EXTRINSIC VISCOUS ANISOTROPY OF NEWTONIAN TWO-PHASE AGGREGATES, FABRIC PARAMETRISATION AND APPLICATION TO MANTLE CONVECTION

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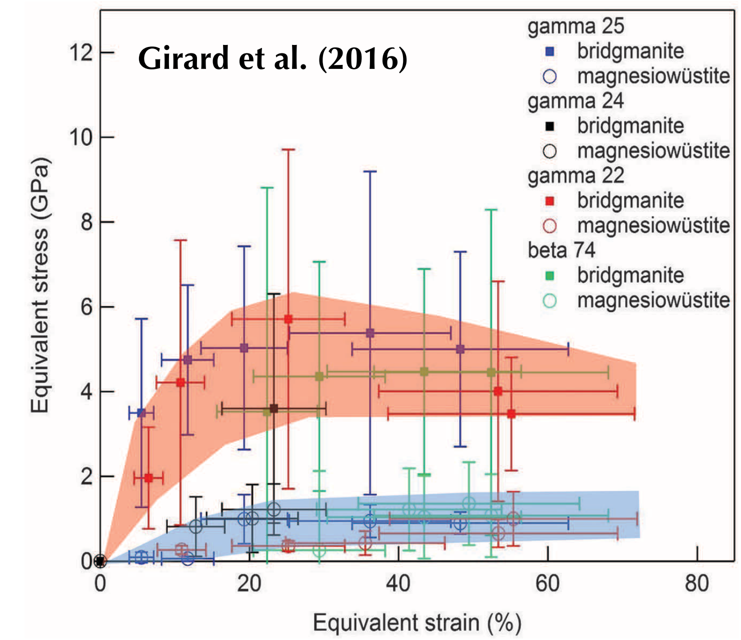
RATIONALE

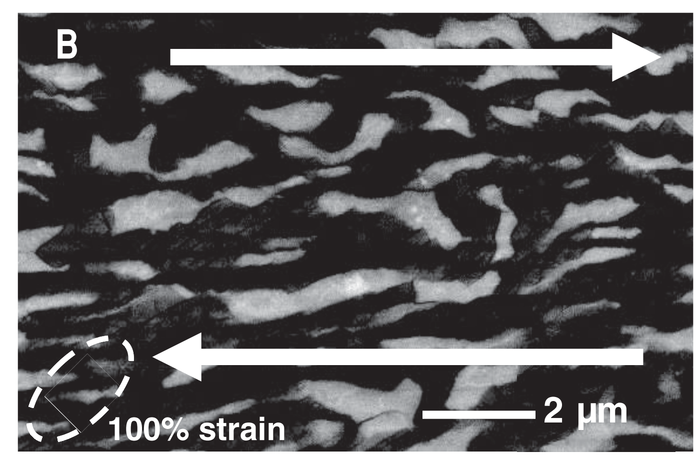
Earth's rock formations are mechanically heterogeneous , i.e. different degrees of extrinsic mechanical anisotropy at different scales

Macro-scale

Folding formed by compression of a layers with distinct visco-elastic properties

Micro-scale





The **bridgemanite/magnesiowüstite** is a lower mantle two-phase aggregate with an abundant strong phase (bridgemanite) and a sparse weak phase (magnesiowüstite)

RATIONALE

PROBLEM

Large scale geodynamic models: anisotropy length scale << resolution

Only for specific shapes and non interactive mineral phases

Large scale geodynamic models: do not predict rock fabrics

SOLUTION

Analytical solutions to predict anisotropic viscous/elastic tensors (e.g. Differential Effective Medium)

Average composite morphology

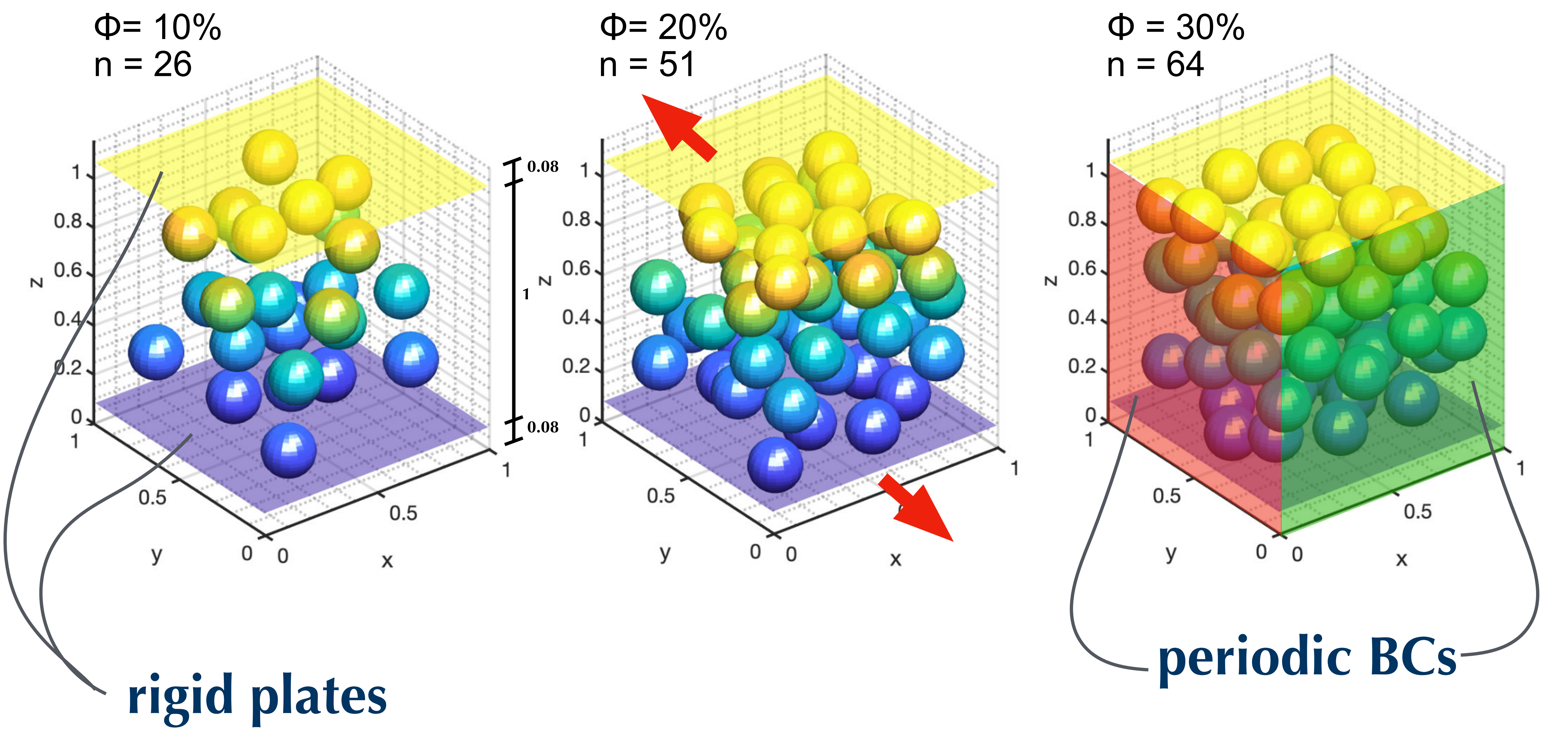
Parametrise fabric

Differential Effective Medium (DEM): $\frac{d\mathbb{C}_{DEM}}{d\phi} = \frac{1}{1-\phi}(\mathbb{C}_I - \mathbb{C}_{DEM})\mathbf{A}$ (McLaughin, 1977)

$\mathbf{A} \rightarrow$ Strain partitioning $4th$ order tensor $\mathbb{C} \rightarrow$ (viscous) stiffness tensor $\phi \rightarrow$ volume fraction

