Testing the improved Integral Suspension Pressure method ISP+ with the PARIO™ device

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Overview

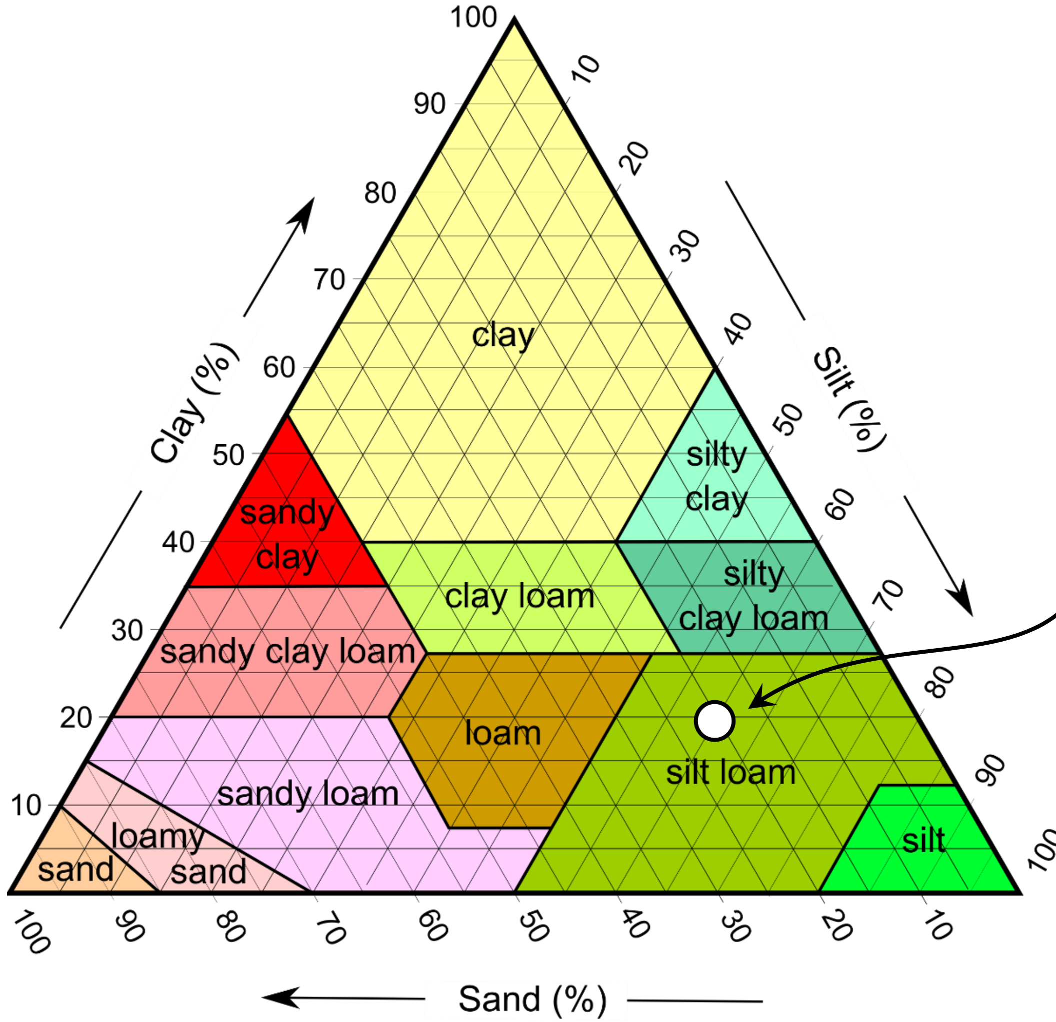
Intro

Theory ISP

Theory ISP$^+$

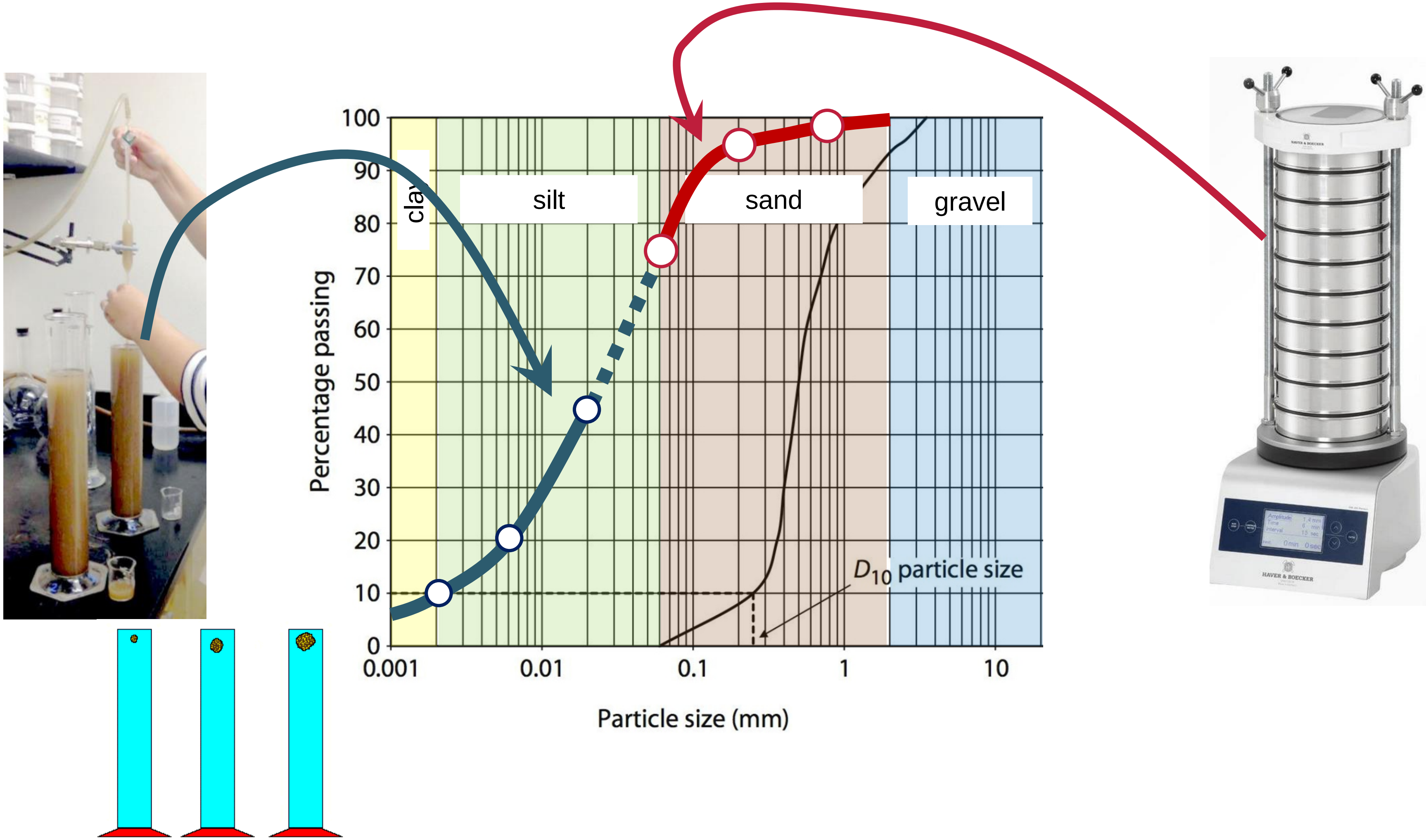
Test with
real data

Particle size distribution determines soil texture



Example:
20 % sand
60 % silt
20 % clay
→
„silt loam“

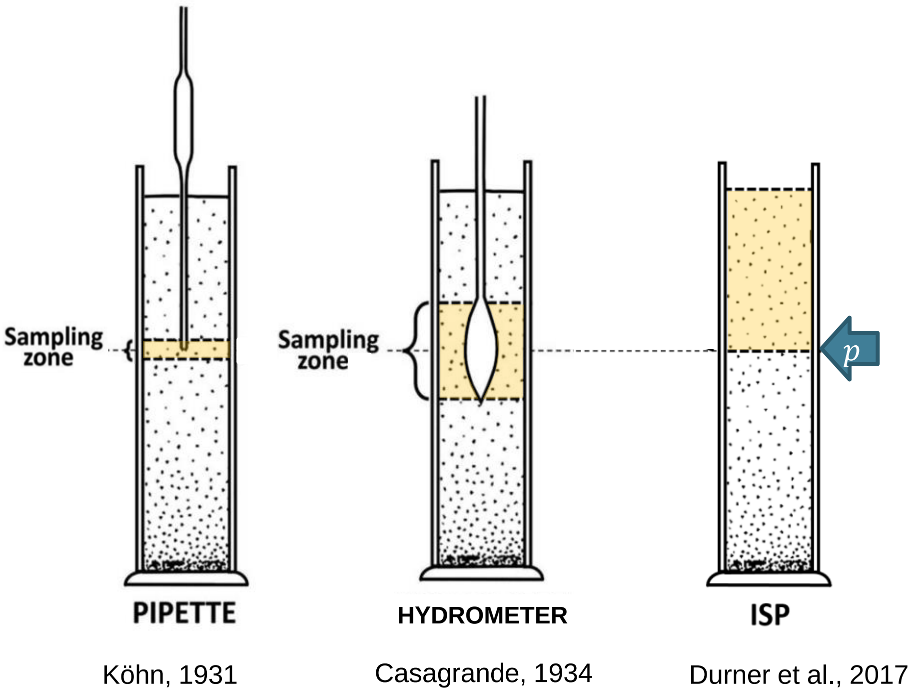
Cumulative particle-size distribution curve is obtained by sieving and sedimentation



Three alternative methods for sedimentation analysis

- The sedimentation of particles is the pedological reference method for the determination of the particle size distribution (PSD) for grain diameters below the sand fraction.

