EGU2020-11522 NP6.2
Vienna, 4-8 May 2020

Magnetoclinicity: Density variance effects in large-scale instability in magnetohydrodynamic turbulence

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Contents

- Magnetoclinicity

  Dynamo and transport in strongly compressible MHD turbulence

- Results from the multiple-scale renormalized perturbation theory for compressible MHD turbulence
- Large-scale instability analysis
- Illustrative applications

Density-variance effects $\langle \rho'^2 \rangle / \overline{\rho}^2$

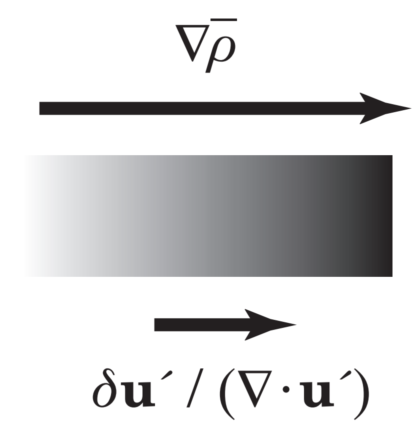
Density variance $$\left(\frac{\partial}{\partial t} + \mathbf{U} \cdot \nabla \right) \langle \rho'^2 \rangle = -2 \langle \rho' \mathbf{u}' \rangle \cdot \nabla \overline{\rho} - 2 \langle \rho'^2 \rangle \nabla \cdot \mathbf{U} + \cdots$$

Magnetoclinicity: $$\langle \mathbf{u}' \times \mathbf{b}' \rangle / \mu_0 = -\chi_\rho \nabla \overline{\rho} \times \mathbf{B} \qquad \chi_\rho \propto \langle \rho'^2 \rangle$$

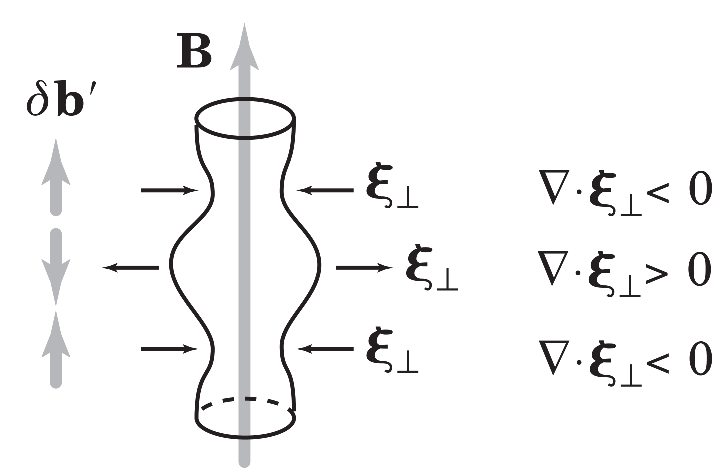
Simplest linear expressions for the density and internal-energy fluctuations

Turbulent dilatation $$\rho' = -\tau_\rho \overline{\rho} \nabla \cdot \mathbf{u}' \qquad q' = -(\gamma_{\rm s} - 1) \tau_q Q \nabla \cdot \mathbf{u}'$$

$$\frac{\partial \mathbf{u}'}{\partial t} = \cdots - (\gamma_{\rm s} - 1) \frac{q'}{\overline{\rho}} \nabla \overline{\rho} + \cdots$$
$$= \cdots + (\gamma_{\rm s} - 1)^2 \tau_q \frac{Q}{\overline{\rho}} (\nabla \cdot \mathbf{u}') \nabla \overline{\rho} + \cdots$$



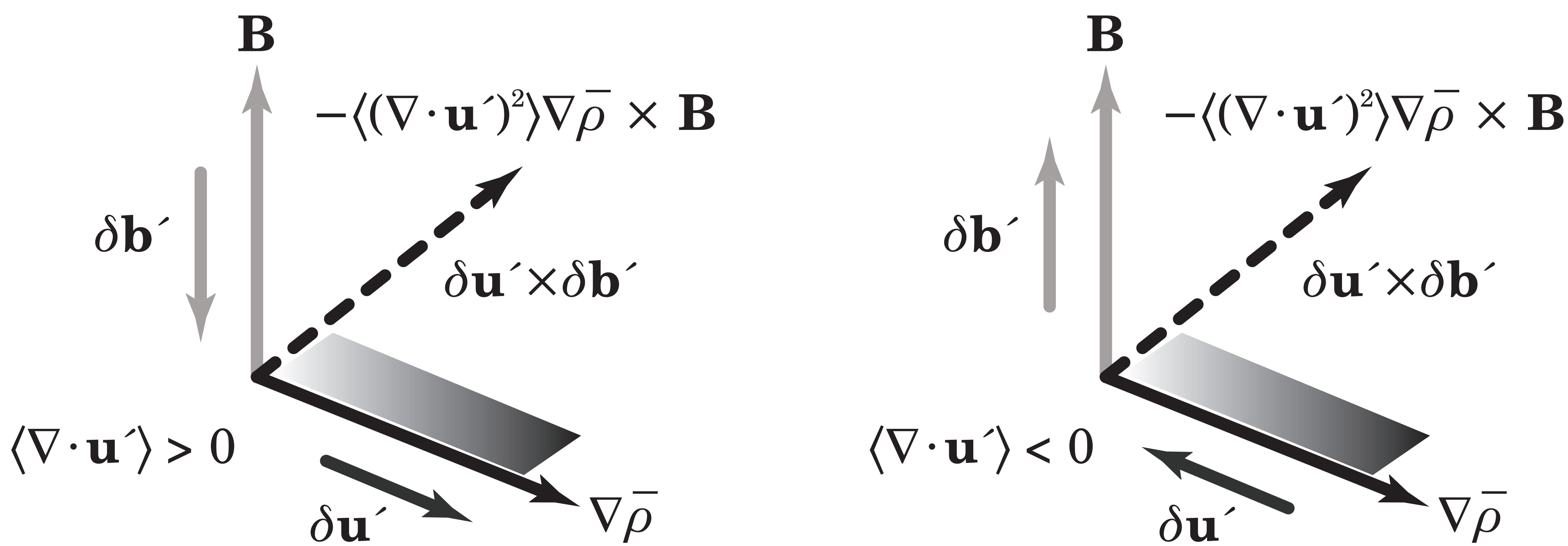
$$\frac{\partial \mathbf{b}'}{\partial t} = \cdots - (\nabla \cdot \mathbf{u}') \mathbf{B} + \cdots$$



