Using spherical scaling functions in scalar and vector airborne gravimetry

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Overview

- Motivation
- Gravity model based on spherical scaling functions
- Application in scalar gravimetry
- Application in vector gravimetry
- Conclusions

Motivation

- ***Scalar gravimetry.*** There are various methods (stochastic, deterministic) for mapping gravity at the flight height given along-line gravity estimates. Gravity is often treated as an isotropic stationary process in 2-D.

  - Actual gravity is not necessarily isotropic.
  - Deterministic methods often ignore gravity spatial correlation and/or simplify the along-line estimate error models.

- ***Vector gravimetry.*** Estimation of gravity vector at the flight path is based on IMU-GNSS integration. Gravity is modeled in time (e.g., as a stationary process).

  - Horizontal components are poorly observable.
  - Additional information is required to separate these from inertial sensor errors.

- *The use of an a-priori spatial gravity model (without assuming isotropy) may improve results in each of the two problems.*

Spatial gravity modeling

- Representation of disturbing potential using spherical scaling functions (SSFs) [1]:

$$T(\boldsymbol{r}) = \sum_k a_k \Phi_j(\boldsymbol{r}, \mathbf{z}_k)$$

where j is the resolution level, $\{a_k\}$ are the unknown scaling coefficients, $\mathbf{z}_k$ is a point of grid on the sphere of radius R.

- Representation of SSF via the Legendre polynomials P_n:

$$\Phi_j(\boldsymbol{r}, \mathbf{z}_k) = \frac{1}{4\pi} \sum_{n=2}^{\infty} (2n+1) \left(\frac{R}{r}\right)^{n+1} b_{j,n} P_n(\hat{\boldsymbol{r}}^T \hat{\mathbf{z}}_k)$$

where $\hat{\boldsymbol{r}}, \hat{\mathbf{z}}_k$ are the unit vectors, $r=|\boldsymbol{r}|$, $b_{j,n}$ is the Legendre coefficient.
For the Abel-Poisson SSF, $b_{j,n}=\exp(-2^{-j}n)$.

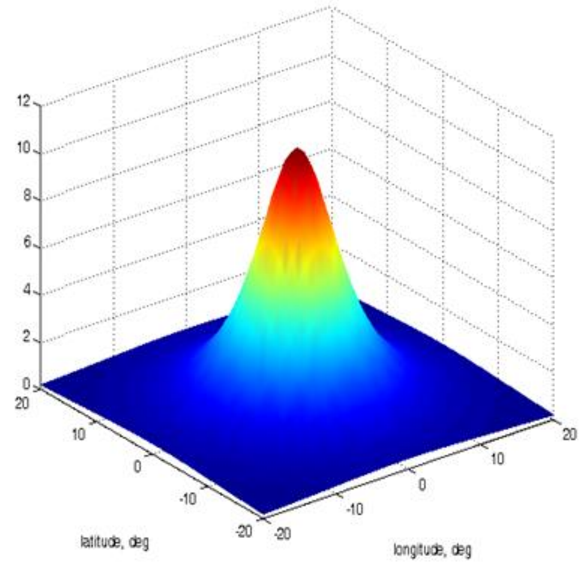


Figure: The Abel-Poisson SSF.

- *Localization in the spatial and frequency domains.*
- *When $\{a_k\}$ are known, functionals of T can be computed at any point outside the sphere.*

Spatial gravity modeling in scalar gravimetry

Scalar gravimetry: along-line estimate model

- Along-line gravity disturbance estimate model:

$$\delta g'(t) = \sum_{\tau} w(t-\tau)\delta g(\tau) + e_g(t)$$

Notation: w is the weight function of a low-pass filter, e_g is the gravity estimate error.

- In the software for the GT-2A gravimeter, gravity is estimated by Kalman filtering [2]. The KF is “close” to the Butterworth filter of order 4 (Fig.1-2). The shaping filter for e_g is

$$W_e(\omega) = \frac{i\omega}{1+(\omega/\omega_c)^4}$$

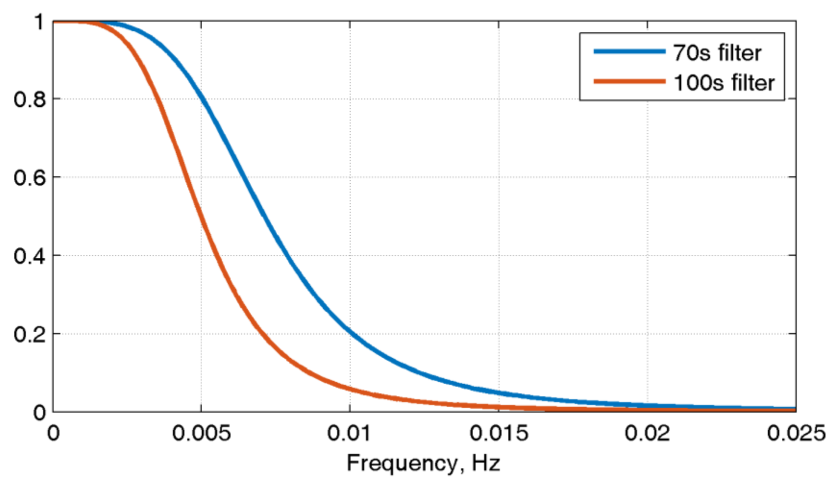


Fig.1. The filter transfer function.

