Earthquake Recurrence Intervals in Complex Seismogenetic Systems

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For detailed information:

- Extended abstract available via this link
- Pre-print available via this link

Standard definition of Recurrence Interval

Recurrence Interval: Statistical estimate of the likelihood of an earthquake to occur

$$\Delta t(\geq M_{th}) = \frac{\text{Number of years on record} + 1}{\text{Number of events } \geq M_{th}}$$

- Assumes Poissonian processes – events of similar size are mutually independent and have a stationary probability of occurrence (Boltzmann-Gibbs thermodynamics) .
- Says ***nothing* about the dynamic state** of fault networks
- *Gross approximation* of the *long-term average (expectation)* of RI.
- Might lead to ***misestimation*** if the dynamics of a seismogenetic system is ***not*** Poissonian

Interevent times

- A parameter ***obviously associated*** with the RI ***and*** the ***dynamic state*** is the ***interevent time*** (IT),
  - ✓ IT: Lapse between consecutive earthquakes over a given area and above a magnitude threshold.

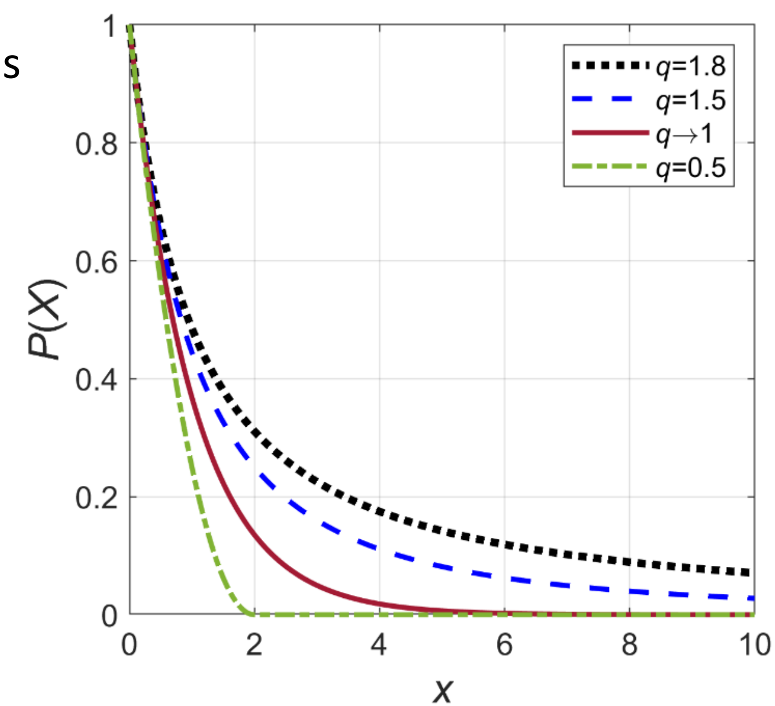
- IT has generally ***not*** been used in estimation of earthquake recurrence intervals.
- In the context of Poissonian processes, **Frequency – Interevent Time (F-T) distributions should** be ***exponential*** whereas they are ***generally not***.
- Empirical F-T distributions usually are ***power laws*** that ***cannot possibly*** fit into the Poissonian (Boltzmann-Gibbs) context.
  - ✓ Attempts to resolve contradiction produced *ad hoc* theories that are generally well formulated and elegant, but unavoidably multi-parametric, unnecessarily complicated and possibly defying the principle of maximum parsimony.