◎ **Incompressible flow + Newtonian rheology**

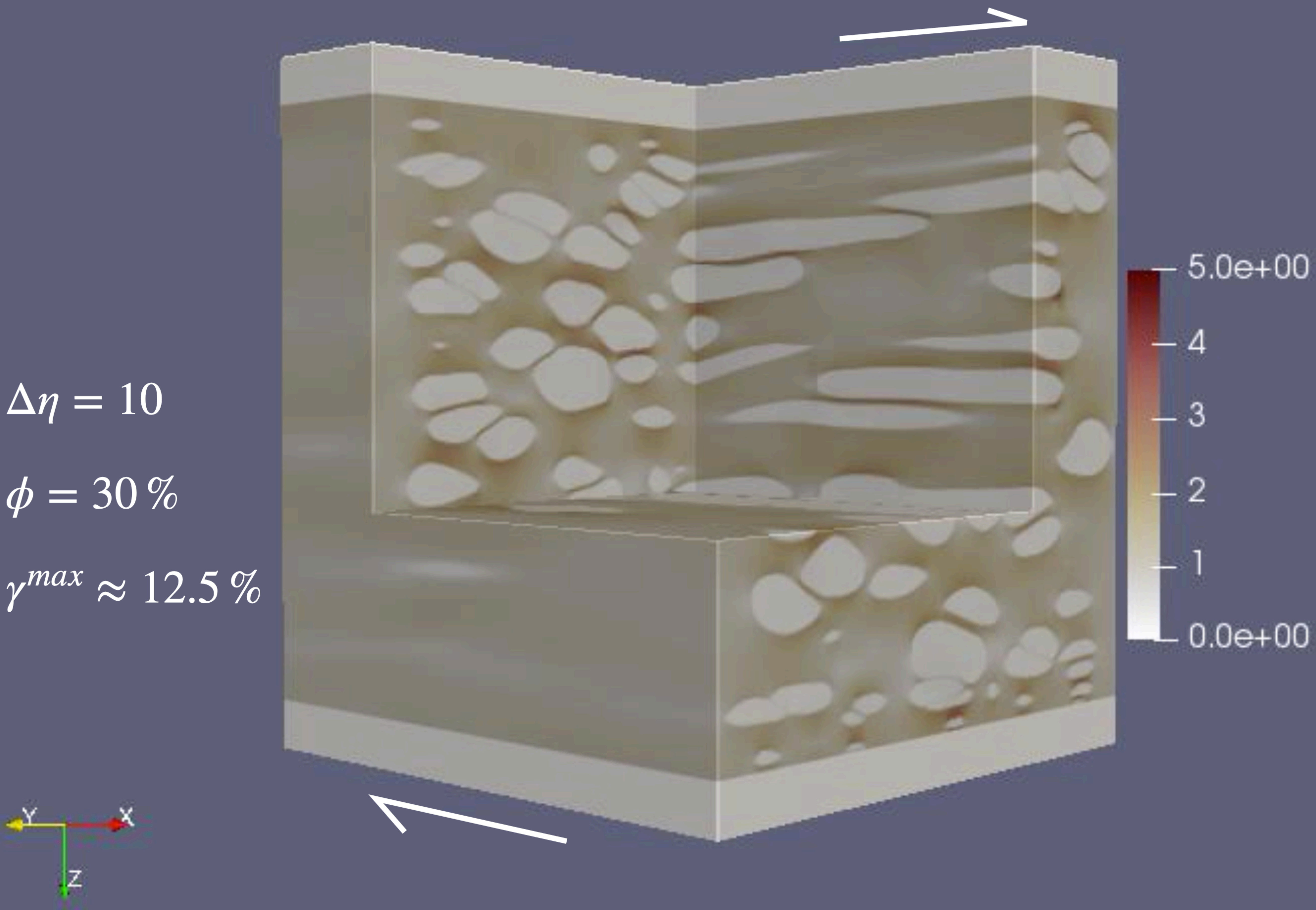
◎ **Stokes equations** → **Finite Differences + particles in cell (Gerya & Yuen, 2007)**