Turbulent electromotive force

$$\frac{\partial}{\partial t}\left\langle \mathbf{u}' \times \mathbf{b}' \right\rangle = \cdots + (\gamma_{\rm s}-1)\frac{1}{\overline{\rho}}\left\langle q'\nabla\cdot\mathbf{u}'\right\rangle \nabla\overline{\rho}\times\mathbf{B}+\cdots$$

$$\simeq \cdots - (\gamma_{\rm s}-1)^2\tau_q\left\langle(\nabla\cdot\mathbf{u}')^2\right\rangle\frac{Q}{\overline{\rho}}\nabla\overline{\rho}\times\mathbf{B}+\cdots$$



Irrespective of the sign of dilatation, the electromotive force is generated in the direction of **B** x $\nabla\overline{\rho}$

$$\rho' = -\tau_\rho\overline{\rho}\nabla\cdot\mathbf{u}' \qquad \langle\rho'^2\rangle = \tau_\rho^2\overline{\rho}^2\langle(\nabla\cdot\mathbf{u}')^2\rangle$$

Transports in strongly compressible magnetohydrodynamic turbulence

Fundamental equations

Mass
$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{u}) = 0$$

Momentum
$$\frac{\partial}{\partial t}\rho u^\alpha + \frac{\partial}{\partial x^a}\rho u^a u^\alpha$$
$$= - \frac{\partial p}{\partial x^\alpha} + \frac{\partial}{\partial x^a}\mu s^{a\alpha} + (\mathbf{j} \times \mathbf{b})^\alpha + f_{\mathrm{ex}}^\alpha$$

$$s^{\alpha\beta} = \frac{\partial u^\beta}{\partial x^\alpha} + \frac{\partial u^\alpha}{\partial x^\beta} - \frac{2}{3}\nabla \cdot \mathbf{u}\delta^{\alpha\beta}$$

Internal energy
$$\frac{\partial}{\partial t}\rho q + \nabla \cdot (\rho \mathbf{u} q) = \nabla \cdot (\kappa \nabla \theta) - p\nabla \cdot \mathbf{u} + \phi$$

$$q = C_V(\theta)\theta$$

$$p = R\rho\theta = (\gamma_0 - 1)\,\rho q$$

Magnetic field
$$\frac{\partial \mathbf{b}}{\partial t} = -\nabla \times \mathbf{e}$$

$$\mathbf{j} = \frac{1}{\mu_0}\nabla \times \mathbf{b} = \sigma\,(\mathbf{e} + \mathbf{u} \times \mathbf{b})$$

Mean-field equations

Means and fluctuations

$$f = F + f', \quad F = \langle f \rangle$$

Density

$$\frac{\partial \overline{\rho}}{\partial t} + \nabla \cdot (\overline{\rho}\mathbf{U}) = -\nabla \cdot \langle \rho' \mathbf{u}' \rangle$$

Momentum

$$\frac{\partial}{\partial t}\overline{\rho}U^\alpha + \frac{\partial}{\partial x^a}\overline{\rho}U^a U^\alpha$$
$$= -(\gamma_0 - 1)\frac{\partial}{\partial x^\alpha}\overline{\rho}Q + \frac{\partial}{\partial x^\alpha}\mu S^{a\alpha} + (\mathbf{J}\times\mathbf{B})^\alpha$$
$$- \frac{\partial}{\partial x^\alpha}\left(\overline{\rho}\langle u'^a u'^\alpha \rangle - \frac{1}{\mu_0}\langle b'^a b'^\alpha \rangle + U^a \langle \rho' u'^\alpha \rangle + U^\alpha \langle \rho' u'^a \rangle\right) + R_U^\alpha$$

