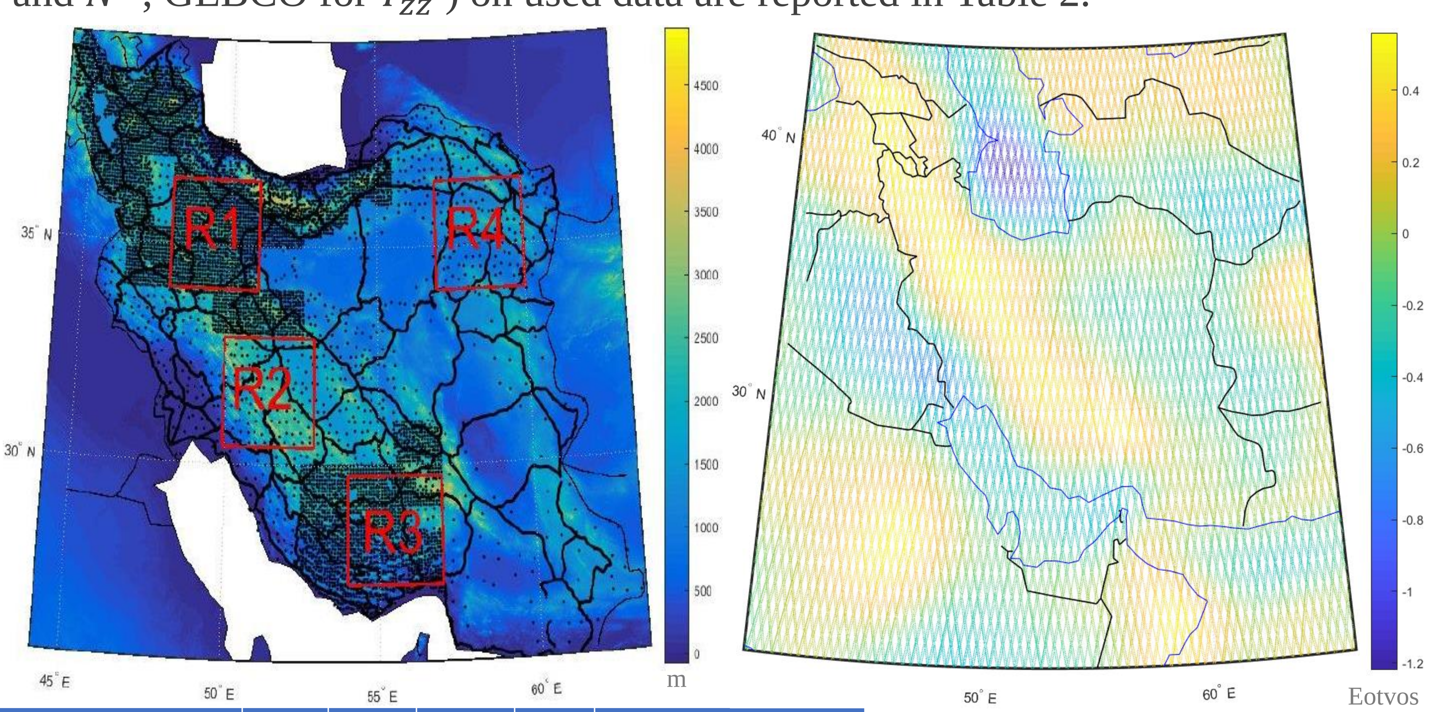

Fig. 1. Distribution of gravity data namely GPS/Leveling and gravity anomaly with the topography as background (left) and residual GOCE T_{zz} gradient (right) over Iran in a dot form.

Table 1. Region specifications and number of observations in each one.

Region		1	2	3	4	5 (Whole Iran)
Min_Phi		34.0	30.4	27.25	34.0	20.0
Max_phi		36.5	32.9	29.75	36.5	44.0
Min_Lam		48.0	49.9	54.0	57.0	40.0
Max_Lam		51.0	52.9	57.0	60.0	66.0
Number of observations	Δg	2062	808	1599	793	120630
	N^G	102	103	19	50	

Table 2. Percentage of removing GGM and RTM effects on the STD of the gravity anomalies, geoid heights and gravity gradient in each region.

Region	1				2				3				4				5 (Whole Iran)	
	Δg		N^G		Δg		N^G		Δg		N^G		Δg		N^G		T_{zz}	
Remove	mGal	%	m	%	mGal	%	m	%	mGal	%	m	%	mGal	%	m	%	Eotvos	%
GGM	6.4	18.9	2.2	85.1	1.6	4.3	2.9	89.6	39	61.1	6.9	95.6	4.5	14.9	1.9	86.3	0.1138	27.29
RTM	12.6	37.3	0.0	0.0	17.8	49	-0.1	-2.7	7.9	12.4	-0.1	-1.0	7.9	26.4	-0.0	-1.0	0.0312	7.480
GGM + RTM	19	56.2	2.2	85.1	19.4	53.3	2.8	86.9	46.9	73.5	6.82	94.6	12.3	41.3	1.9	84.9	0.1371	32.880

Table 3. Accuracy of the LSC gradient estimation before and after the covariance improvement in optimized grid size and sample interval.