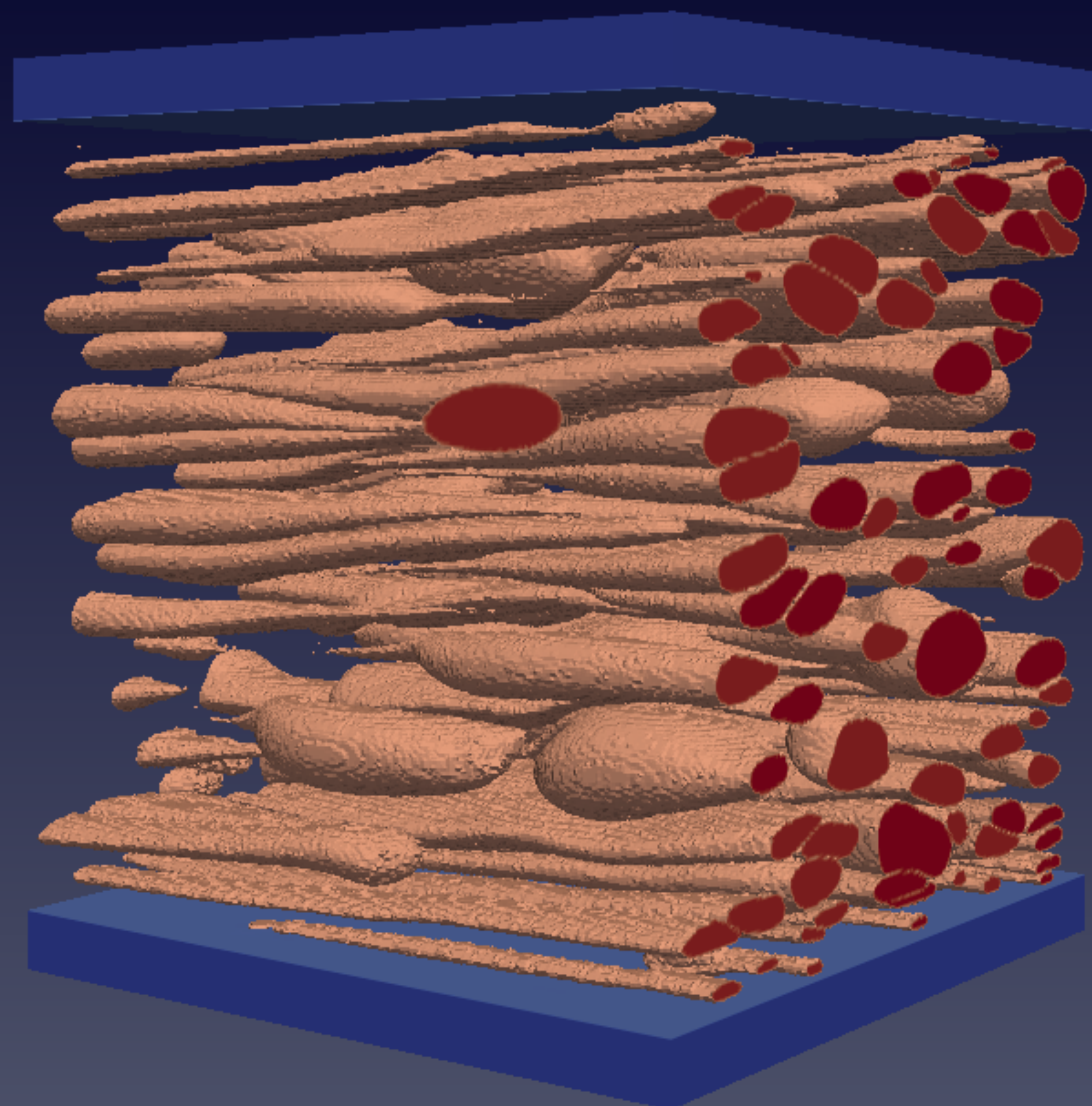
$\Delta\eta = 10$

$\phi = 30\,\%$

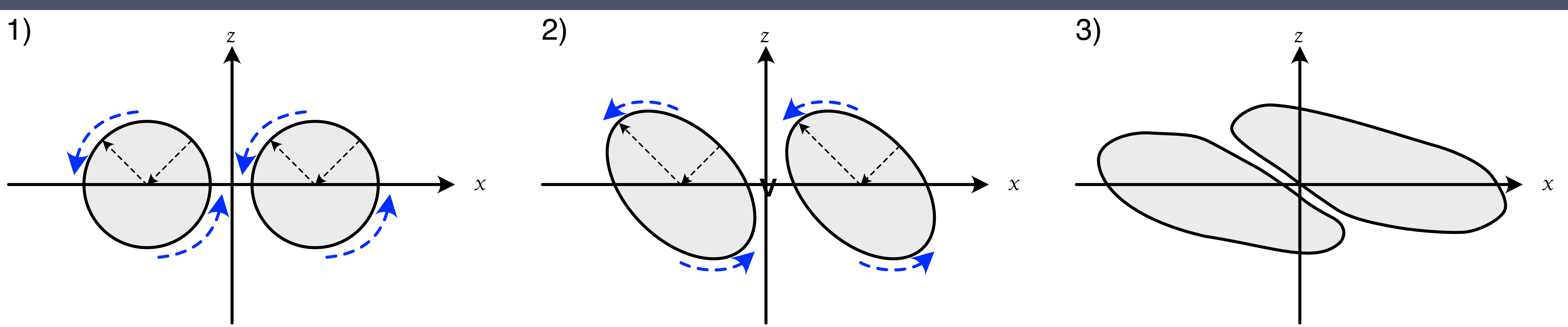
$\gamma^{max} \approx 12.5\,\%$



3D Models: Evolution - strong inclusions



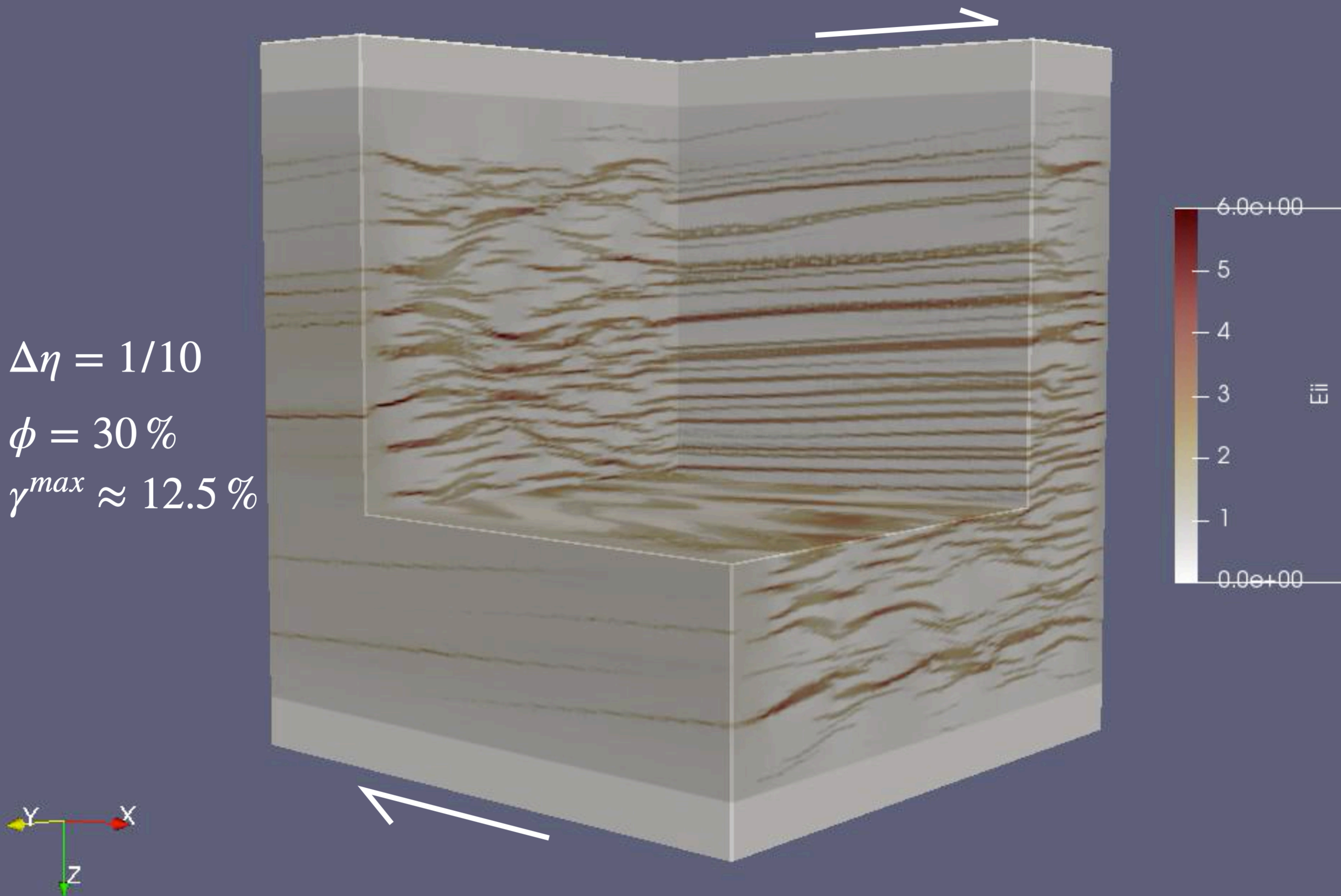
- **Tiling effect between inclusions**
- **Cigar-shaped inclusions**