- Soil material is dispersed and transferred into an aqueous suspension. Subsequently, the particles are sedimented in the earth's gravity field in a sedimentation cylinder. Due to the particle size dependent sedimentation velocity, sinking fronts are formed and the particle size distribution can be determined by taking an aliquot of the suspension (PIPETTE method) or by measuring the buoyancy of a body (HYDROMETER method).



- A recent variant is the **integral suspension pressure method (ISP)**, in which the suspension pressure is measured continuously at one depth and the particle size distribution is determined by an inverse simulation of the change in pressure signal over time.

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In a nutshell: The ISP theory

In the ISP method, the integral of the cumulative mass distribution function is related to the measured pressure at depth L by

$$p(L,t) = \rho_{\text{liq}}\, g\, L + \rho_{\text{add}}\, g \int_0^L F(D)dz$$

Here, $p(L,t)$ is the suspension pressure, measured at time t in depth L, ρ_{liq} is the liquid density, g is gravitational acceleration, ρ_{add} is the density increase by the presence of all particles in the suspension, and $F(D)$ is the cumulative particle mass distribution function of particles up to diameter D. The term $\rho_{\text{liq}}\, g\, L$ expresses the constant pressure component provided by the liquid, the most right term the time-variable part.

The integral can be solved analytically for some distribution functions. In the general case, i.e. for arbitrary distribution functions, the calculation of the pressure $p(L,t)$ must be done by numerical integration.