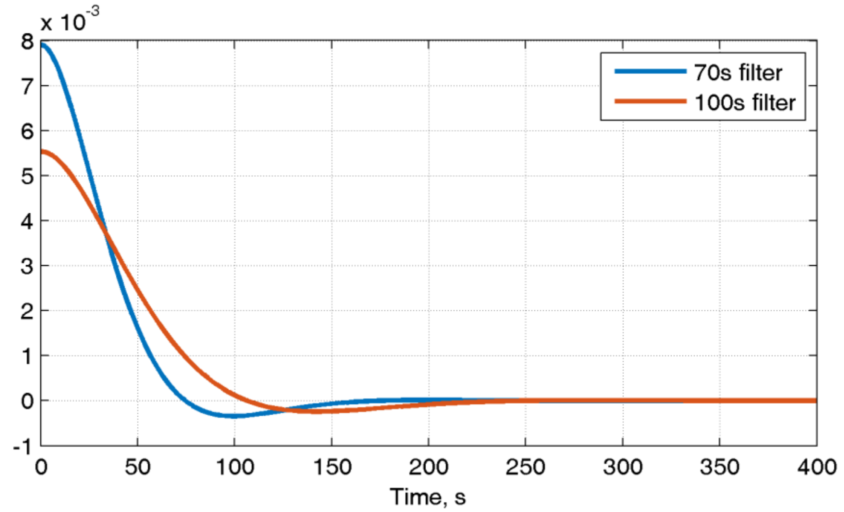


Fig.2. The filter impulse response function.

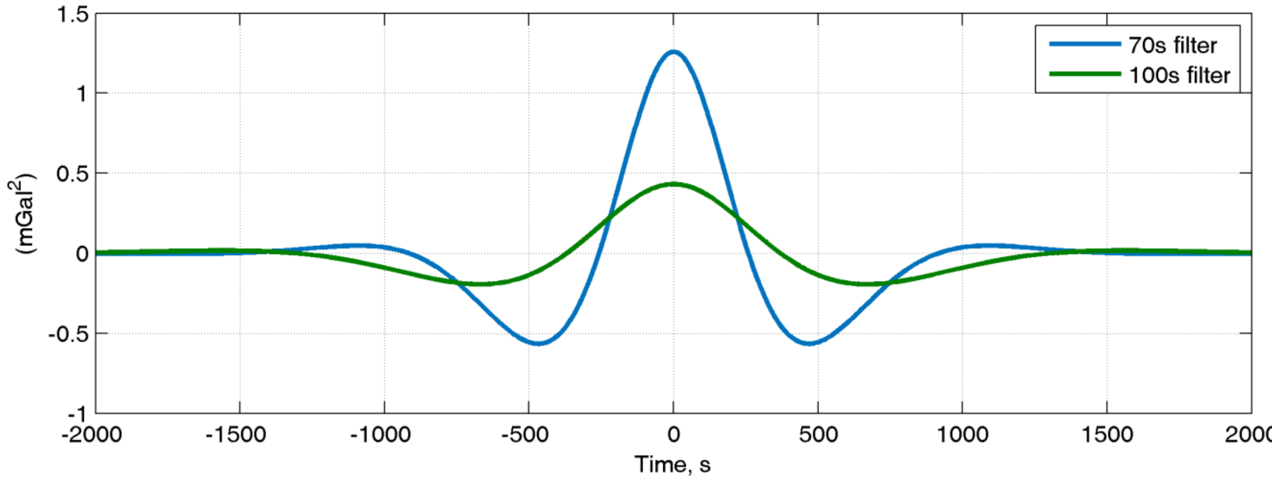


Fig.3. The autocorrelation function of the along-line gravity estimate errors.

- *The estimate error e_g is strongly correlated at a line.*
- *The estimate errors at adjacent lines are not correlated.*

Scalar gravimetry: LS problem

- The along-line gravity estimate model:

$$\delta g'(t) = \sum_{\tau} w(t-\tau)\delta g(\boldsymbol{r}(\tau)) + e_g(t), \quad C_e(t) = E[e_g(s)e_g(s+t)]$$

- Gravity disturbance representation based on SSFs:

$$\delta g(\boldsymbol{r}(\tau)) = \sum_{k} a_k \frac{\partial \Phi_j(\boldsymbol{r}(\tau), \mathbf{z}_k)}{\partial r}$$

- The along-line estimate model in matrix form (l is the nr. of a line):

$$\boldsymbol{g}_l' = \mathbf{H}_l \boldsymbol{a} + \boldsymbol{e}_l, \quad \mathbf{C}_l = E[\boldsymbol{e}_l \boldsymbol{e}_l^T], \quad l = 1, \ldots, L$$

Here, $\boldsymbol{a}$ is the SC vector for the computation area, $\boldsymbol{g}_l'$ is the vector of observations at the l-th line, $\mathbf{H}_l$ is the matrix of the SSF derivative and weight function values, L is the nr. of lines.

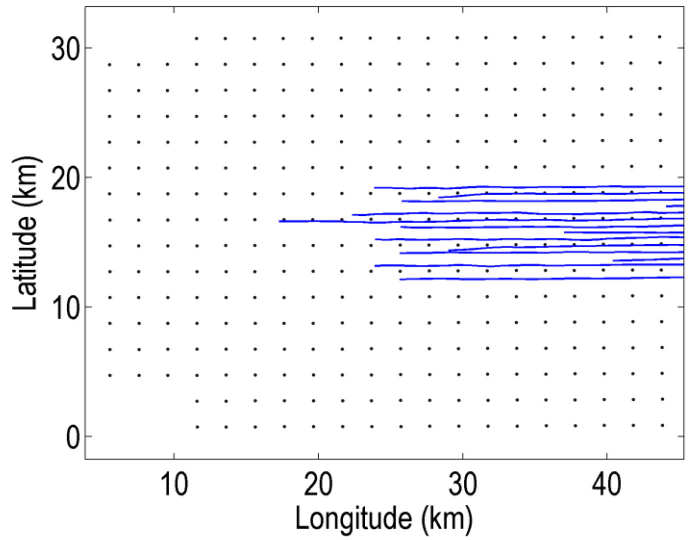


Figure: grid points and line fragments.