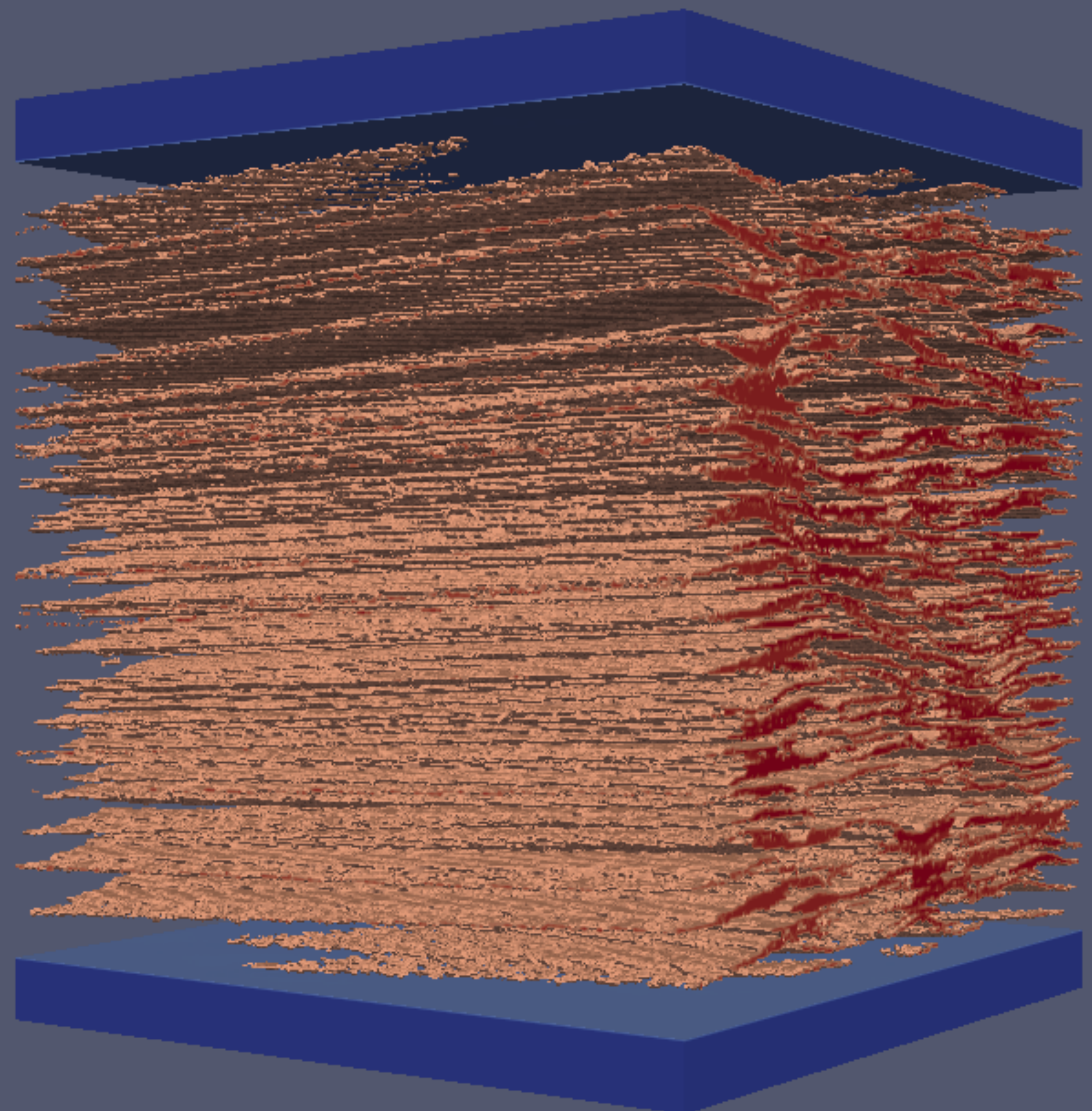


$\Delta\eta = 1/10$

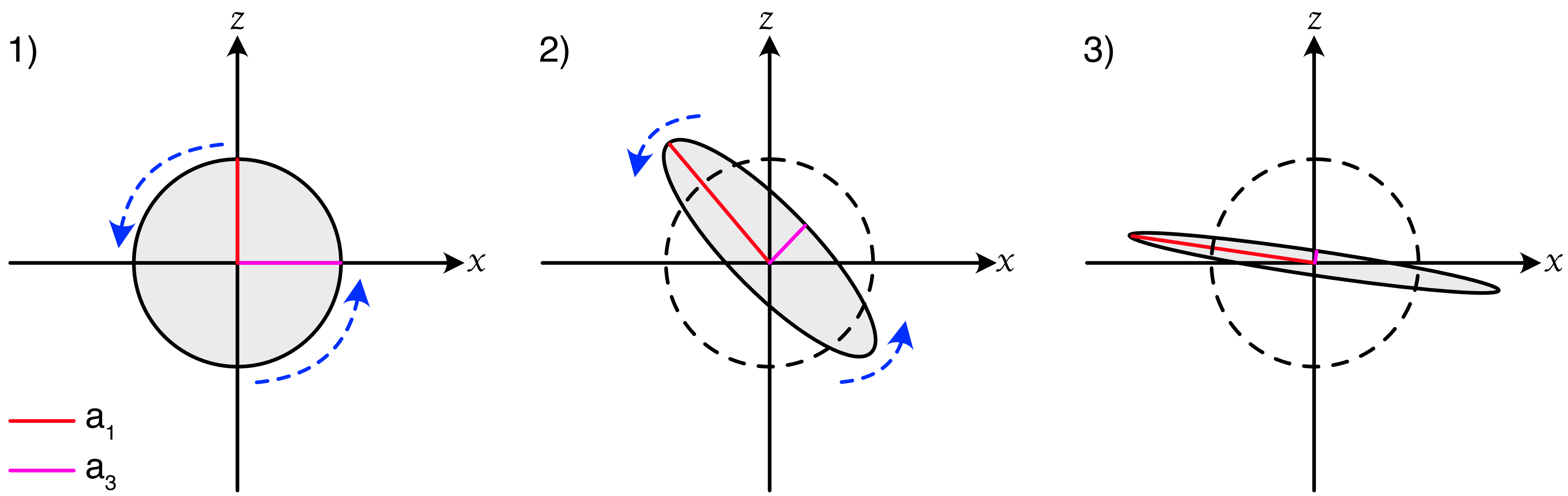
$\phi = 30\,\%$

$\gamma^{max} \approx 12.5\,\%$



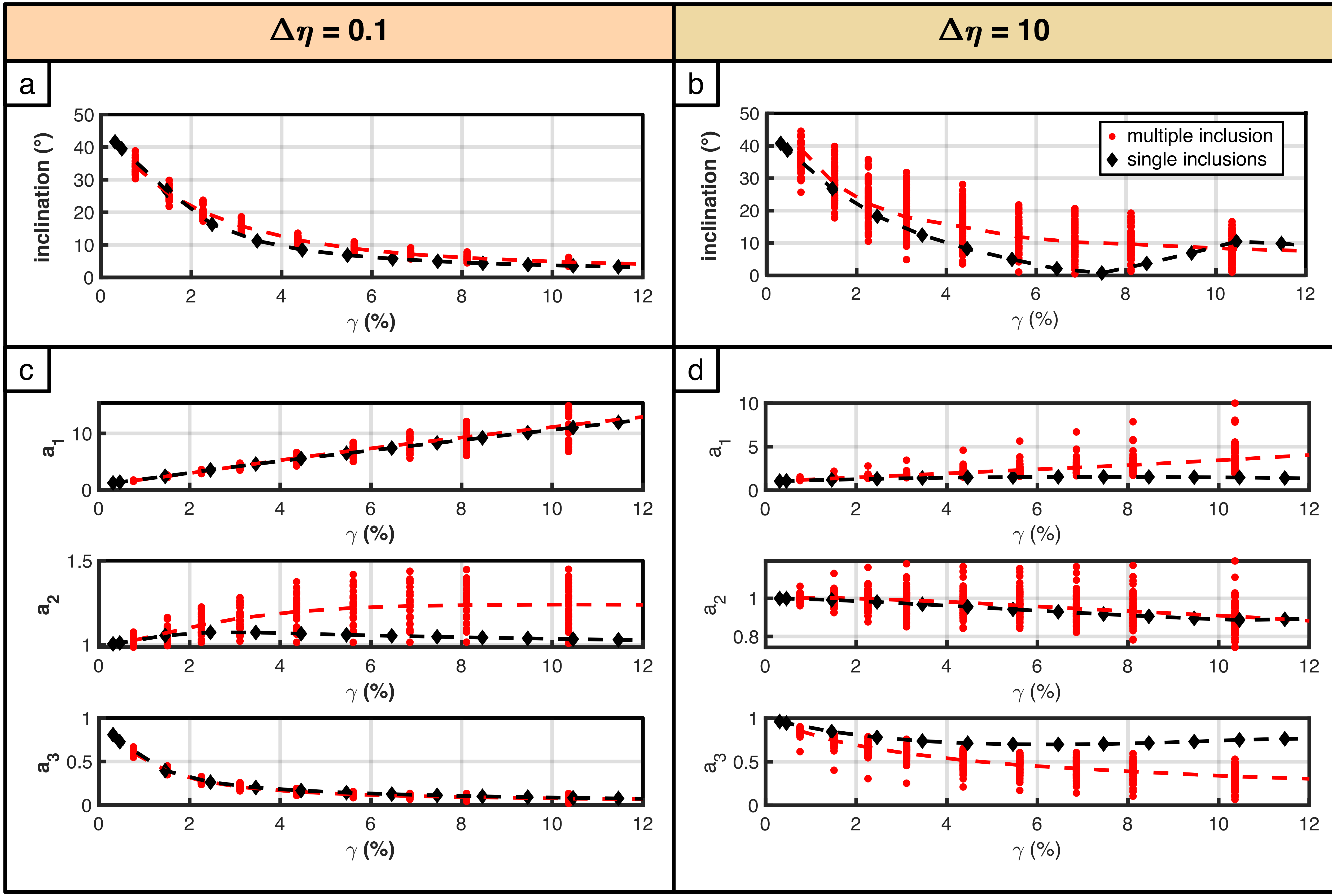


- **Laminar fabric**
- **Boudinage**



DEM vs AVERAGED FABRIC

Inclusion shape/inclination (ϕ = 10%)