ISP: Estimation of F(D) by inverse modelling

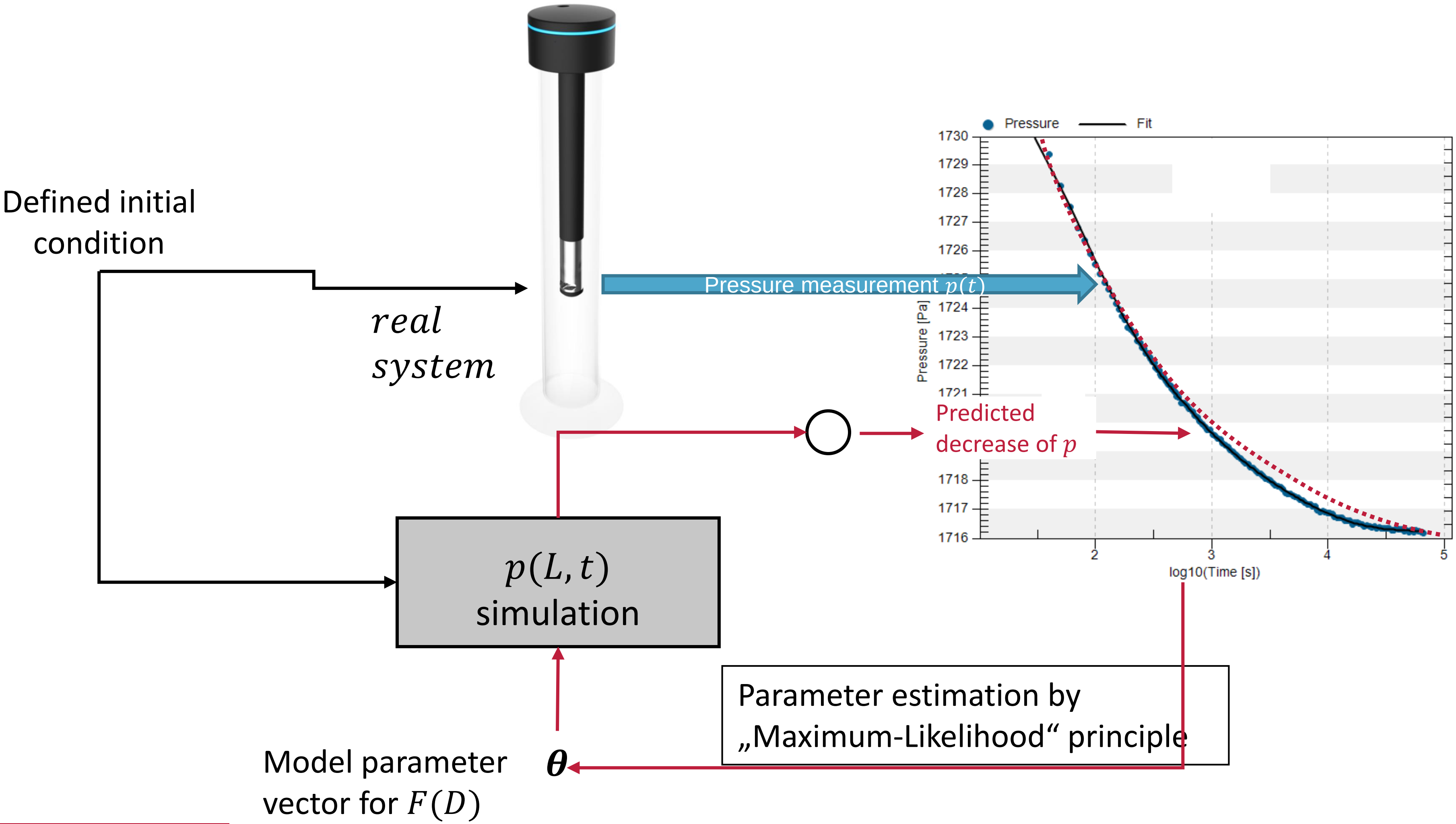
$F(D)$ is represented by an Hermitian Spline that covers the range $F = 0$ at 0.01 µm to $F = 1$ at 2000 µm with 12 support points.

The spline‘s parameters are obtained by matching a simulated pressure decrease with the observed one, i.e., by minimizing an objective function that contains the measured pressure data and the sand fractions data.

$$\mathrm{O}(\boldsymbol{\theta}) = \sum_{i=1}^{m_p} \frac{\left[p_i - p(t_i, \boldsymbol{\theta})\right]^2}{\sigma_p^2} + \sum_{i=1}^{m_F} \frac{\left[F_i - F(D_i, \boldsymbol{\theta})\right]^2}{\sigma_F^2}$$