	Before Improv	After Improv
Grid size	10 min	10 min
Sample interval	6.5 min	6.5 min
Mean (Eotvos)	0.000344	0.000306
STD (Eotvos)	0.014910	0.014904

Table 4. Accuracy of the LSC geoid estimation based on different COV models in each region.

	Uniform	Local		Uniform	Local	
		Simple	Improved		Simple	Improved
Region	1			2		
Mean	0.4	0.39	0.4	0.49	0.42	0.43
STD	0.255	0.257	0.255	0.158	0.158	0.143
Region	3			4		
Mean	0.5	0.5	0.5	0.5	0.51	0.5
STD	0.234	0.233	0.214	0.114	0.181	0.111

Table 5. Simple and improved combined COV parameters and their LSC geoid determination's statistics in each region.

Region	1		2		3		4	
strategy	Simple	Improved	Simple	Improved	Simple	Improved	Simple	Improved
Mean	0.40	0.40	0.40	0.41	0.53	0.53	0.50	0.49
STD	0.256	0.255	0.145	0.142	0.228	0.225	0.148	0.118
Distr. Topo.	Dense Smooth		Sparse Rough		Dense Rough		Sparse Smooth	

Results of Covariance Analysis

One way to construct such COV functions is involving two steps, first, calculation of an empirical COV function from residual observations and then fitting the TR1974 model to it for finding the optimized values for α, R_B, and A. To this aim, we have considered five regions (R1 to R5) with different data distribution and topography (Fig. 2) and two strategies: First, One Quantity-Derived COV (OQDC) (T_{zz}-derived COV for Tzz estimation and Δg-derived COV for geoid height (N) estimation). Second, Combined Quantities-Derived COV (CQDC) which includes Δg and N^G to derive COV for N estimation. It is worth mentioning that the work area in case of using T_{zz} data is the whole of Iran.

For T_{zz} as it is discussed in HH Shali et al. (2019), the residual data were divided into two datasets namely, observations and control points. The observation points served as input data within the LSC procedure using the TR1974 COV model and the control points used to evaluate the accuracy of the LSC gravity gradient estimation. It was resulted that the TR1974 COV model has a better performance for T_{zz} in comparison with terrestrial gravity anomalies. And the implementation of COV improvement could not enhance the result of T_{zz} modeling remarkably at the satellite altitude. Moreover, in satellite data usage, the α parameter has no impact either in theory or functionally in computation. Results of the LSC gradient estimation before and after the covariance improvement in optimized grid size and sample interval are reported in Table 3.