inclination = angle with respect to horizontal plane

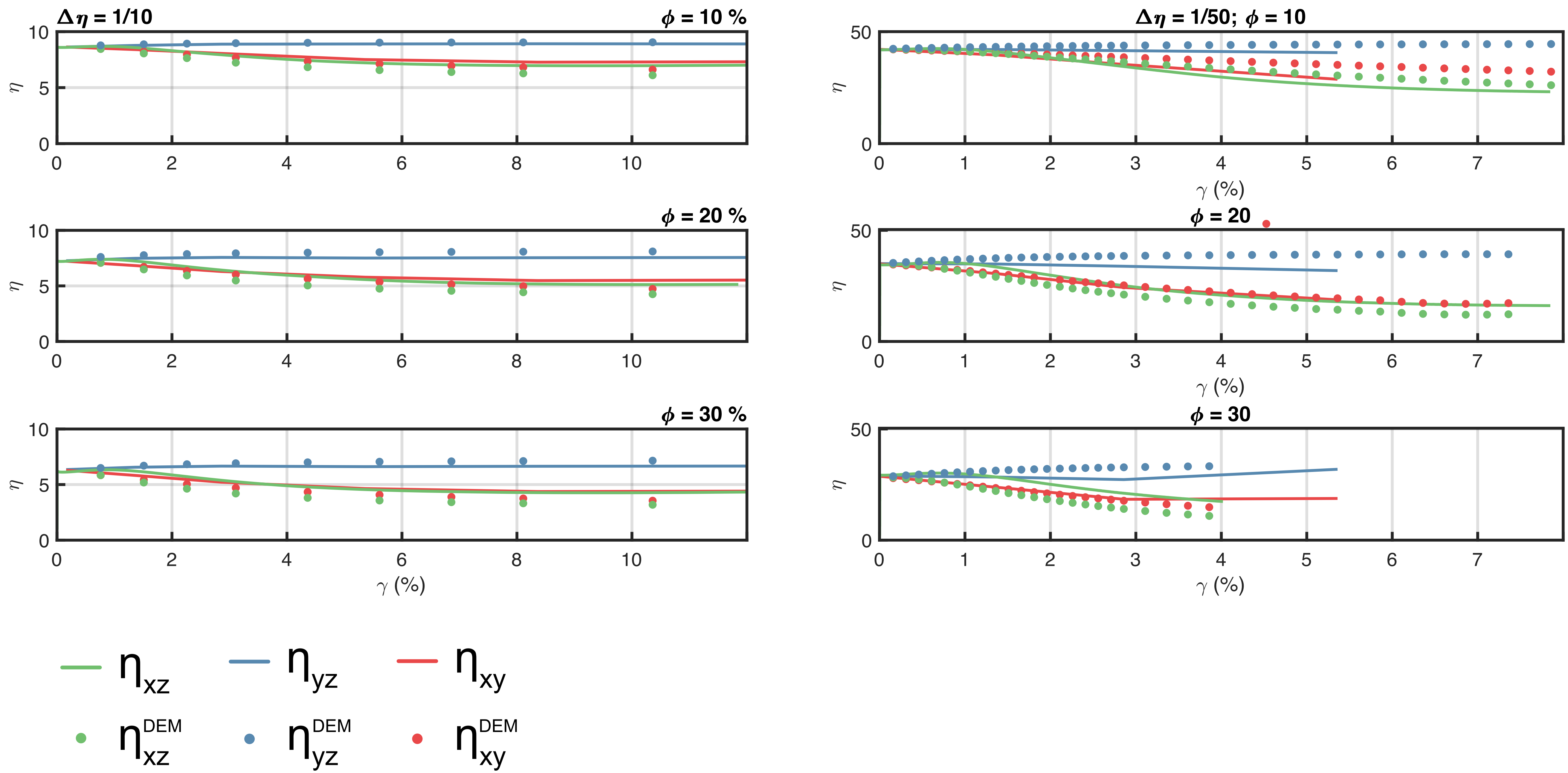
$a_i = i$th principal axis

ϕ = volume fraction

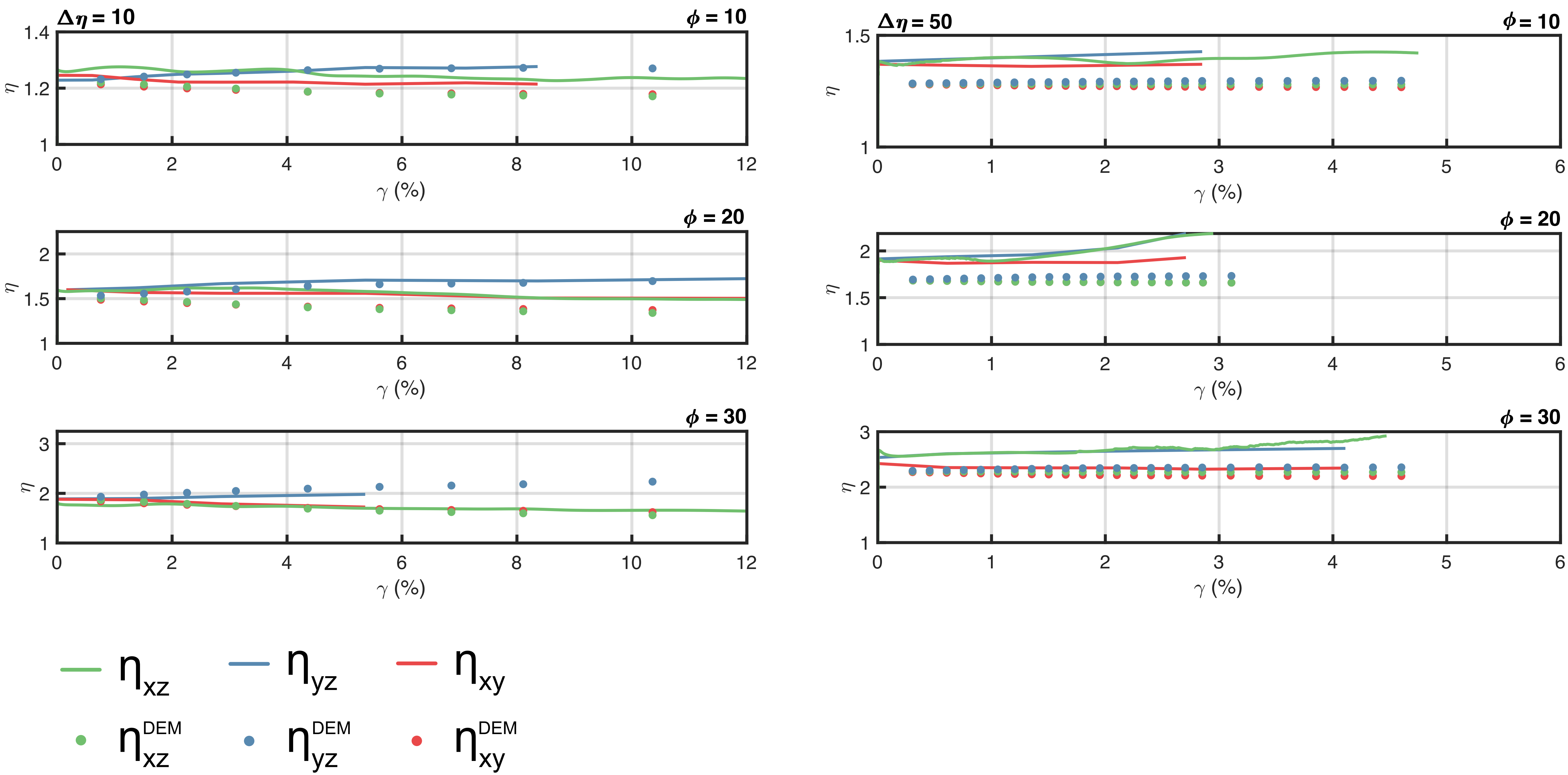
γ = shear strain

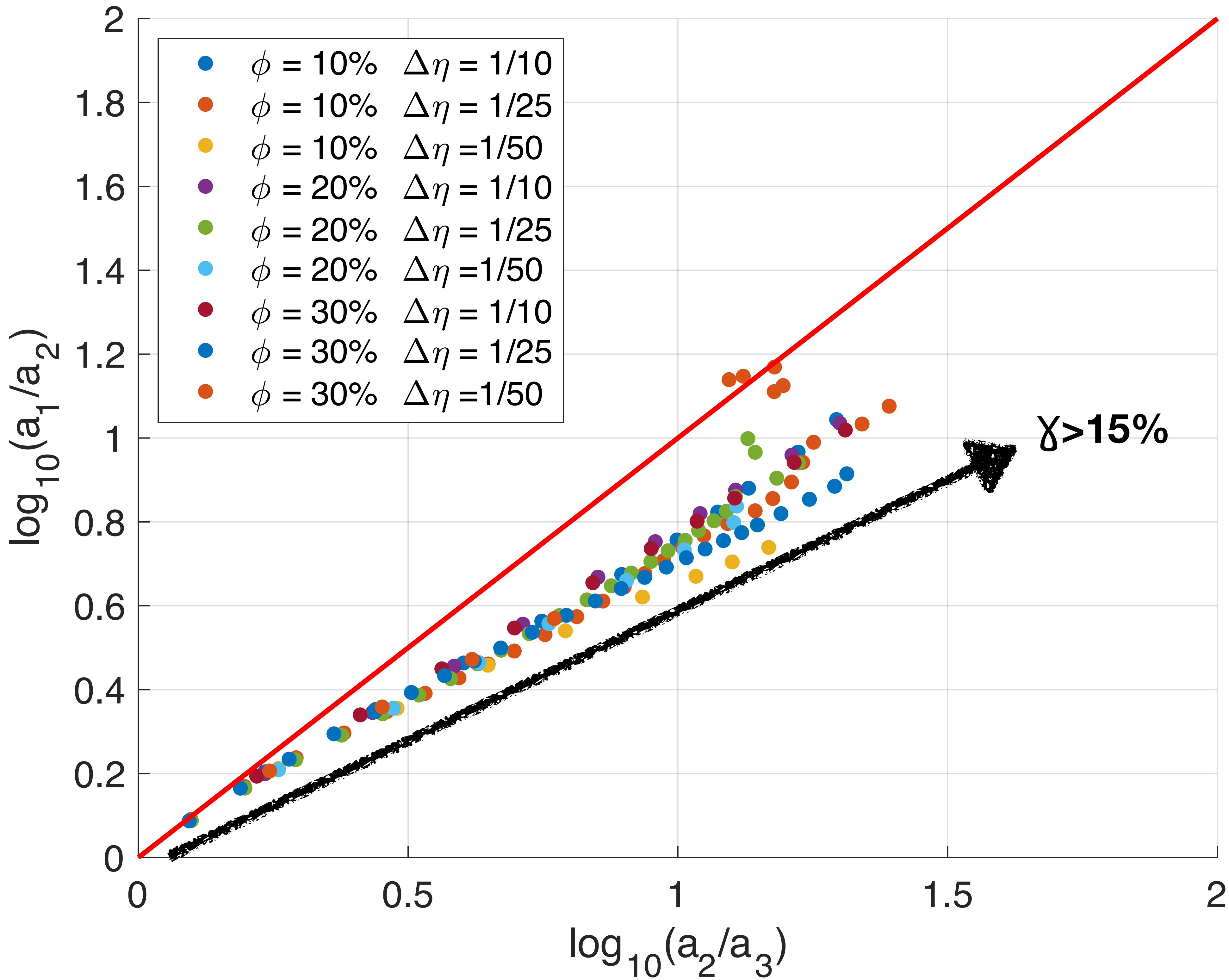
DEM vs AVERAGED FABRIC

Effective viscosities: weak inclusions