Enter Complexity…

- Seismicity expresses a fault network (system) that *evolves* in a *fractal-like spacetime* and may be sustainably *non-equilibrating* (Complex), *sustainably equilibrating* (Poissonian), or may transition between equilibrating and non-equilibrating (Complex) states.
- Complex States require a significant proportion of successive earthquakes to be *dependent* through *short and long range interaction* (***correlation***) introducing *delayed feedback*: confers memory manifested by power-law distributions.
- The statistical properties of Complex States can be studied with **Non-Extensive Statistical Physics** (NESP) → direct generalization of Boltzmann-Gibbs thermodynamics to non-equilibrating systems
- In NESP, for ***real*** dynamic variables $x \in [0,\infty)$ the CDF is

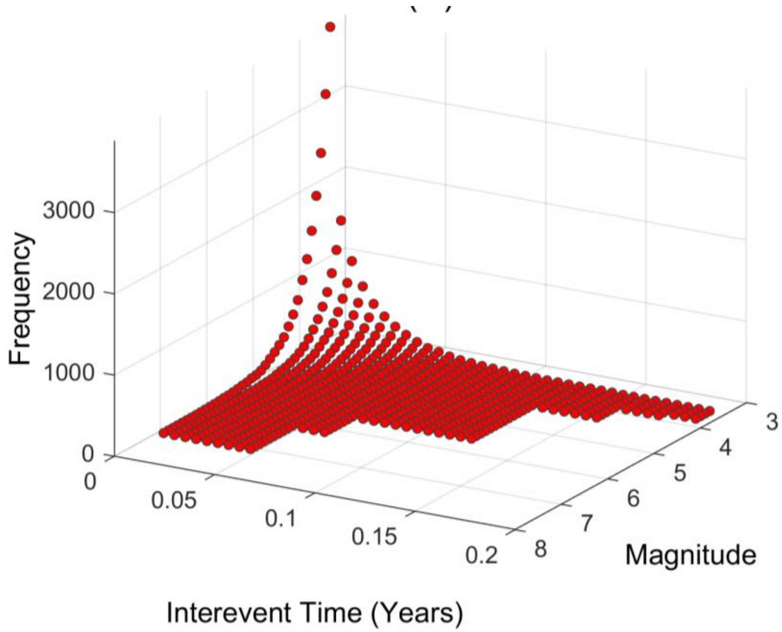
$$P(>x) = \exp_q\left(-x \cdot x_0^{-1}\right) = \left[1-(1-q)\left(-x \cdot x_0^{-1}\right)\right]^{(1-q)^{-1}}$$

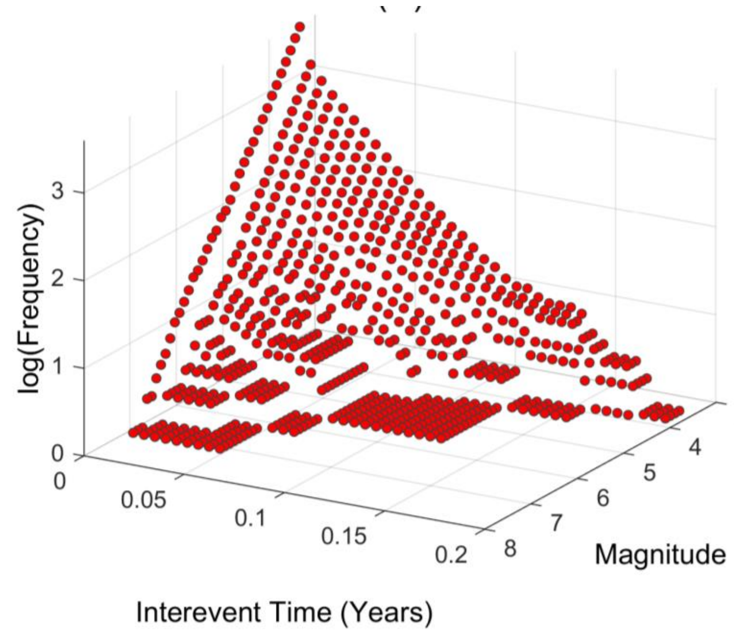
- $\exp_q$ is the ***q-exponential function***
- x_0: ***q-relaxation constant***;
- q: ***entropic index (level of correlation)***
- For $q_T \neq 1$ $\exp_q$ is a *Zipf-Mandelbrot power law*
- For $q_T = 1$ $\exp_q(-x/x_0) = \exp(-x/x_0)$, i.e. exponential distribution – Poissonian process.