- LS problem for the scaling coefficient vector $\boldsymbol{a}$:

$$\sum_{l=1}^{L} \left\| \boldsymbol{g}_l' - \mathbf{H}_l \boldsymbol{a} \right\|^2_{\mathbf{C}_l^{-1}} \to \min_{\boldsymbol{a}}$$

Scalar gravimetry: recursive LSE algorithm

1. Setting the SSF model parameters (resolution level; grid density and size).
2. LS estimation of scaling coefficients:

- Recursive implementation (as along-line estimate errors at any two lines are not correlated).
- Information form of LSE (due to rank deficiency of the problem at recursion steps).

<u>Steps of recursive LSE:</u>

- Determining grid points for the l-th line;
- Designing the coefficient matrix $\mathbf{H}_l$ and the covariance matrix $\mathbf{C}_l$;
- LS update of the information vector $\boldsymbol{y}_l$ and information matrix $\mathbf{P}_l^{-1}$:

$$\boldsymbol{y}_l = \boldsymbol{y}_{l-1} + \mathbf{C}_l^{-1}\mathbf{H}_l\boldsymbol{g}_l', \quad \mathbf{P}_l^{-1} = \mathbf{P}_{l-1}^{-1} + \mathbf{H}_l^T\mathbf{C}_l^{-1}\mathbf{H}_l$$

  where $\boldsymbol{y}_l = \mathbf{P}_l^{-1}\boldsymbol{a}, \quad \boldsymbol{y}_0 = 0,\ \mathbf{P}_0^{-1} = \mathbf{0}.$

- At the last step ($l=L$), the covariance matrix of the SC estimate errors $\mathbf{P}_L$ is computed (via inversion of the information matrix and Tikhonov regularization), and the estimate of SCs is obtained from $\boldsymbol{y}_L = \mathbf{P}_L^{-1}\boldsymbol{a}.$

Scalar gravimetry: results

Survey area and data

- Stavropol region, Russia (2005)
- 14 lines spaced at ~500 m
- flight altitude is 540 m (+/-50 m)
- GT-2A gravimeter
- along-line gravity estimates (sampled at 0.1 Hz)
- 100-s Kalman filter, equiv. spatial res. 4 km
- STD of the along-line estimate errors: 0.6 mGal.

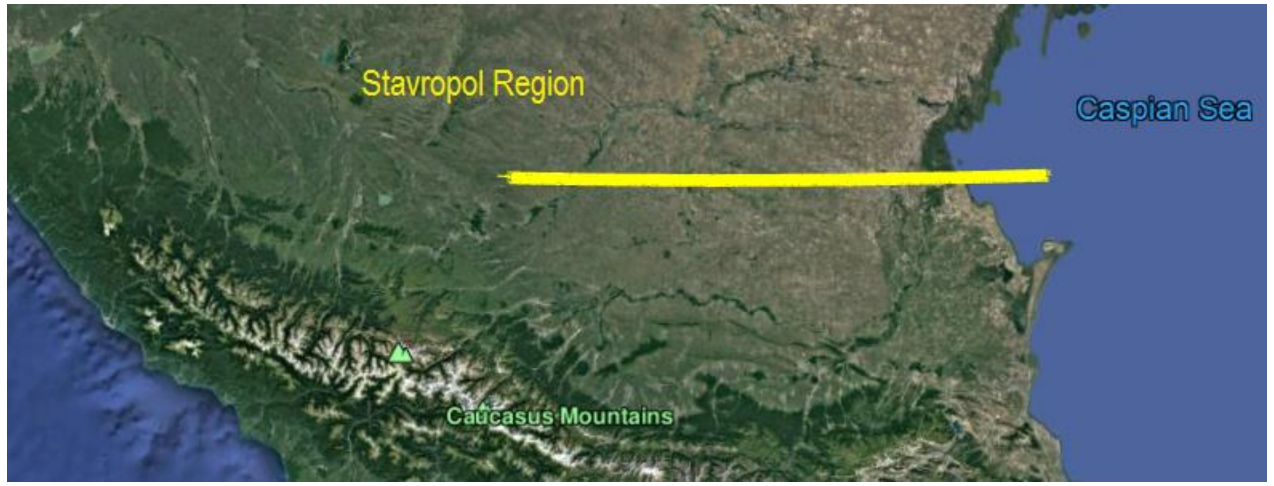


Fig.1. Flight tracks on top of the ground image.