Effective viscosities: strong inclusions

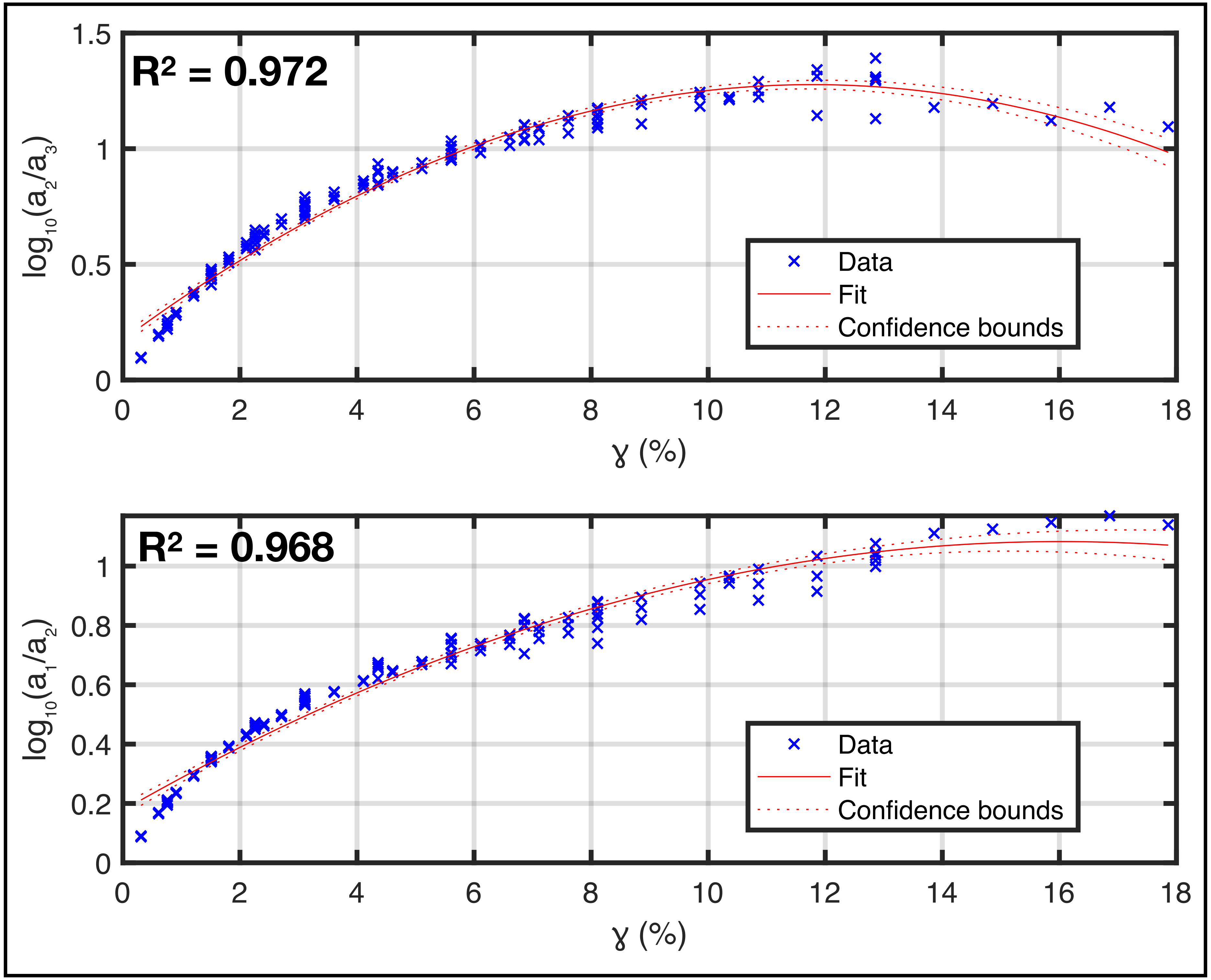




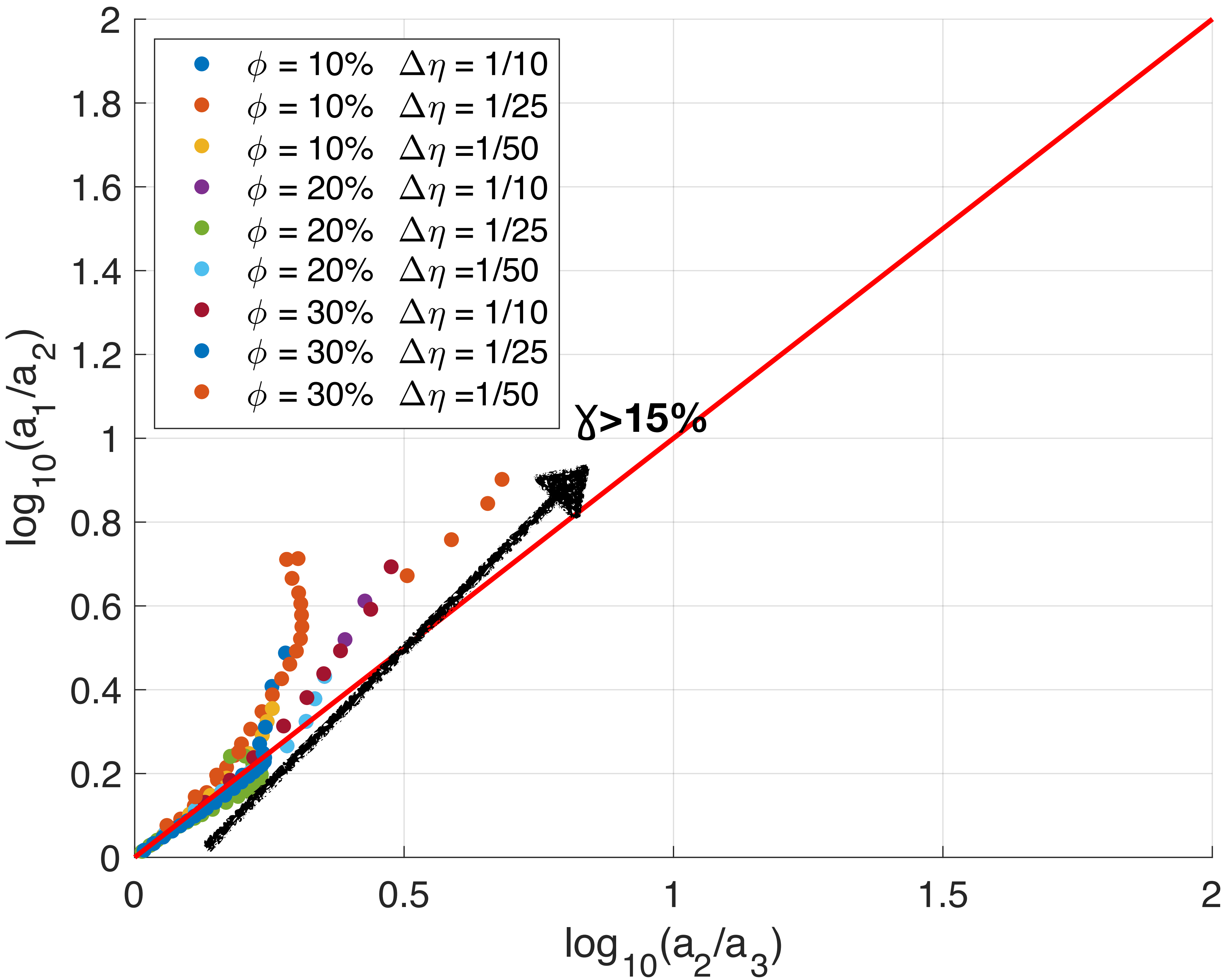
FLINN DIAGRAM:

Every circle represents the average shape of the inclusion phase at a given shear strain

$$\log_{10} \textbf{shape} \approx a + b\log_{10}(a_1/a_2)_{FSE} + c\log_{10}(a_2/a_3)_{FSE}$$