Internal energy

$$\frac{\partial}{\partial t}\overline{\rho}Q + \nabla \cdot (\overline{\rho}\mathbf{U}Q) = \nabla \cdot \left(\frac{\kappa}{C_V}\nabla Q\right) - \nabla \cdot (\overline{\rho}\langle q' \mathbf{u}' \rangle + Q\langle \rho' \mathbf{u}' \rangle + \mathbf{U}\langle \rho' q' \rangle)$$
$$- (\gamma_0 - 1)\left(\overline{\rho}Q\nabla \cdot \mathbf{U} + \overline{\rho}\langle q' \nabla \cdot \mathbf{u}' \rangle + Q\langle \rho' \nabla \cdot \mathbf{u}' \rangle\right) + R_Q$$

Magnetic field

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{U}\times\mathbf{B} + \langle \mathbf{u}' \times \mathbf{b}' \rangle) + \eta \nabla^2 \mathbf{B}$$

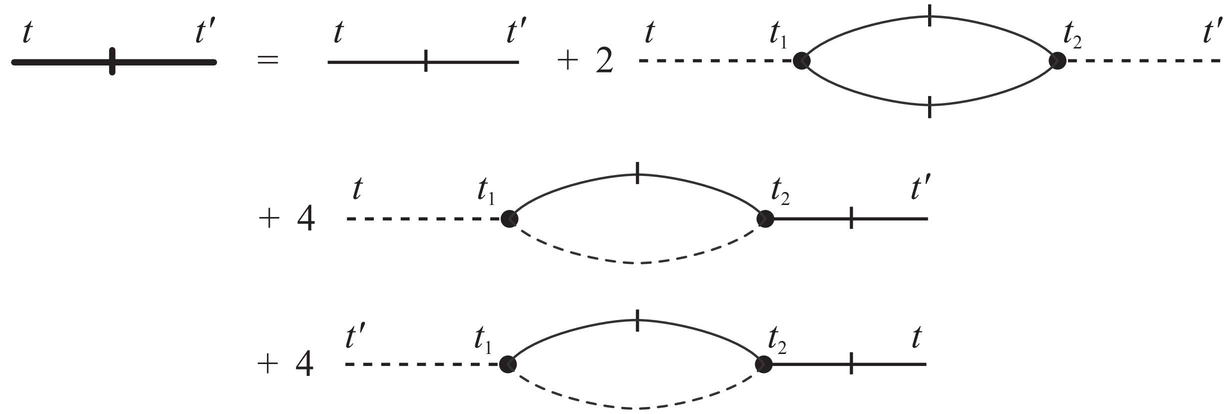
where

$$R_U^\alpha = -\frac{\partial}{\partial t}\langle \rho' u'^\alpha \rangle - \frac{\partial}{\partial x^a}\langle \rho' u'^a u'^\alpha \rangle$$
$$- (\gamma_0 - 1)\frac{\partial}{\partial x^\alpha}\langle \rho' q' \rangle - \frac{1}{2\mu_0}\frac{\partial}{\partial x^\alpha}\langle \mathbf{b}'^2 \rangle \qquad \text{etc.}$$

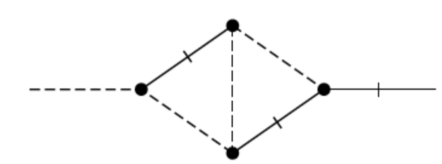
Theoretical framework

Direct-interaction approximation

Correlation function $Q_{\alpha\beta}(\mathbf{k},\mathbf{k}';t,t')$

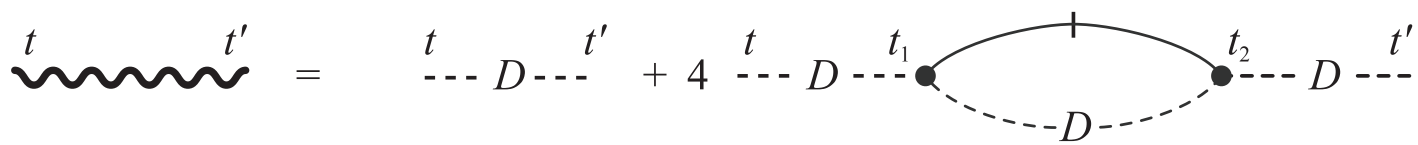


Higher-order vertex parts



not included

Response function $G_{\alpha\beta}(\mathbf{k},\mathbf{k}';t,t')$



Multiple-scale analysis

Fast and slow variables $\boldsymbol{\xi}=\mathbf{x},\ \mathbf{X}=\delta\mathbf{x};\ \tau=t,\ T=\delta t$

Slow variables ***X*** and *T* change only when ***x*** and *t* change much.

$$f=F\left(\mathbf{X};T\right)+f'\left(\boldsymbol{\xi},\mathbf{X};\tau,T\right)$$
$$\nabla=\nabla_{\boldsymbol{\xi}}+\delta\nabla_{\mathbf{X}};\quad \frac{\partial}{\partial t}=\frac{\partial}{\partial\tau}+\delta\frac{\partial}{\partial T}$$