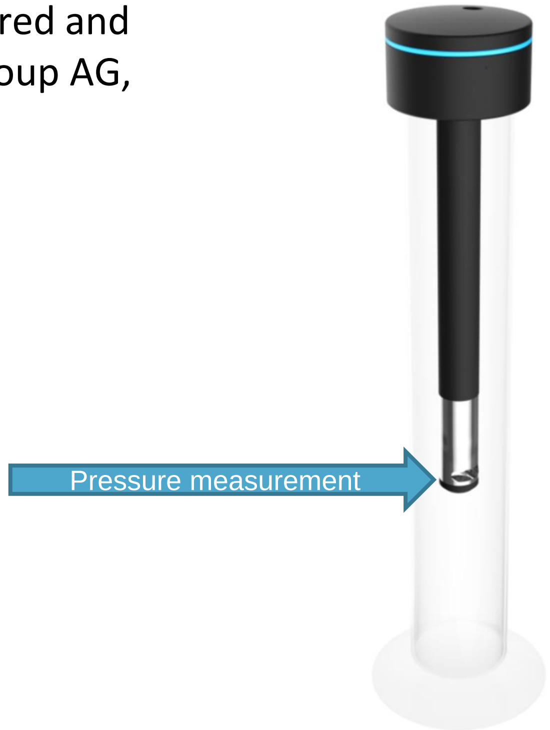
Details are given in Durner, W., S.C. Iden, and G. von Unold (2017): The integral suspension pressure method (ISP) for precise particle-size analysis by gravitational sedimentation, *Water Resources Research, 53, 33-48, doi:10.1002/2016WR019830* (open access).

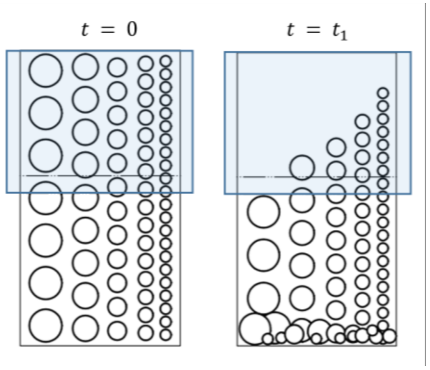
Illustration of ISP Parameter identification



Implementation of the method in PARIO

The ISP method was made commercially available by a measuring instrument called PARIO™, manufactured and traded by METER Group AG, Munich.





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Why ISP$^+$?

Practical tests have shown that the accuracy of the ISP is less than theoretically expected (±1.5 %) and that the measurement time required to determine the clay content exceeds it’s theoretical expectation (6 h).

For practical experiences, see e.g. the contribution of Nemes et al. (2020)*

*EGU2020-9832 | Displays | SSS6.10/HS13.26
Measurement of soil particle-size distribution by the PARIO measurement system: lessons learned and comparison with two other measurement techniques

Attila Nemes, Anna Angyal, Andras Mako, Jan Erik Jacobsen, and Eszter Herczeg
Mon, 04 May, 08:30–10:15 | D2278

What is the idea of ISP$^+$?

- ISP$^+$ (read "ISP plus") is an extension of the experimental protocol which makes the inverse problem better-posed.
- At the end of an ISP$^+$ measurement run, part of the suspension is released laterally from the sedimentation cylinder through an outlet, collected in a beaker, and oven-dried.
- The dry mass of the collected soil particles is integrated into the objective function of the inverse problem.

ISP measurement

Release of suspension

Object function in ISP+

Integration of the collected dry mass, m_{obs}, into the objective function of the inverse problem $\text{O}(\boldsymbol{\theta})$ regularizes the inverse problem of estimating the parameter vector $\boldsymbol{\theta}$ and reduces the uncertainty of the identified PSD in the fine particle range (clay and fine silt fractions).

$$\text{O}(\boldsymbol{\theta}) = \underbrace{\sum_{i=1}^{m_p} \frac{\left[p_i - p(t_i, \boldsymbol{\theta})\right]^2}{\sigma_{\text{p}}^2}}_{\text{simulated vs. observed pressure data}} + \underbrace{\sum_{i=1}^{m_F} \frac{\left[F_i - F(D_i, \boldsymbol{\theta})\right]^2}{\sigma_{\text{F}}^2}}_{\text{simulated vs. observed sand data}} + \underbrace{\frac{(\text{m}_{\text{sim}} - m_{\text{obs}})^2}{\sigma_{\text{m}}^2}}_{\text{simulated vs. observed drained particle mass}}$$

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Test of method: PARIO$^+$ experiments

Sedimentation vessel	PARIO sedimentation cylinders. 6 cm ID, 40 cm height
Total suspension volume	1.0 L
Depth of pressure measurement	18 cm
Depth of suspension outlet	12 cm
Duration of measurement: ISP standard method (reference) ISP+ variants	 12 h 6 h, 4 h, and 2.5 h
Soil material	25 g quartz flower, no organic content
Initiation	60 seconds mixing by overhead shaking
No. of replicates	3 replicates for each variant

