

When does the Lorenz Model exhibit the Signal-to-Noise Paradox?

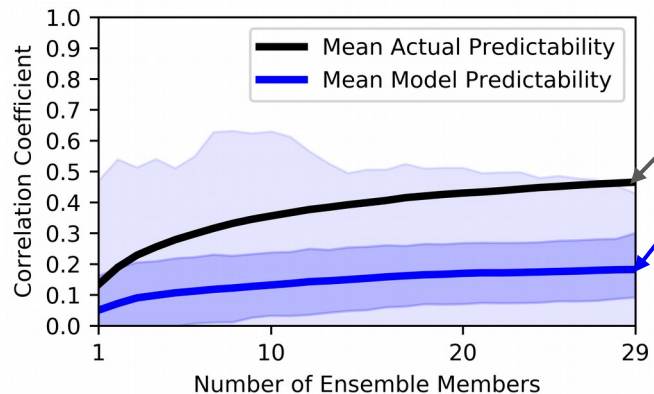
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The SNP in seasonal predictions with a comprehensive Earth System Model



The Signal-to-Noise Paradox (SNP, Dunstone et al., 2016 & Scaife et al., 2018):

Actual Predictability: Model predicting actual world

Model Predictability: Model predicting itself

$$\text{Ratio of Predictable Components (RPC)} = \frac{\text{Actual Predictability}}{\text{Model Predictability}} > 1$$

RPC expected to be 1 but empirically $\text{RPC} > 1 \rightarrow$ Signal-to-Noise Paradox

Figure 1. Mean actual and model predictability for seasonal predictions with MPI-ESM for different ensemble sizes. The dark shading indicates the interquartile range, while the light shading indicates the minimum and maximum over 1000 random permutations of the ensemble.

Current Hypotheses:

1. Model deficiencies (e.g., Smith et al., 2016; O'Reilly et al., 2019; Stockdale et al., 2015)
2. Statistical uncertainties (e.g. Weisheimer et al., 2019)

Here, we formulate an alternative hypothesis by investigating under which conditions the SNP occurs in a simple dynamical model, the Lorenz 1963 Model (Lorenz, 1963). Using a simple model allows us to explicitly formulate a perfect model framework, where model deficiencies can be ruled out and conduct computationally cheap experiments with different model setups.

Seasonal Predictions in the Lorenz 1963 Model - a conceptual Framework

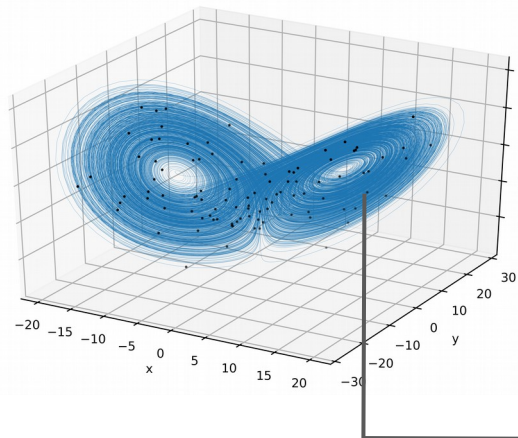


Figure 2. Pool of initial conditions (blue line) acquired by integrating the model after a initial spinup phase for 10000 time units. Black dots denote the 100 randomly chosen initial states for the 100 forecasts.

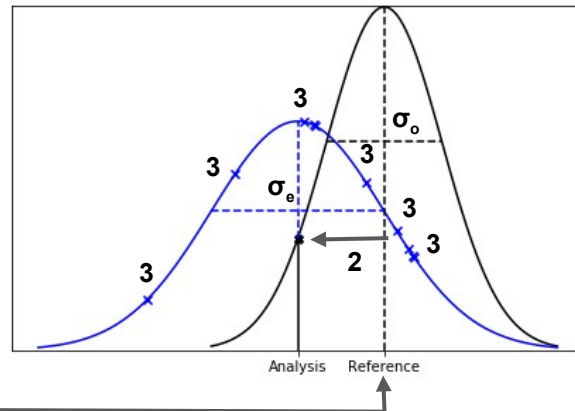


Figure 3. Illustration of the generation of initial states. The black line denotes the observational probability density function which is centered around the true value (reference). From this PDF the analysis is drawn (black dot). Around the analysis an ensemble (blue crosses) is generated by sampling from the ensemble PDF (blue line) with standard deviation σ_e .