Statistical assumptions on the lowest-order (basic) fields

Basic fields are homogeneous isotropic

$$\frac{\langle \rho'_{\mathrm{B}}(\mathbf{k};\tau)\rho'_{\mathrm{B}}(\mathbf{k}';\tau')\rangle}{\delta(\mathbf{k}+\mathbf{k}')} = \left\langle Q'_{\rho}(k;\tau,\tau')\right\rangle = Q_{\rho}(k;\tau,\tau')$$

$$\frac{\langle {\vartheta'_{\mathrm{B}}}^{\alpha}(\mathbf{k};\tau){\chi'_{\mathrm{B}}}^{\beta}(\mathbf{k}';\tau')\rangle}{\delta(\mathbf{k}+\mathbf{k}')}$$

$$= D^{\alpha\beta}(\mathbf{k})Q_{\vartheta\chi\mathrm{S}}(k;\tau,\tau') + \Pi^{\alpha\beta}(\mathbf{k})Q_{\vartheta\chi\mathrm{C}}(k;\tau,\tau') + \frac{i}{2}\frac{k^c}{k^2}\epsilon^{\alpha\beta c}H_{\vartheta\chi}(k;\tau,\tau')$$

$$\frac{\langle q'_{\mathrm{B}}(\mathbf{k};\tau)q'_{\mathrm{B}}(\mathbf{k}';\tau')\rangle}{\delta(\mathbf{k}+\mathbf{k}')} = \left\langle Q'_{q}(k;\tau,\tau')\right\rangle = Q_{q}(k;\tau,\tau')$$

with solenoidal and dilatational projection operators

$$D^{\alpha\beta}(\mathbf{k}) = \delta^{\alpha\beta} - \frac{k^\alpha k^\beta}{k^2}, \quad \Pi^{\alpha\beta}(\mathbf{k}) = \frac{k^\alpha k^\beta}{k^2}$$

Turbulent electromotive force

$$
\begin{aligned}
\langle \mathbf{u}' \times \mathbf{b}' \rangle^\alpha &= \epsilon^{\alpha ab} \langle u'^a b'^b \rangle \\
&= \epsilon^{\alpha ab} \left(\langle {u'_0}^a {b'_0}^b \rangle + \langle {u'_0}^a {b'_1}^b \rangle + \langle {u'_1}^a {b'_0}^b \rangle + \cdots \right) \\
&= \epsilon^{\alpha ab} \left(\langle {u'_\mathrm{B}}^a {b'_\mathrm{B}}^b \rangle + \langle {u'_\mathrm{B}}^a {b'_{01}}^b \rangle + \langle {u'_\mathrm{B}}^a {b'_{10}}^b \rangle + \cdots \right. \\
&\qquad \left. + \langle {u'_{01}}^a {b'_\mathrm{B}}^b \rangle + \cdots + \langle {u'_{10}}^a {b'_\mathrm{B}}^b \rangle + \langle {u'_{10}}^a {b'_{01}}^b \rangle + \cdots \right).
\end{aligned}
$$

Results

$$
\begin{aligned}
\langle \mathbf{u}' \times \mathbf{b}' \rangle = &- (\beta + \zeta) \nabla \times \mathbf{B} + \alpha \mathbf{B} - (\nabla \zeta) \times \mathbf{B} + \gamma \nabla \times \mathbf{U} \\
&\boxed{- \chi_{\overline{\rho}} \nabla \overline{\rho} \times \mathbf{B} - \chi_Q \nabla Q \times \mathbf{B} - \chi_D \frac{D\mathbf{U}}{Dt} \times \mathbf{B}}
\end{aligned}
$$

"magnetoclinicity"

Transport coefficients

$$
\begin{aligned}
\chi_{\overline{\rho}} = &\frac{1}{3} (\gamma_s - 1)^2 \frac{Q}{\overline{\rho}} \int d\mathbf{k}\ k^2 \int_{-\infty}^{\tau} d\tau_1 \int_{-\infty}^{\tau_1} d\tau_2 \int_{-\infty}^{\tau_2} d\tau_3 \\
&\times G_{u\mathrm{C}}(k;\tau,\tau_1) G_q(k;\tau,\tau_2) G_b(k;\tau,\tau_3) Q_{u\mathrm{C}}(k;\tau_2,\tau_3),
\end{aligned}
$$

$$
\begin{aligned}
\chi_Q = &\frac{1}{3} (\gamma_s - 1) \int d\mathbf{k}\ k^2 \int_{-\infty}^{\tau} d\tau_1 \int_{-\infty}^{\tau_1} d\tau_2 \int_{-\infty}^{\tau_2} d\tau_3 \\
&\times G_{u\mathrm{C}}(k;\tau,\tau_1) G_\rho(k;\tau,\tau_2) G_b(k;\tau,\tau_3) Q_{u\mathrm{C}}(k;\tau_2,\tau_3),
\end{aligned}
$$

$$
\begin{aligned}
\chi_D = &\frac{1}{3} \int d\mathbf{k}\ k^2 \int_{-\infty}^{\tau} d\tau_1 \int_{-\infty}^{\tau_1} d\tau_2 \int_{-\infty}^{\tau_2} d\tau_3 \\
&\times G_{u\mathrm{C}}(k;\tau,\tau_1) G_\rho(k;\tau,\tau_2) G_b(k;\tau,\tau_3) Q_{u\mathrm{C}}(k;\tau_2,\tau_3).
\end{aligned}
$$