Results

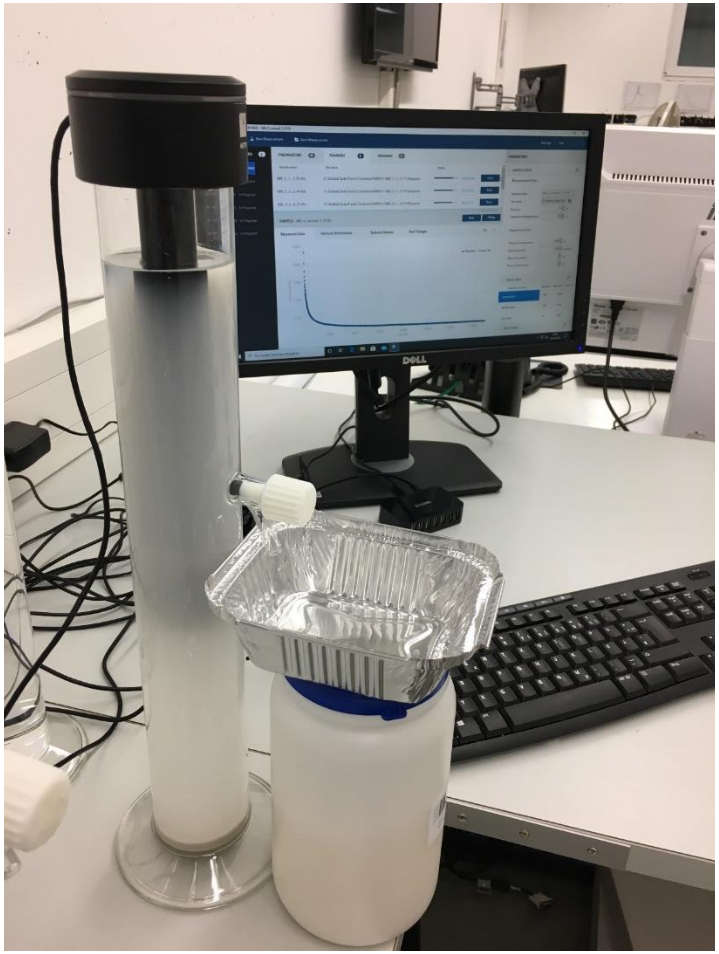
- The table lists the sand, silt, and clay fractions identified by the method variants. The standard deviation of the replicates are given as ± values.

Variant	duration	Sand	Silt	Clay
ISP	12 h	20.7% ± 0.3%	75.0% ± 5.2%	4.2% ± 5.1%
ISP+	6 h	20.7% ± 0.1%	73.9% ± 0.8%	5.4% ± 1.0%
ISP+	4 h	20.7% ± 0.1%	72.9% ± 0.1%	6.4% ± 0.1%
ISP+	2.5 h	20.7% ± 0.1%	70.5% ± 0.3%	8.8% ± 0.2%

- For the original ISP, and for the investigated material, the standard deviation of the clay estimate is in the same magnitude as the clay content. This is not satisfying.
- For all ISP^+ variants, the uncertainty of the identified clay fractions is greatly reduced.
- The sand fractions are unaffected by the type of analysis, since they are externally measured.
- The ISP^+ measurements show a tendency toward higher identified clay content with shorter measurement time. The reason for this is not clear yet.

Conclusions

- First practical test of the ISP+ method showed that the principle of the method works well.
- The ISP+ measurements showed a greatly reduced uncertainty of estimating the clay fraction.
- Also, measurement time for the particle size distribution analysis could be shortened down to 2.5 hrs.
- One problem that has not yet been solved is that the ISP+ evaluations simultaneously led to higher estimates of the clay content. This effect was more pronounced with the shorter measurement times.
- Further work will focus on revealing the reason for the observed bias in the estimation of the clay fraction.