Ensemble Generation:

- 1) Random initial state on the Lorenz Attractor chosen as initial state for the **reference**.
- 2) The initial **analysis** is drawn from a normal distribution around the initial reference with standard deviation σ_o (**observational spread**).
- 3) 100 initial states for the ensemble are drawn from a normal distribution around the initial analysis with a standard deviation of σ_e (**initial ensemble spread**).

Time Integration:

- 4) Model is integrated for 180 days starting from the initial reference and from each of the 100 ensemble members.
- 5) Analysis is generated by adding normally distributed noise on the reference run with standard deviation σ_o .
- 6) Ensemble simulations and analysis are averaged over one season (90 days) with 30 day lead-time.

Repeated 100 times to yield 100-year 100-member ensemble hindcast

Equal Initial Spread experiment

We conduct 100 Hindcast Experiments each comprising 100 years and 100 ensemble members with initial ensemble spread σ_e being equal to the observational spread σ_o but using different randomly chosen initial states.

The RPC as well as the actual predictability vary between the experiments, but the average over all experiments is close to the expected behaviour of $\text{RPC}=1$.

For all ensemble sizes the mean actual predictability is within the interquartile range of the model predictability for the experiment that is close to the average behaviour in Figure 4 a).

The SNP does not occur in the Lorenz Framework if the initial ensemble spread represents the observational spread.



What happens if we change the initial ensemble spread ?

Equal initial ensemble spread $\sigma_e = 0.01$, $\sigma_o = 0.01$

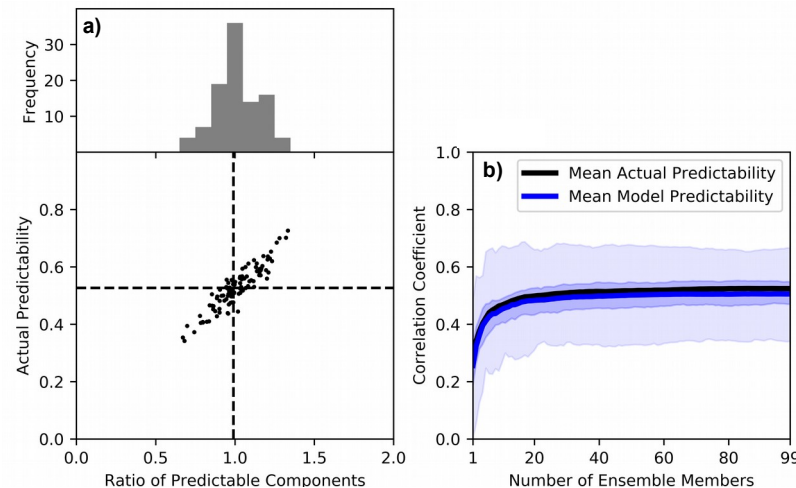


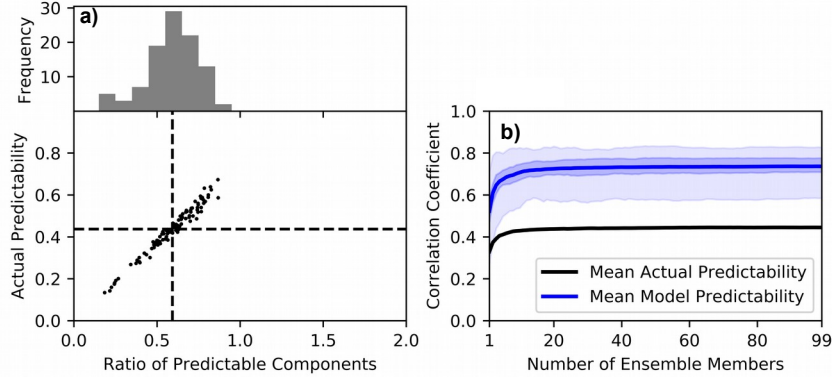
Figure 4.

a): Actual Predictability against RPC for 100 seasonal hindcast experiments at one month lead-time with equal initial ensemble spread and observational spread. The horizontal (vertical) dashed line indicates the mean of the RPC (Actual Predictability) over all hindcast experiments.

b): Mean Actual Predictability and Mean Model Predictability in dependence of the ensemble size for a representative experiment. The dark shading indicates the interquartile range, while the light shading indicates the minimum and maximum over 100 random permutations of the ensemble.