Turbulence model in terms of density variance

$$\int d\mathbf{k}\ k^2 Q_{u\mathrm{C}}(k;\tau_1,\tau_2) = \frac{1}{\tau_\rho^2}\frac{\langle \rho'^2\rangle}{\overline{\rho}^2} \longleftarrow ik^a {u'_{\mathrm{B}}}^a(\mathbf{k};\tau) = \frac{1}{\tau_\rho \overline{\rho}}\rho'_0(\mathbf{k};\tau)$$

$$\langle \mathbf{u}' \times \mathbf{b}'\rangle = -(\beta+\zeta)\nabla\times\mathbf{B} + \alpha\mathbf{B} - (\nabla\zeta)\times\mathbf{B} + \gamma\nabla\times\mathbf{U}$$
$$-\chi_{\overline{\rho}}\nabla\overline{\rho}\times\mathbf{B} - \chi_Q\nabla Q\times\mathbf{B} - \chi_D\frac{D\mathbf{U}}{Dt}\times\mathbf{B}$$

Transport coefficients

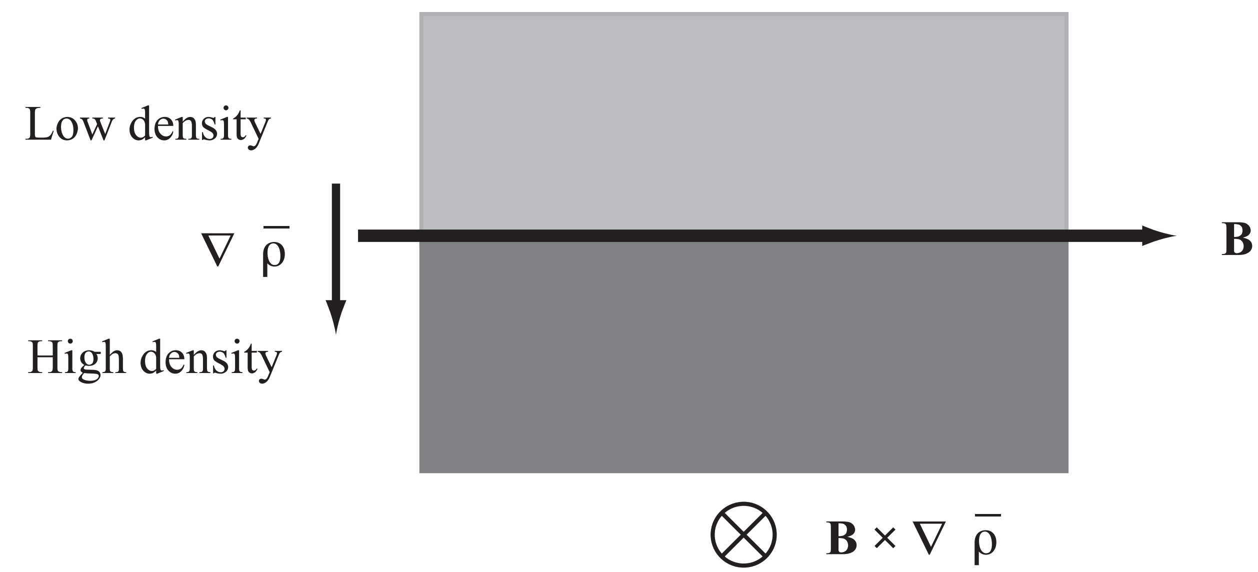
	$\beta = \tau_b\langle \mathbf{u}'^2\rangle/2 + \tau_u\langle \mathbf{b}'^2\rangle/(2\mu_0\overline{\rho})$	MHD energy
	$\zeta = \tau_b\langle \mathbf{u}'^2\rangle/2 - \tau_u\langle \mathbf{b}'^2\rangle/(2\mu_0\overline{\rho})$	Residual energy
	$\alpha = \tau_b\langle -\mathbf{u}'\cdot\boldsymbol{\omega}'\rangle + \tau_u\langle \mathbf{b}'\cdot\mathbf{j}'\rangle/\overline{\rho}$	Residual helicity
	$\gamma = (\tau_u+\tau_b)\langle \mathbf{u}'\cdot\mathbf{b}'\rangle$	Cross helicity

$$\left.\begin{aligned}
\chi_\rho &= (\gamma_{\mathrm{s}}-1)^2\frac{\tau_u\tau_q\tau_b}{\tau_\rho^2}\frac{Q}{\overline{\rho}}\frac{\langle\rho'^2\rangle}{\overline{\rho}^2}\\
\chi_Q &= (\gamma_{\mathrm{s}}-1)\frac{\tau_u\tau_b}{\tau_\rho}\frac{\langle\rho'^2\rangle}{\overline{\rho}^2}\\
\chi_D &= \frac{\tau_u\tau_b}{\tau_\rho}\frac{\langle\rho'^2\rangle}{\overline{\rho}^2}
\end{aligned}\right\} \text{Density variance}$$