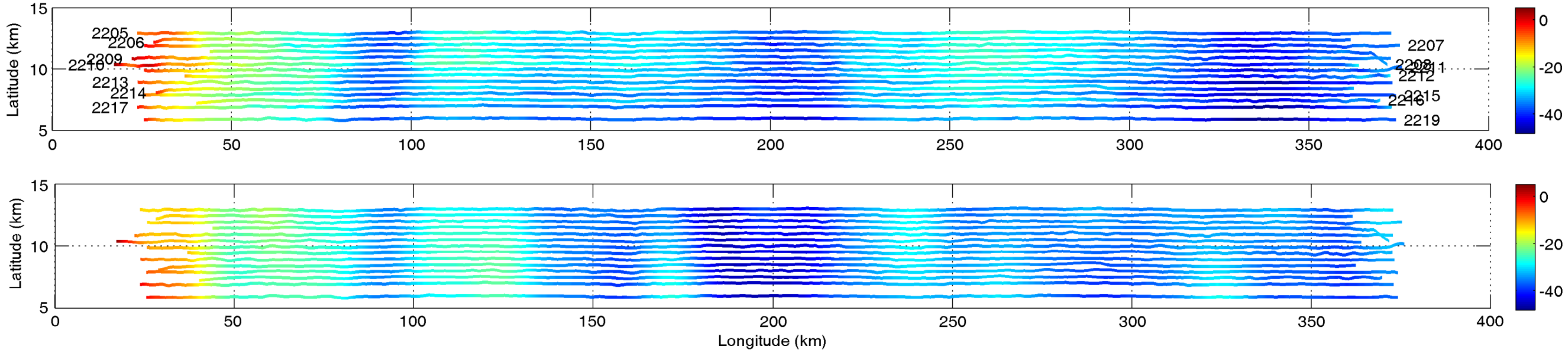


Fig.2. Airborne gravity at lines and ID numbers of lines (upper). EGM08 gravity at the same lines (lower), shown for comparison. All values in mGal. EGM08 data has poor quality in this area due to “fill-in” (in particular, near the coastline).

Scalar gravimetry: results (2)

- LS algorithm (SSF res. level j=12; 1.5 km x 1.5 km lat-lon grid; 4509 grid points).
- Reconstruction of gravity disturbances at lines (overall RMS: 1.5 mGal):

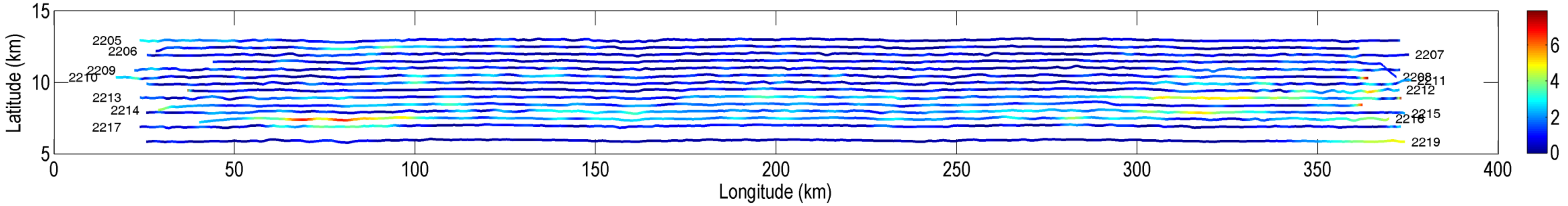


Fig.1. Absolute values of the differences between the original airborne gravity and gravity reconstructed from the scaling coefficient estimates, mGal.

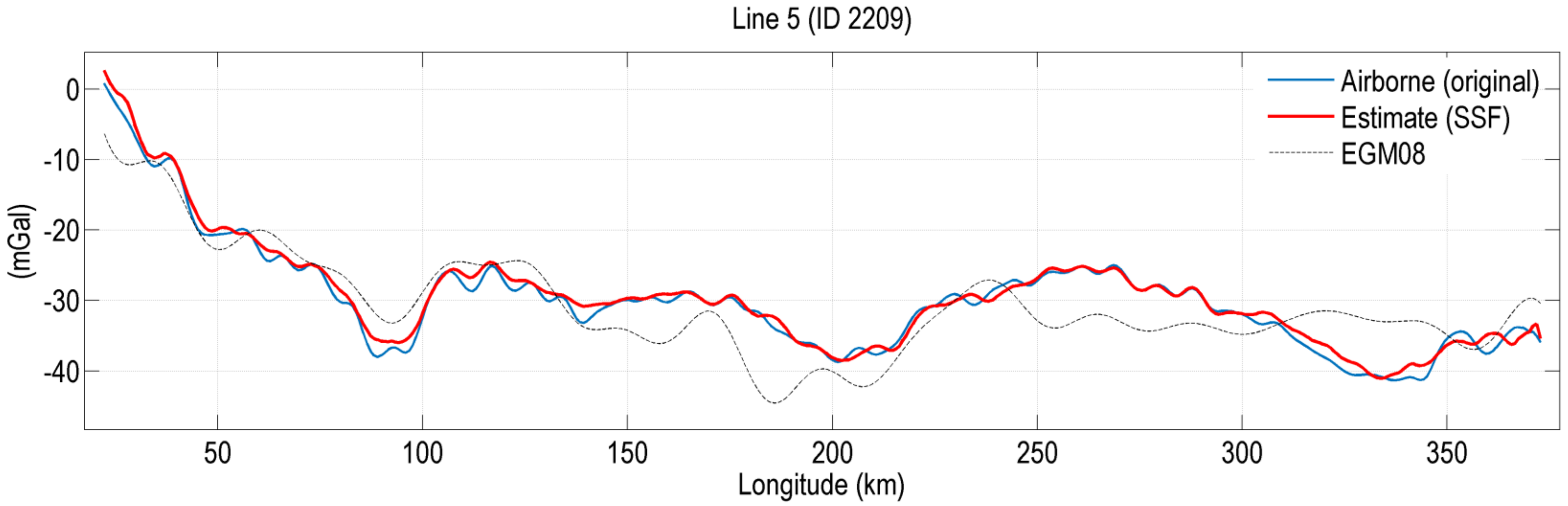


Fig.2. The original airborne gravity (blue), reconstructed gravity (red), EGM08 (black), mGal.