Unequal initial Spread Experiments

Low initial ensemble spread $\sigma_e = 0.001$, $\sigma_o = 0.01$



High initial ensemble spread $\sigma_e = 0.1$, $\sigma_o = 0.01$

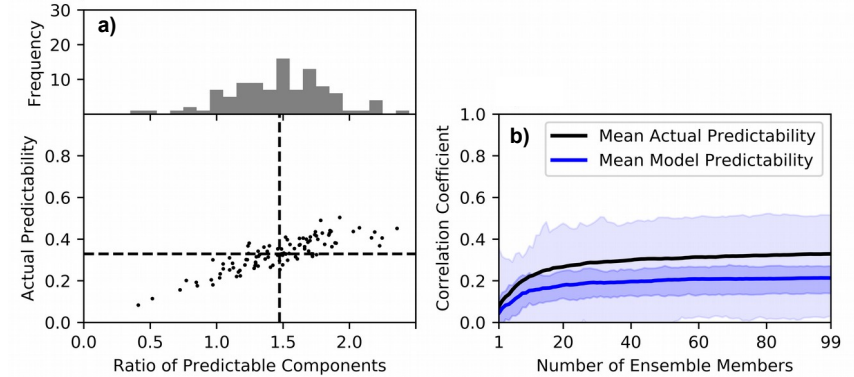


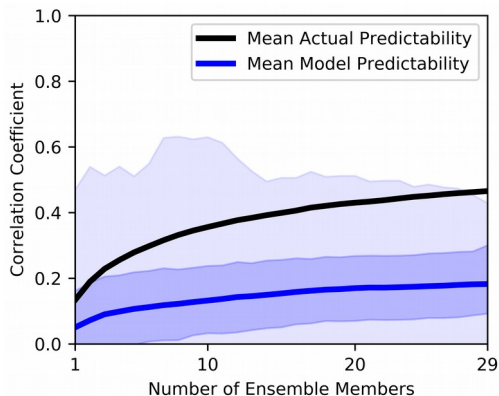
Figure 5.
a): Actual predictability against RPC for 100 seasonal hindcast experiments at one month lead-time with a 10 times lower (left) and higher (right) initial ensemble spread compared to the observational spread. The horizontal (vertical) dashed line indicates the mean of the RPC (Actual Predictability) over all hindcast experiments.
b): Mean Actual Predictability and mean model predictability in dependence of the ensemble size for a representative experiment. The dark shading indicates the interquartile range, while the light shading indicates the minimum and maximum over 100 random permutations of the ensemble.

Even when the model is perfect, i.e. analysis and forecasts are produced by the same model with the same parameterization, the average model behaviour can appear to be overconfident (RPC < 1) or underconfident (RPC > 1) depending on the ratio of initial ensemble spread to observational spread.

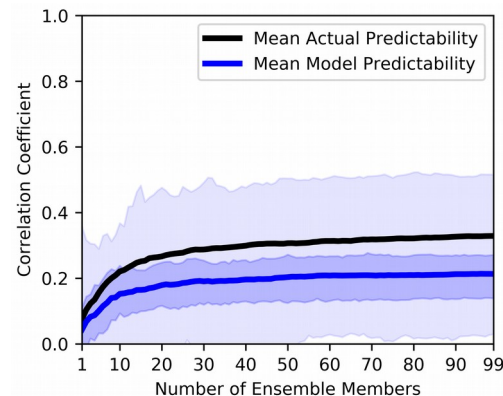
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Based on our experiments, we conclude that the Signal-to-Noise Paradox can occur if in the process of initialization the ensemble spread is overestimated compared to the observational spread even in a perfect model framework. Our results suggest that the magnitude of the initial ensemble spread relative to the observational spread could be an alternative hypothesis for the origin of the SNP.

Comprehensive ESM



Conceptual Model



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