“Magnetoclinicity” instability

$$\langle \mathbf{u}' \times \mathbf{b}' \rangle / \mu_0 = +\chi_\rho \mathbf{B} \times \nabla \overline{\rho} + \cdots$$

Large-scale instability analysis



Unperturbed + Perturbation fields

$$F = F_0 + \delta F \qquad \delta F \ll F_0$$

Mean density $\overline{\rho} = \rho_0 + \delta\rho$

Mean velocity $\mathbf{U} = \mathbf{U}_0 + \delta\mathbf{U} = \delta\mathbf{U} = (\delta U^x, \delta U^y, \delta U^z)$

Mean internal energy $Q = Q_0 + \delta Q$

Mean magnetic field $\mathbf{B} = \mathbf{B}_0 + \delta\mathbf{B} = (B_0, 0, 0) + (\delta B^x, \delta B^y, \delta B^z)$

Perturbation equations

Density
$$\frac{\partial \delta\rho}{\partial t} + (\delta\mathbf{U}\cdot\nabla)\rho_0 + \rho_0\nabla\cdot\delta\mathbf{U} = -\nabla\cdot\langle\rho'\mathbf{u}'\rangle_1$$

Momentum
$$\begin{aligned}\frac{\partial}{\partial t}\rho_0\delta U^\alpha =& -(\gamma_\mathrm{s}-1)\frac{\partial}{\partial x^\alpha}\left(\rho_0\delta Q + \delta\rho Q_0\right)\\ &+ \frac{\partial}{\partial x^a}\mu\left(\frac{\partial\delta U^\alpha}{\partial x^a}+\frac{\partial\delta U^a}{\partial x^\alpha}\right) + (\mathbf{J}_0\times\delta\mathbf{B})^\alpha + (\delta\mathbf{J}\times\mathbf{B}_0)^\alpha\\ &- \frac{\partial}{\partial x^a}\left(\delta\rho\langle u'^a u'^\alpha\rangle_1 + \delta U^a\langle\rho' u'^\alpha\rangle_1 + \delta U^\alpha\langle\rho' u'^a\rangle_1\right)\end{aligned}$$

Internal energy
$$\begin{aligned}&\frac{\partial}{\partial t}\left(\rho_0\delta Q + \delta\rho Q_0\right) + \nabla\cdot\left(\rho_0\delta\mathbf{U}Q_0\right)\\ &= \nabla\cdot\left(\frac{\kappa}{C_v}\nabla\delta Q\right) - \nabla\cdot\left(\delta\overline{\rho}\langle q'\mathbf{u}'\rangle_1 + \delta Q\langle\rho'\mathbf{u}'\rangle_1 + \delta\mathbf{U}\langle\rho' q'\rangle_1\right)\\ &- (\gamma_s-1)\left[\rho_0 Q_0\nabla\cdot\delta\mathbf{U} + \delta\rho\langle q'\nabla\cdot\mathbf{u}'\rangle_1 + \delta Q\langle\rho'\nabla\cdot\mathbf{u}'\rangle_1\right]\end{aligned}$$

Magnetic field
$$\frac{\partial\delta\mathbf{B}}{\partial t} = \nabla\times(\delta\mathbf{U}\times\mathbf{B}_0) + \eta\nabla^2\delta\mathbf{B} + \nabla\times\langle\mathbf{u}'\times\mathbf{b}'\rangle_1$$

with
$$\begin{aligned}\langle\mathbf{u}'\times\mathbf{b}'\rangle_1 =& -\eta_\mathrm{T}\delta\mathbf{J} + \alpha\delta\mathbf{B} + \gamma\delta\mathbf{\Omega}\\ &+ \chi_\rho\mathbf{B}_0\times\nabla\delta\rho + \chi_\rho\delta\mathbf{B}\times\nabla\rho_0\end{aligned}$$
etc.