Supplementary material

Theory of the ISP method

Theory (1): Settling of particles by Stokes‘ law

Stokes‘ law describes the terminal sedimentation velocity v [m s^{-1}] of spherical particles as a function of their diameter D [m]

$$v = \frac{g(\rho_p - \rho_{sol})}{18\,\eta} D^2$$

g = 9.81 m s^{-2} is the acceleration due to gravity

ρ_p [kg m^{-3}] is the density of the suspended particles

ρ_{sol} [kg m^{-3}] is the density of the liquid

η [kg s^{-1} m^{-1}] is the dynamic viscosity of the liquid

Theory (2): Diameter of particles reaching z after settling time t

The equivalent diameter of the particles which have reached depth z after time t is calculated by noting that $z = v\,t$

$$D(z,t) = \sqrt[2]{\frac{18\,\eta\,z}{g\,t\,(\rho_{\mathrm{p}} - \rho_{\mathrm{sol}})}} \qquad (1)$$

Theory (3): relating mass fraction F to hydrodynamic diameter D

The corresponding mass fraction is given by the cumulative particle mass distribution function $F(D)$ [-] which is defined as:

$$F(D) = \frac{m(x \leq D)}{m_p} = \int_0^D f(x)dx \qquad (2)$$

x is a dummy variable for the particle diameter

$m(x \leq D)$ is the mass of particles with an effective diameter smaller or equal to D

$f(x)$ is the statistical density function describing the particle-size distribution.

Theory (4): Pressure in a suspension at a detain depth z

$$p(L,t) = \int_0^L \rho(z,t)\, g\, dz$$

z [m] is the vertical coordinate, positive downward

$\rho(z,t)$ [kg m^{-3}] is suspension density above the measurement depth L

g = 9.81 m s^{-2} is the acceleration due to gravity

Theory (5): Initial density of suspension

$$\rho_0 = \rho_{sol} + \frac{m_p}{V_{tot}}\left(1 - \frac{\rho_{sol}}{\rho_p}\right) = \rho_{sol} + \rho_{add}$$

ρ_0 [kg m^{-3}] is the suspension density right after mixing

ρ_{sol} [kg m^{-3}] is the liquid density

ρ_p [kg m^{-3}] is the particle density

ρ_{add} [kg m^{-3}] is the additional density caused by suspended particles

V_p [m^3] is the volume of particles

V_tot [m^3] is the total volume of the suspension m_p [kg] is the mass of particles

Theory (6): relating density at z to F(D)

The density of the suspension at depth z and at time t is obtained by extending Eq. (2) by the mass fraction given by $F(D)$

$$\rho(z,t) = \rho_{\mathrm{sol}} + F(D)\,\rho_{\mathrm{add}} \qquad (3)$$

Theory (7): relating pressure at z to F(D)

This relates the integral of the cumulative mass distribution function to the measured pressure at depth L.

$$p(L,t) = \rho_{\text{sol}}\, g\, L + \rho_{\text{add}}\, g \int_0^L F(D)dz \qquad (4)$$

The integral in Eq. (4) can be solved analytically for some distribution functions. In the general case, i.e. for arbitrary distribution functions, the calculation of the pressure $p(L,t)$ with Eq. (4) must be done by numerical integration.

Estimation of F(D) by inverse modelling

$F(D)$ is parameterized as Hermitian spline with 12 support points, ranging from $F = 0$ at 0.01 µm to $F = 1$ at 2000 µm.

$$\vec{F} = [0;\ \theta_1;\ \theta_2;\ \theta_3;\ \theta_4; \theta_5;\ \theta_6; \theta_7;\ \theta_8;\ \theta_9;\ \theta_{10}; \theta_{11}; 1.0]$$

$$\vec{D} = [0.01; 0.63; 2.0; 3.6; 6.3; 11.3; 20; 36; 63; 125; 200; 630; 2000]\ \ \mu\mathrm{m}$$

Object function contains measured pressure data and measured data for sand

$$\mathrm{O}(\boldsymbol{\theta}) = \sum_{i=1}^{m_p} \frac{[p_i - p(t_i, \boldsymbol{\theta})]^2}{\sigma_p^2} + \sum_{i=1}^{m_F} \frac{[F_i - F(D_i, \boldsymbol{\theta})]^2}{\sigma_F^2}$$