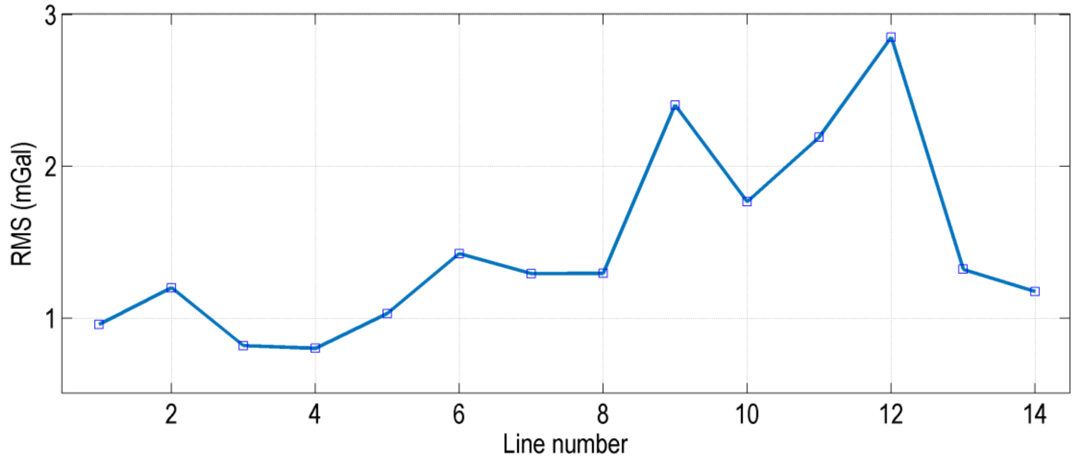


Fig.3. RMS of the differences between the reconstructed and original gravity. Greater RMS (lines 9-12) is due to bias in the original airborne data.

- *Reconstructed gravity shows better repeatability at adjacent lines than the original data.*
- *Bias of 2 mGal in the original airborne data was revealed (lines 9-12).*

Scalar gravimetry: results (3)

- Comparison with EGM08:

  - Bias of 2 mGal in the airborne data is confirmed (at the same lines 9-12).
  - The reconstructed gravity shows no bias at these lines.

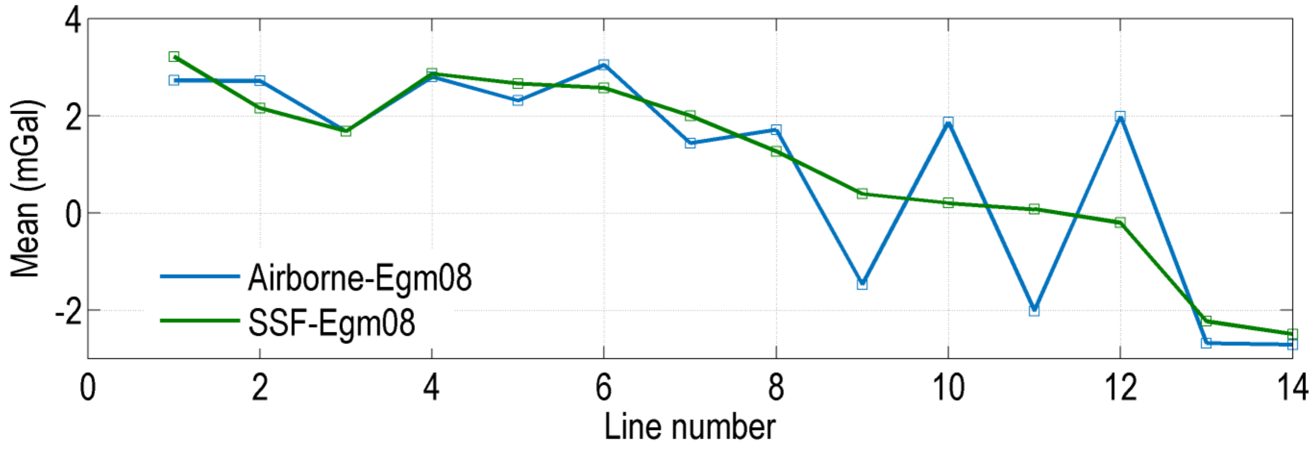


Fig.1. Mean values of the differences between the original gravity and EGM08 (blue) and between the reconstructed gravity and EGM08 (green), mGal.

- Cross validation:

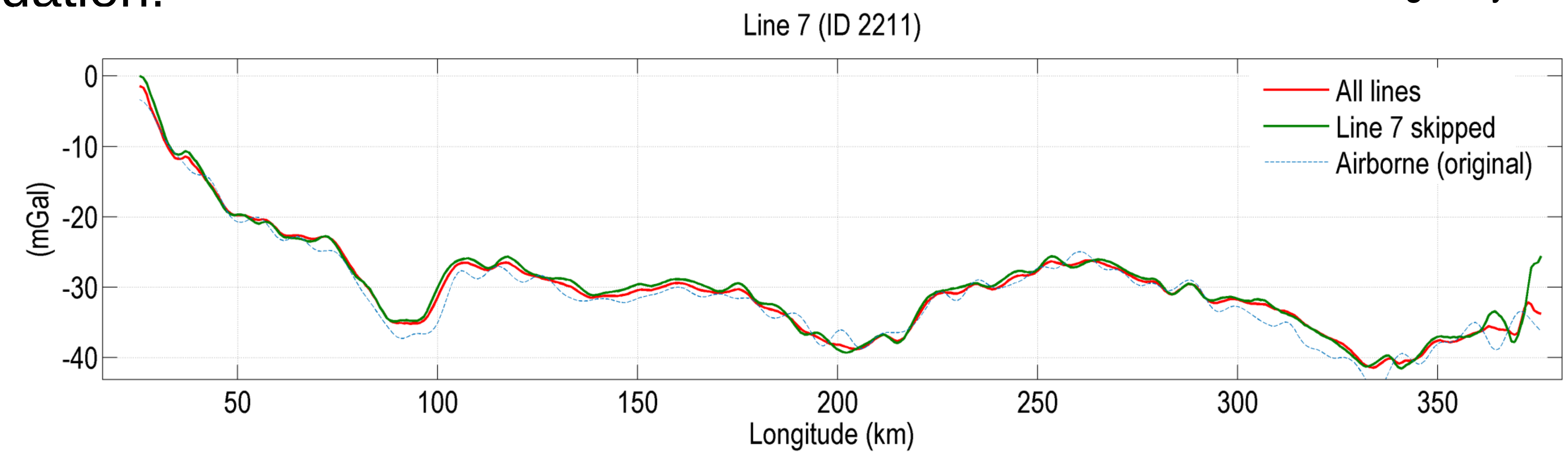
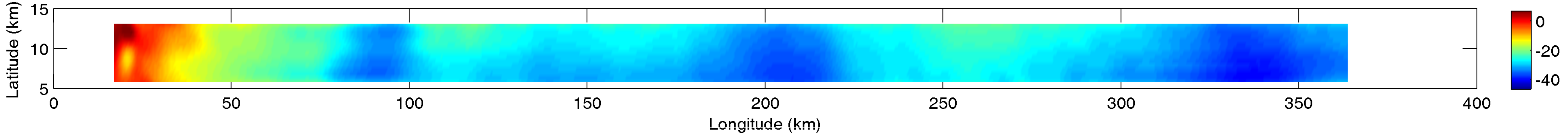


Fig.2. Gravity reconstructed at line 7 from two estimates of scaling coefficients: computed using data at all lines (red); computed without using data at line 7 (green).

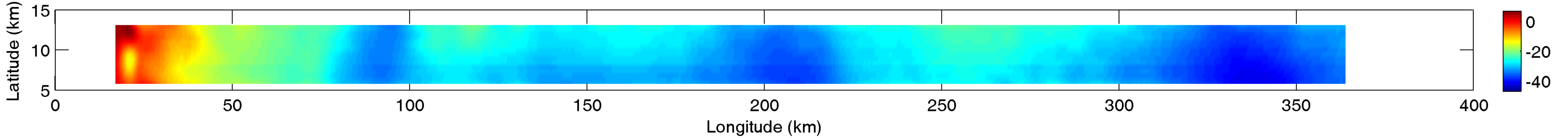
➢ *The gravity estimation accuracy from the cross validation is 0.5 mGal (RMS).*

Scalar gravimetry: results (4)

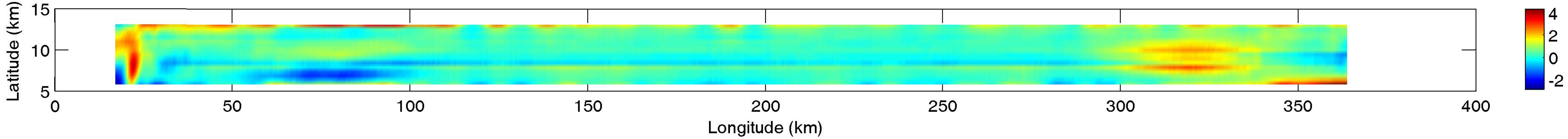
- Taking into account autocorrelation of along-line estimate errors (full covariance matrices $\mathbf{C}_l$):



- Neglecting autocorrelation of along-line gravity estimate errors (diagonal covariance matrices $\mathbf{C}_l$):



- The difference between the two maps:



Figures: Gravity interpolated at 0.5 km x 0.5 km lat-lon grid at h=590 m above the ellipsoid: full error covariance matrices were used (upper), diagonal error covariance matrices were used (middle), the difference between the two results (lower). All values in mGal.

- *In the case of diagonal covariance matrices, gravity has a more pronounced isotropy ("blurring"). When using full covariance matrices, the result is smoothed in the W-E direction (along the lines).*
- *The difference between the two results is 0.8 mGal (RMS).*

Spatial gravity modeling in vector gravimetry

Vector gravimetry: gravity estimation using SSFs

- Based on INS-GNSS integration and Kalman filtering. Gravity vector is modeled in *time* (general approach).
- Difficult to separate gravity horizontal components from INS systematic errors (in general approach). To improve separation, we use the *spatial* model based on the SSFs.
- Key features of the proposed estimation algorithm [3]:
  - Along-path gravity vector estimation using the SSFs and Kalman filter.
  - Scaling coefficients of the gravity model are incorporated in the KF via:

    $$\dot{a}_k(t) = 0, \quad k = 1, \dots, N$$

    where t is the survey flight time, N is the total number of SCs in the area.
  - Information form of KF is used (due to rank deficiency at recursion steps).
  - Scaling coefficient estimate is obtained at the last step of the KF recursions (using regularization due to the ill-conditioned problem).