Normal mode analysis

$$\delta F = \hat{f}(z)\exp[i(k^x x + k^y y) - i\omega_{\mathbf{k}} t]$$

$$\begin{pmatrix}
-k^2\kappa_\rho + i\omega_{\mathbf{k}} & ik^x\rho_0 & ik^y\rho_0 & \dfrac{d\rho_0}{dz} & 0 & 0 & 0 \\
-ik^x c_s^2 & \kappa_\rho \dfrac{d^2\rho_0}{dz^2} + i\omega_{\mathbf{k}}\rho_0 & 0 & 0 & 0 & 0 & 0 \\
-ik^y c_s^2 & 0 & \kappa_\rho \dfrac{d^2\rho_0}{dz^2} + i\omega_{\mathbf{k}}\rho_0 & 0 & -ik^y B_0 & ik^x B_0 & 0 \\
0 & ik^x\kappa_\rho & ik^y\kappa_\rho & \kappa_\rho \dfrac{d^2\rho_0}{dz^2} + i\omega_{\mathbf{k}}\rho_0 & 0 & 0 & ik^x B_0 \dfrac{d\rho_0}{dz} \\
0 & k^2\gamma & -ik^y B_0 & 0 & -k^2\eta_{\mathrm{T}} + \chi_\rho \dfrac{d^2\rho_0}{dz^2} + i\omega_{\mathbf{k}} & 0 & ik^y\alpha \\
0 & 0 & k^2\gamma + ik^x B_0 & 0 & 0 & -k^2\eta_{\mathrm{T}} + \chi_\rho \dfrac{d^2\rho_0}{dz^2} + i\omega_{\mathbf{k}} & -ik^x\alpha \\
0 & ik^x B_0 & 0 & k^2\gamma & -ik^y\alpha & ik^x\alpha & -k^2\eta_{\mathrm{T}} + i\omega_{\mathbf{k}}
\end{pmatrix}
\begin{pmatrix}
\hat{\rho} \\ \hat{u}^x \\ \hat{u}^y \\ \hat{u}^z \\ \hat{b}^x \\ \hat{b}^y \\ \hat{b}^z
\end{pmatrix} = \mathbf{0}$$

Dispersion relation

$$\chi_\rho \frac{d^2\rho_0}{dz^2} - \eta_{\mathrm{T}} k^2 + i\omega_{\mathbf{k}} = 0$$

$$\mathrm{Im}\ \omega_{\mathbf{k}} = -\eta_{\mathrm{T}} k^2 + \chi_\rho \frac{d^2\rho_0}{dz^2}$$

$$\delta B^\alpha = \hat{b}^\alpha \exp\left(-\eta_{\mathrm{T}} k^2 + \chi_\rho \frac{d^2\rho_0}{dz^2}\right)\exp[i(k^x x + k^y y)]$$