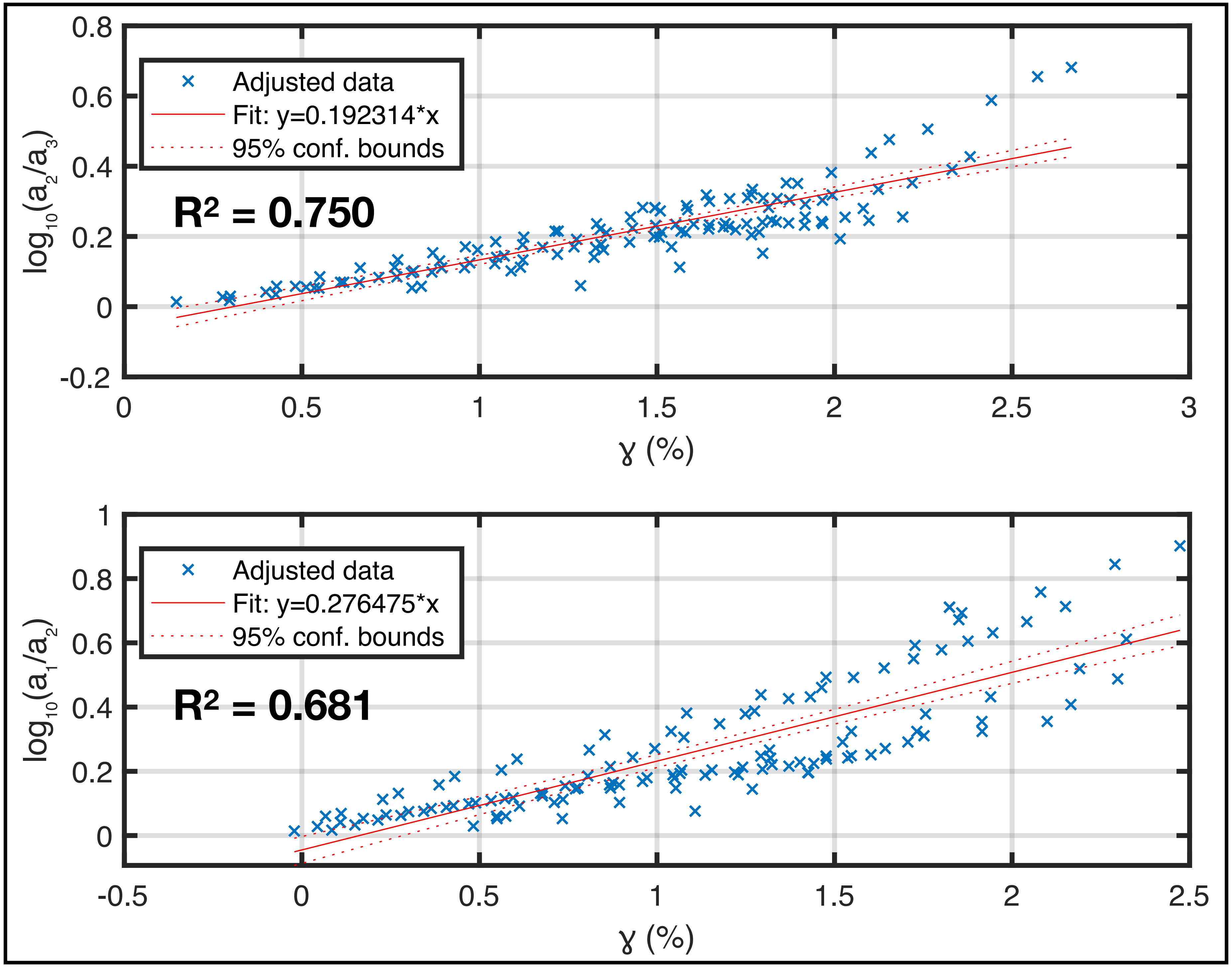
Inclusion average shape parametrised as function of the bulk Finite Strain Ellipsoid (FSE) and **~ independent of** ϕ and $\Delta\eta$



FLINN DIAGRAM:

Every circle represents the average shape of the inclusion phase at a given shear strain

$$\log_{10} \textbf{shape} \approx a + b\log_{10}(a_1/a_2)_{FSE} + c\log_{10}(a_1/a_2)^2_{FSE} + d\log_{10}(a_2/a_3)_{FSE} + e\log_{10}(a_2/a_3)^2_{FSE} + f\phi + g\Delta\eta + h\Delta\eta^2$$



Inclusion average shape parametrised as function of the bulk Finite Strain Ellipsoid (FSE), volume fraction ϕ and viscosity contrast Δη

- Weak inclusions → planar fabric → strain localisation
- Strong inclusions → cigar-shaped
- The fabric can be parameterised as a function of shear strain, volume fraction and viscosity contrast
- Anisotropic viscous tensor of Newtonian-isotropic two-phase composites can be computed averaging the inclusion shape and using the DEM (max. errors ~10-15%)

APPLICATION TO GEODYNAMIC MODELS

t = 0 : solve Stokes -> velocity, strain,…
(isotropic material)

t > 0 : solve Stokes -> velocity, strain,…
(anisotropic material)

predict fabric

DEM $\rightarrow \eta_{ijkl}$

Finite Strain Ellipsoid

Rotation matrix → **Orientation**

$$R = [v_1^T, v_2^T, v_3^T]$$

Particle shape → **Fabric**

$$\{a_1, a_2, a_3\}$$

$a_i = i$**th eigenvalue of the FSE**

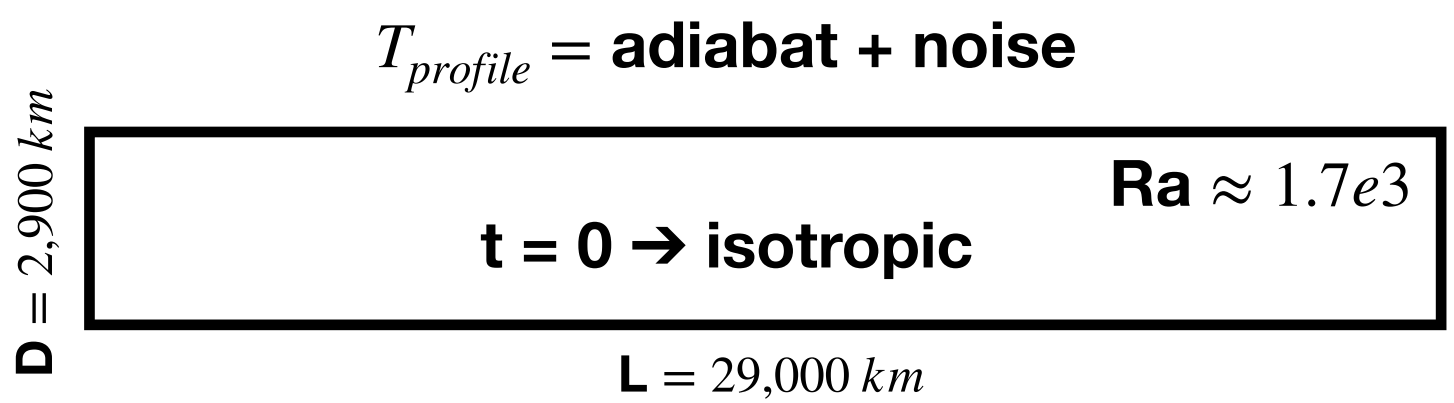
$v_i = i$**th eigenvector of the FSE**

Stokes equations:

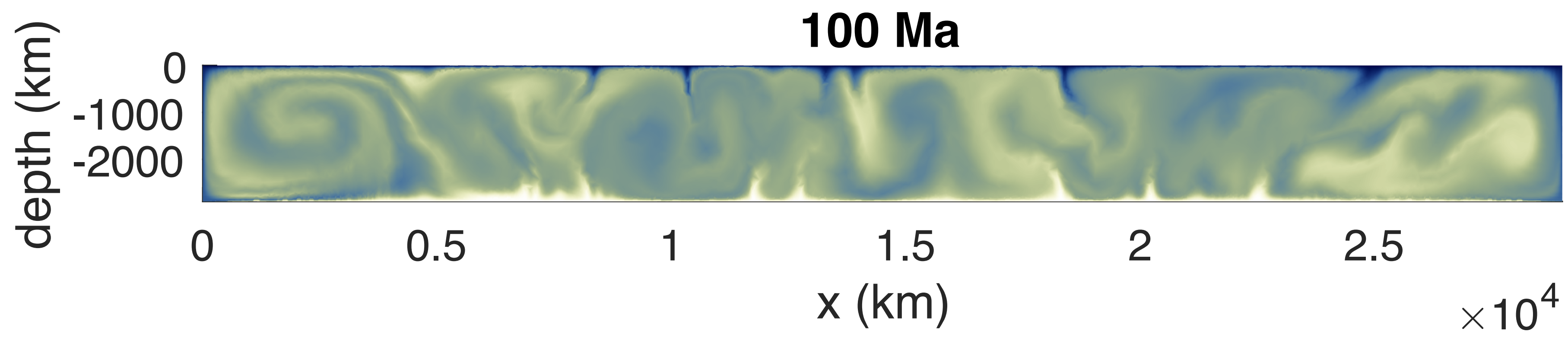
- Incompresible
- Lagrangian Finite Element Method
- Newtonian Rheology
- $\rho = \rho_o(1 - \alpha\Delta T)$