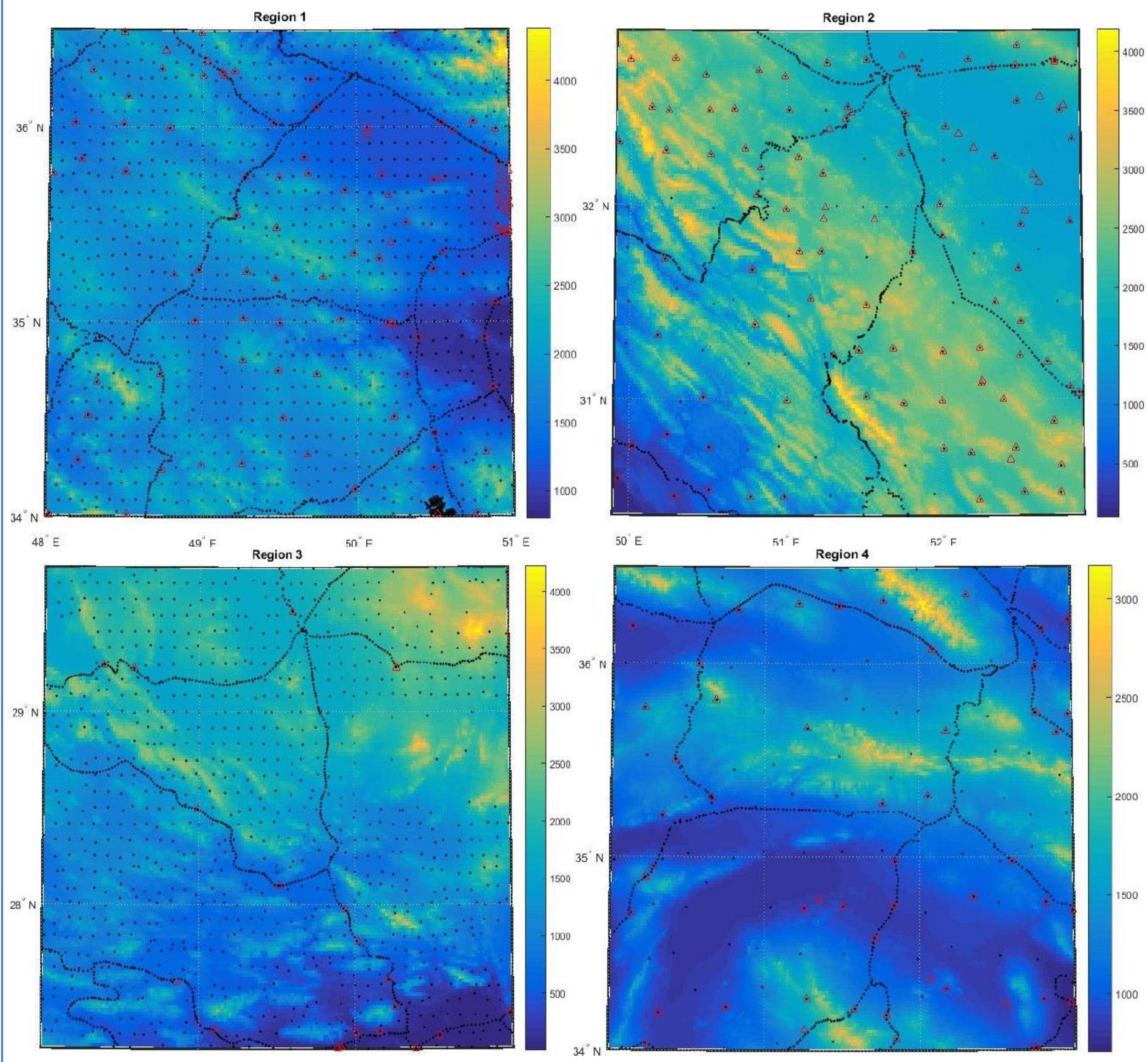

Fig. 2. Observations (black dots) and control points (red triangles) in A) R1, B) R2, C) R3 and D) R4. Topography as background (m).