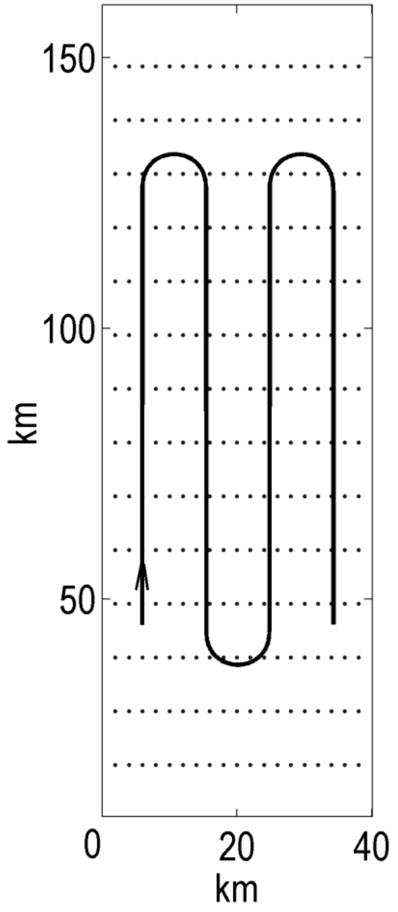
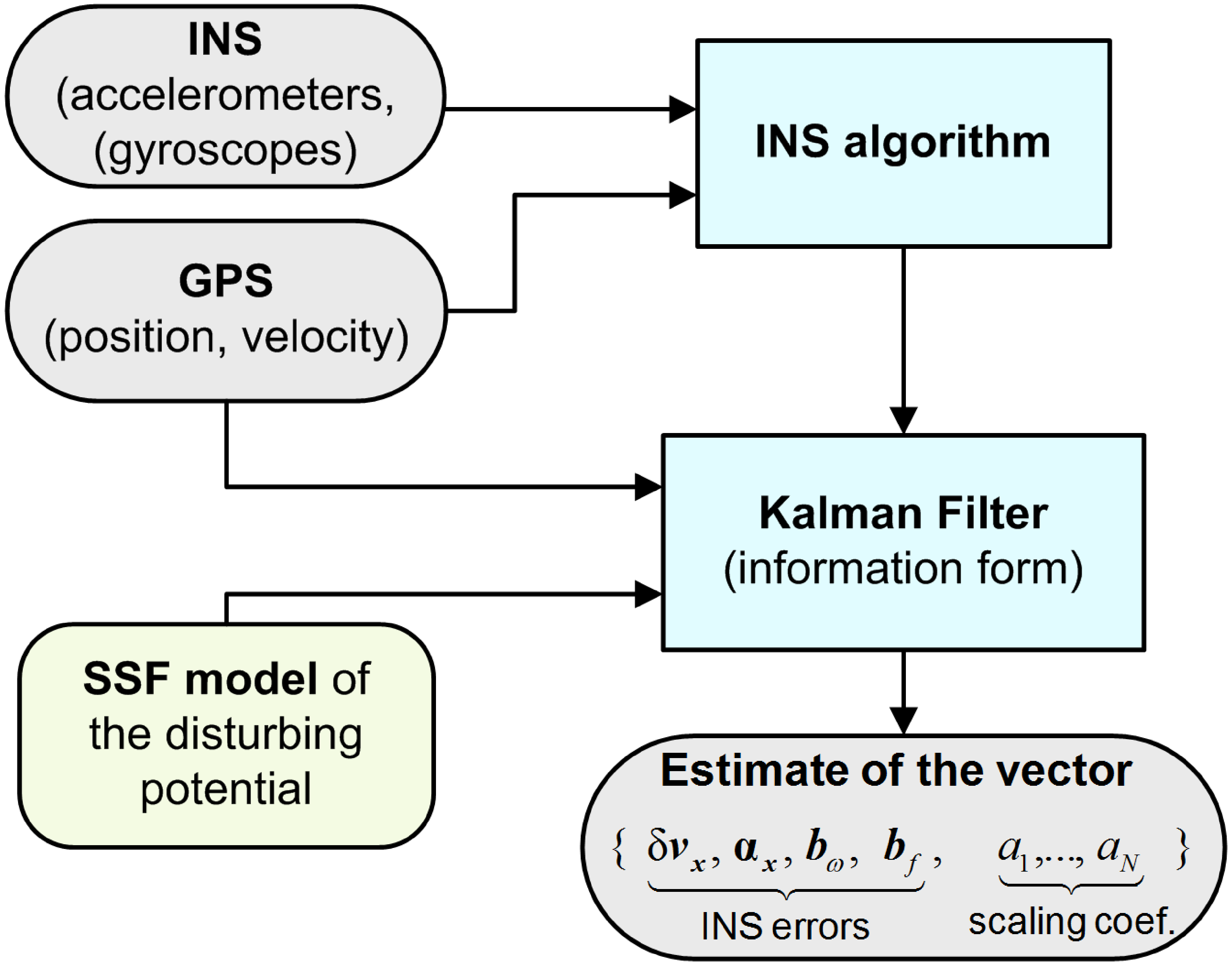


Figure: simulated flight path and lat-lon grid points.