Methods:

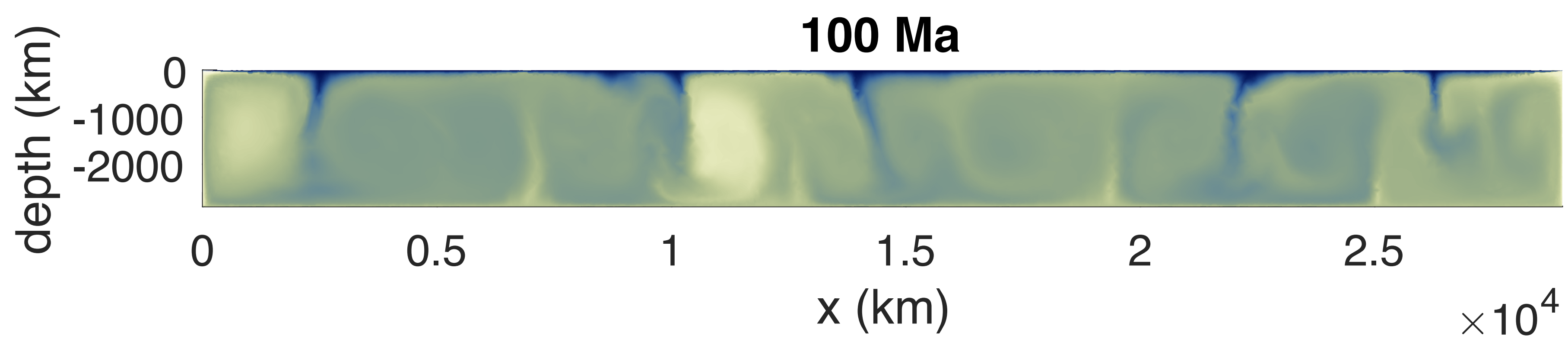
1. Isotropic
2. Anisotropic (fabric defined @ t=0; does not evolve with time)
3. Anisotropic (isotropic @ t=0; fabric evolves following parametrisation))



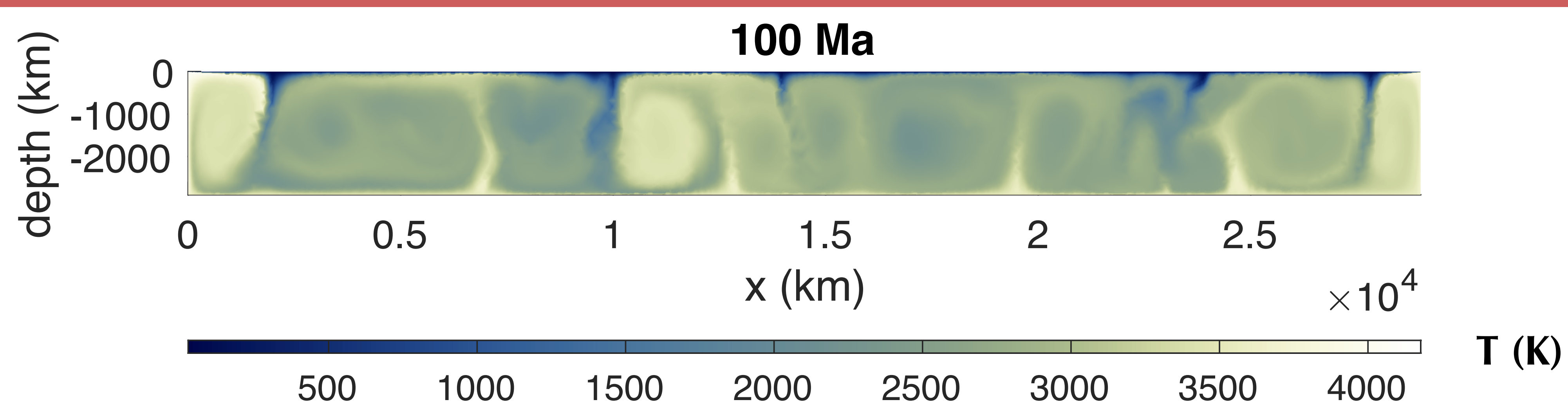
(1) Isotropic

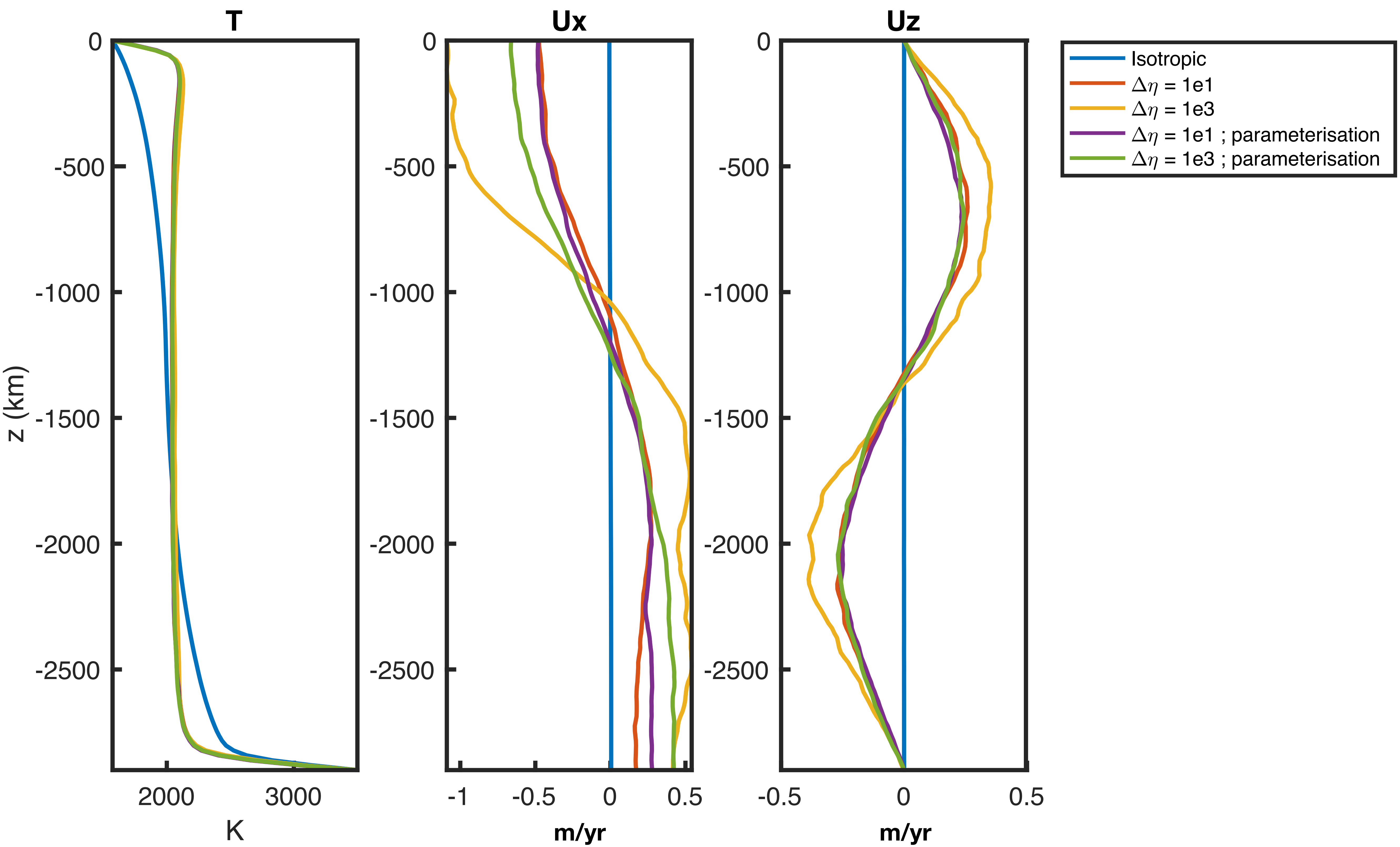


(2) Anisotropic (constant fabric)



(3) Anisotropic (evolving fabric)




T
Ux
Uz
z (km)
0
-500
-1000
-1500
-2000
-2500
2000
3000
K
-1
-0.5
0
0.5
m/yr
-0.5
0
0.5
m/yr
Isotropic
$\Delta\eta$ = 1e1
$\Delta\eta$ = 1e3
$\Delta\eta$ = 1e1 ; parameterisation
$\Delta\eta$ = 1e3 ; parameterisation