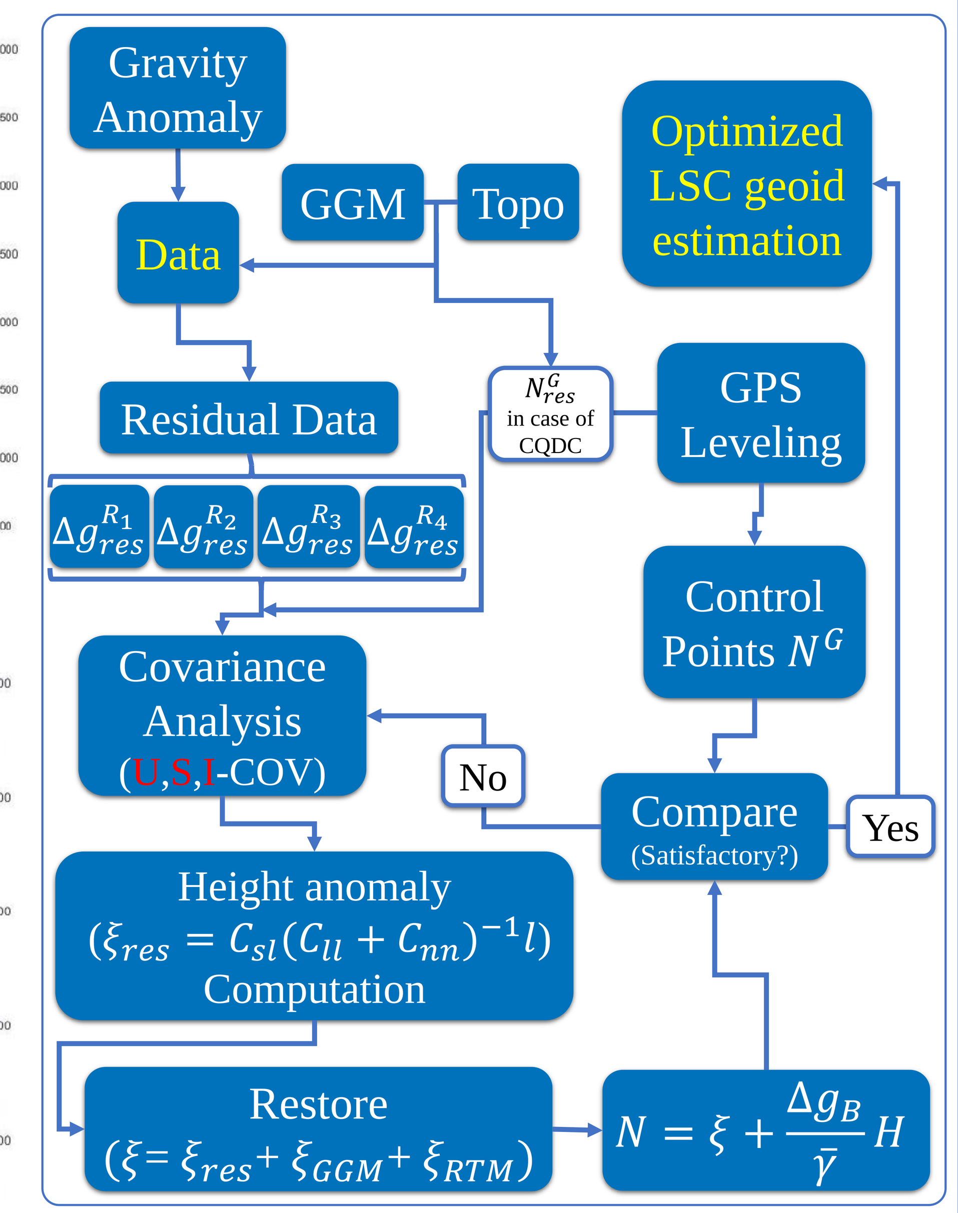

Fig. 3. Flow diagram of recursive concept to find the best estimate of the Problem

As it is depicted in Fig. 2, residual gravity anomalies (in OQDC case) used as input data for COV estimation and GPS/Leveling data used as control points to asses the LSC geoid estimation and then COV quality. To this end, different solutions are considered regarding COV modeling namely, Uniform-COV (U_COV) by using all data over Iran to construct COV function, Simple-COV (S_COV) through building the COV model using the data provided locally in each region, and Improved-COV (I_COV) as like as S_COV, but improve the estimation of the COV parameters by means of a recursive procedure. The same theory was applied for CQDC case, except using both residual gravity anomaly and GPS/Leveling as input data for COV estimation. The whole procedure is depicted in the flow diagram of Fig. 3 and the results of these cases are reported in Table 4, and 5.

Conclusion and Future work

In this study, we analyzed the TR1974 COV model with different strategies and case studies using gravity gradient, gravity anomaly and GPS/Leveling data for gravity modeling. According to Table 2, removing GGM and RTM effects from Δg reduce it up to averagely 28.9 and 29.9 percent respectively. On the other hand, GGM and RTM effects for N^G reduction reach up to averagely 87.7 and -1.3 percent respectively which means that a big part of the N^G signal's energy is in its long-wavelength contribution. Therefore, removing global effect from N^G has significant a influence on the accuracy of the gravity modeling. While, topographic effect has considerable contribution in Δg signal and required detailed analysis through gravity reduction. With respect to T_{zz} reduction, one should note that GOCE observations cannot include all the short-wavelengths of T_{zz} signal of the Earth's gravity field. So, the computed RTM effect is limited to its long-wavelengths.

According to Table 3, TR1974 COV model shows a better performance in using T_{zz} as a satellite product rather than the terrestrial ones (Δg and N). Thus, the idea of COV improvement is not beneficial to the satellite data as much as it is for terrestrial data. By considering the better performance in regions with rougher topography, we believe that the COV improvement procedure lead to enhancement in modeling the higher D/O of the TR1974 COV. So that, its proficiency is not as much as our expectation for GOCE products though additional investigations are required in order to verify this claim. The accuracy of gravity modeling in Ramouz et al. (2020) obviously depends on first, data distribution and second, topography harshness within the work area. But these patterns have not been observed clearly in this study. Our prediction for this inconsistency is related to the different used control points. That is to say, control and observation points in Ramouz et al. (2020) are from unique source with the same accuracy while the control points of this study are GPS/Leveling observation from Iranian Height Datum with different accuracy and includes tilt bias in North-South and East-West directions based on our ongoing study.

Fig. 4. Graph of the std of the LSC geoid estimation based on different COV models in each region.

Similar to Ramouz et al (2020), Table 4 and Fig. 4 show that I_COV (OQDC strategy) is more efficient in R2 and R3, which have rough topography in comparison with R1 and R4. As regard to CQDC strategy, comparing its S_COV results with the U_COV and S_COV of OQDC strategy demonstrates enhancement in geoid determination in regions with rough topography (R2 and R3). Moreover, implementing I_COV on CQDC strategy lead to more accurate geoid estimation. In case of adequate well-distributed N observations, CQDC strategy specially in regions with sparse gravity anomalies data distribution could be advantageous.

And finally, the maximum accuracy of the data provided by the GOCE mission ranges from 50 to 280, denoted as the Measurement Bandwidth (MBW). On the other hand, GOCE data have good coverage over Iran while terrestrial data are not well distributed in some parts of this area. Therefore, mixing these two kinds of data could have a great benefit but needs a comprehensive and detailed investigation for Iran which is considered as a future work

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