Vector gravimetry: proposed algorithm



Vector gravimetry: results

- Simulated data (assuming a navigation-grade INS and differential carrier-phase mode of GNSS).
- Comparing the general approach (*modeling in time*) with the proposed approach (*spatial modeling*):

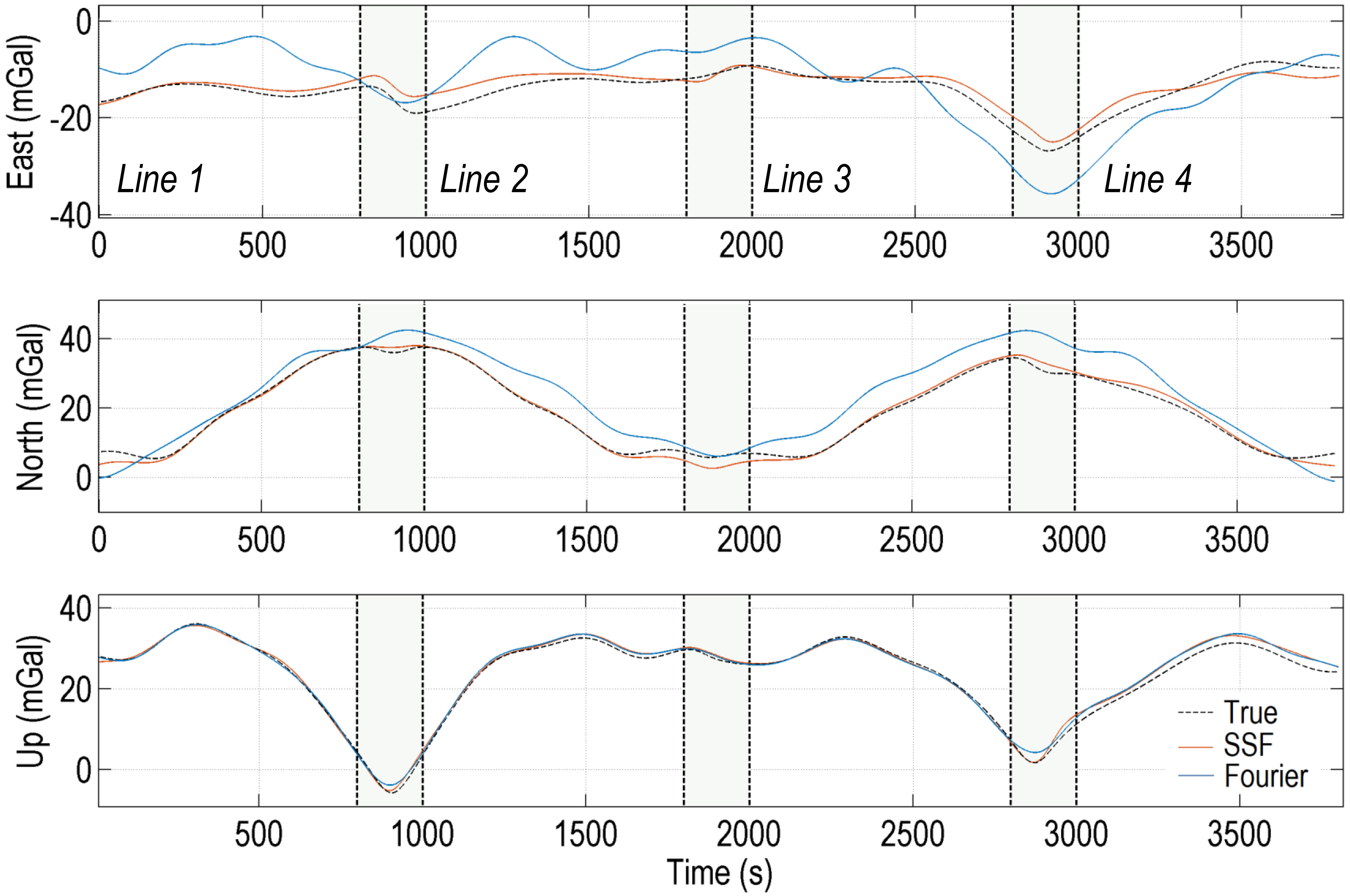


Figure: Gravity vector components: estimated using the proposed approach (red), estimated using the general approach (blue), true gravity (black), mGal.

➢ *The proposed approach showed significantly better accuracy for the horizontal components (1.0-1.2 mGal STD and 0.4-1.0 mGal Mean) than the general approach.*

Conclusions

- Spatial gravity modeling based on spherical scaling functions is capable to improve results in scalar and vector airborne gravimetry.

- *In scalar gravimetry:*
  - the gravity mapping algorithm based on LS technique was developed;
  - the algorithm takes into account gravity spatial behavior
  - and uses full statistical information about along-line gravity estimate errors;
  - the algorithm has a numerically effective recursive (line-by-line) implementation.

- *In vector gravimetry:*
  - an approach to gravity vector estimation based on Kalman filtering and spatial gravity modeling is proposed;
  - the approach shows significantly better results for gravity horizontal components than the general approach based on modeling in time (from processing simulated data).

References

[1] W. Freeden, V. Michel (2004) Multiscale potential theory (with applications to geoscience). Birkhauser Verlag.

[2] Y.V. Bolotin, A.A. Golovan (2013) Methods of inertial gravimetry. Moscow Univ. Mech. Bull. 68, 117–125. https://doi.org/10.3103/S0027133013050026

[3] V.S. Vyazmin (2020) New algorithm for gravity vector estimation from airborne data using spherical scaling functions. International Association of Geodesy Symposia. Springer Int. Publ. AG. (In press.)

Contact

G1.2 session's chat on Tuesday, May 5, 14:00-15:45

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