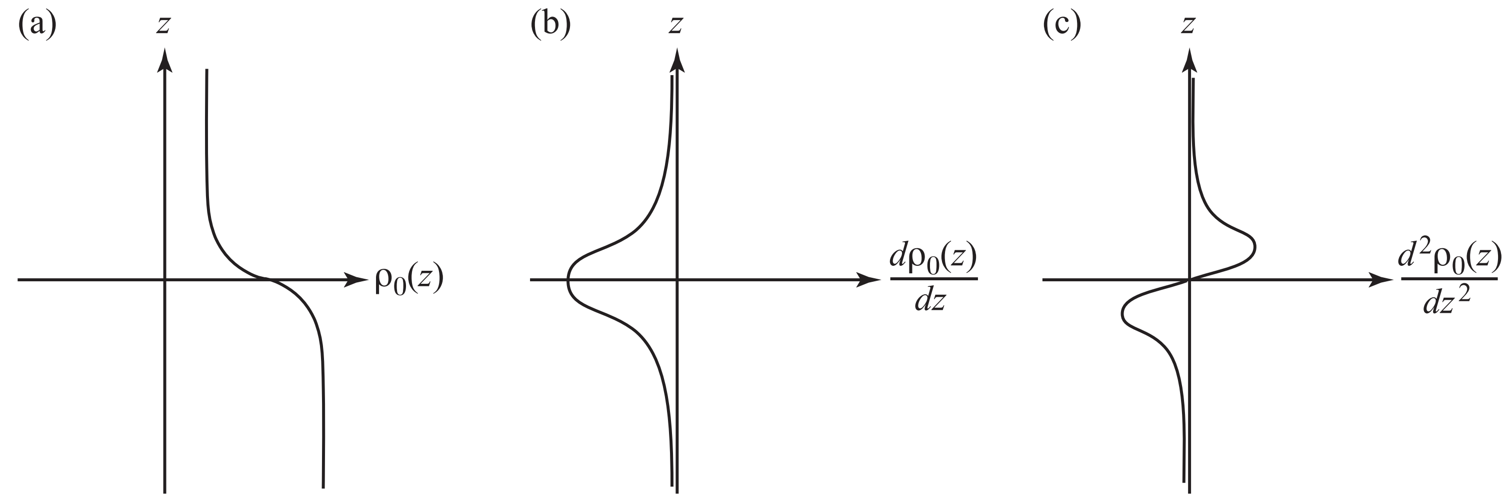
Density modulation

$$\overline{\rho} = \rho_0 + \delta\rho$$

$$\rho_0(z) = \rho_r - \rho_c \tanh\left(z/z_c\right)$$

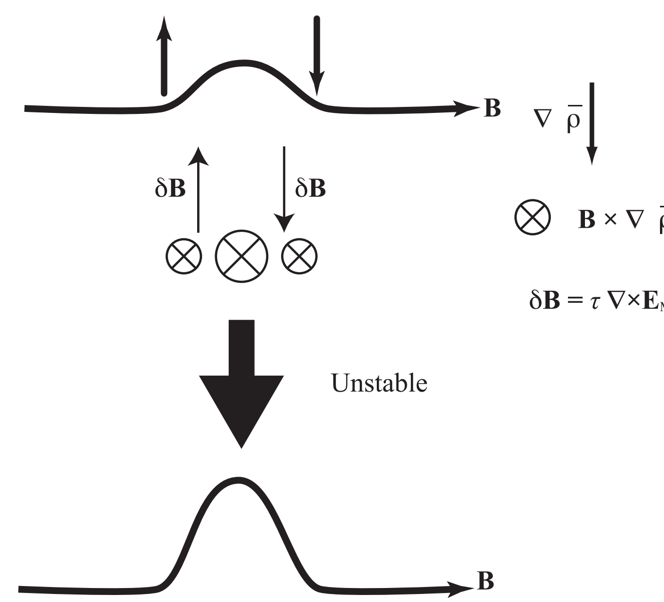
$$\frac{d\rho_0(z)}{dz} = -\frac{\rho_c}{z_0}\frac{1}{\cosh^2\left(z/z_c\right)}$$

$$\frac{d^2\rho_0(z)}{dz^2} = +\frac{2\rho_c}{z_c^2}\frac{\tanh(z/z_c)}{\cosh^2\left(z/z_c\right)}$$



$$\left\langle \mathbf{u}' \times \mathbf{b}' \right\rangle / \mu_0 = +\chi_\rho \mathbf{B} \times \nabla\overline{\rho} + \cdots$$

$$\chi_\rho \propto \langle \rho'^2 \rangle$$



$$\mathrm{Im}\ \omega = -\beta k^2 + \chi_\rho \frac{d^2\rho_0}{dz^2}$$

$$\delta B^\alpha = \hat{b}^\alpha \exp\left(-\eta_{\mathrm{T}} k^2 + \chi_\rho \frac{d^2\rho_0}{dz^2}\right) \exp[i(k^x x + k^y y)]$$

Self-consistent model (closure)

$$\langle \mathbf{u}' \times \mathbf{b}' \rangle = -(\beta + \zeta)\nabla \times \mathbf{B} + \alpha \mathbf{B} - (\nabla \zeta) \times \mathbf{B} + \gamma \nabla \times \mathbf{U}$$

$$- \chi_{\overline{\rho}} \nabla \overline{\rho} \times \mathbf{B} - \chi_Q \nabla Q \times \mathbf{B} - \chi_D \frac{D\mathbf{U}}{Dt} \times \mathbf{B}$$

$$\langle \rho' \mathbf{u}' \rangle = -\kappa_{\overline{\rho}} \nabla \overline{\rho} - \kappa_Q \nabla Q - \kappa_D \frac{D\mathbf{U}}{DT} - \kappa_B \mathbf{B}$$

Turbulent energy

$$\left(\frac{\partial}{\partial t} + \mathbf{U} \cdot \nabla \right) \langle \mathbf{u}'^2 + \mathbf{b}'^2 \rangle / 2 = -\langle \mathbf{u}'\mathbf{u}' - \mathbf{b}'\mathbf{b}' \rangle \cdot \nabla \mathbf{U} - \langle \mathbf{u}' \times \mathbf{b}' \rangle \cdot \nabla \times \mathbf{B} - \varepsilon_K + \cdots$$

Turbulent cross helicity

$$\left(\frac{\partial}{\partial t} + \mathbf{U} \cdot \nabla \right) \langle \mathbf{u}' \cdot \mathbf{b}' \rangle = -\langle \mathbf{u}'\mathbf{u}' - \mathbf{b}'\mathbf{b}' \rangle \cdot \nabla \mathbf{B} - \langle \mathbf{u}' \times \mathbf{b}' \rangle \cdot \nabla \times \mathbf{U} - \varepsilon_W + \cdots$$

Density variance

$$\left(\frac{\partial}{\partial t} + \mathbf{U} \cdot \nabla \right) \left\langle \rho'^2 \right\rangle = -2 \left\langle \rho' \mathbf{u}' \right\rangle \cdot \nabla \overline{\rho} - 2 \left\langle \rho'^2 \right\rangle \nabla \cdot \mathbf{U} + \cdots$$

Turbulent residual helicity

$$\left(\frac{\partial}{\partial t} + \mathbf{U} \cdot \nabla \right) \langle \mathbf{b}' \cdot \mathbf{j}' - \mathbf{u}' \cdot \boldsymbol{\omega} \rangle = -\langle \mathbf{u}'\mathbf{u}' - \mathbf{b}'\mathbf{b}' \rangle \cdot \nabla \boldsymbol{\Omega} - \frac{1}{\tau \beta} \langle \mathbf{u}' \times \mathbf{b}' \rangle \cdot \mathbf{B} - \varepsilon_H + \cdots$$

$$\mathbf{j}' = \nabla \times \mathbf{b}', \;\; \boldsymbol{\omega}' = \nabla \times \mathbf{u}', \;\; \boldsymbol{\Omega} = \nabla \times \mathbf{U}$$

Summary

- **Magnetoclinicity: Dynamo in strong compressibility**
- **Transports in strongly compressible MHD turbulence**
- **Large-scale instability**
- **Turbulence modelling based on analytical theory**

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