BIVARIATE MAGNITUDE – INTEREVENT TIME DISTRIBUTIONS

- Earthquake magnitudes and interevent times are related.
  - ✓ The larger the magnitude scale, the longer the recurrence interval *and* interevent time.
- Joint evaluation of Frequency – Magnitude – Interevent Time distributions ensures observance of this important details.
- Frequency distribution of Interevent Time should be evaluated ***conditionally*** on the frequency distribution of magnitudes





NESP-COMPATIBLE STATISTICAL MODEL

Statistical properties of seismogenetic space-times represented with generalized **bivariate *q-exponential* Frequency-Magnitude distributions** of Gutenberg-Richter type:

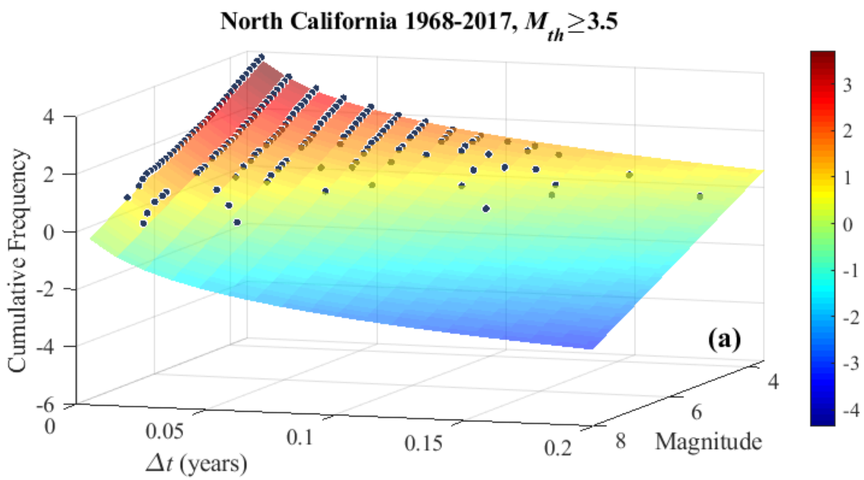
$$\underbrace{\frac{N\left(>\{M \geq M_{th},\ \Delta t : M \geq M_{th}\}\right)}{N_0}}_{P(M \geq M_{th},\ \Delta t\ :\ M \geq M_{th})} = \underbrace{\left(1 - \frac{1-q_M}{2-q_M} \cdot \frac{10^M}{\alpha^{2/3}}\right)^{\left(\frac{2-q_M}{1-q_M}\right)}}_{P(M \geq M_{th})} \cdot \underbrace{\left(1-(1-q_T)\cdot\frac{\Delta t}{\Delta t_0}\right)^{\frac{1}{1-q_T}}}_{P(\Delta t\ :\ M \geq M_{th})}$$

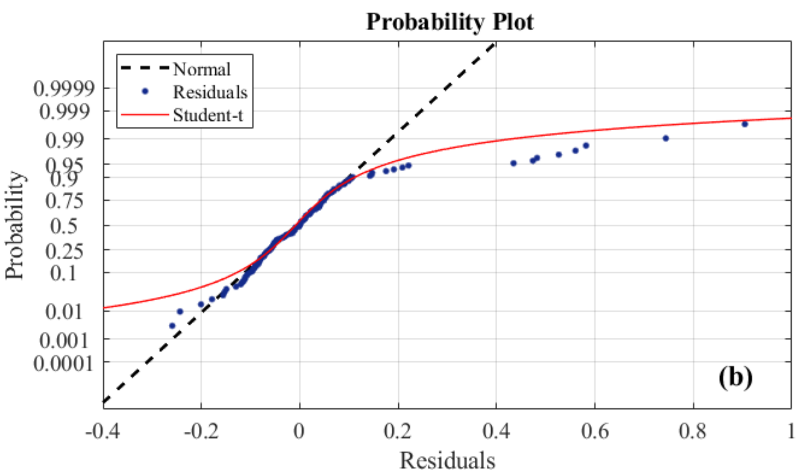
or

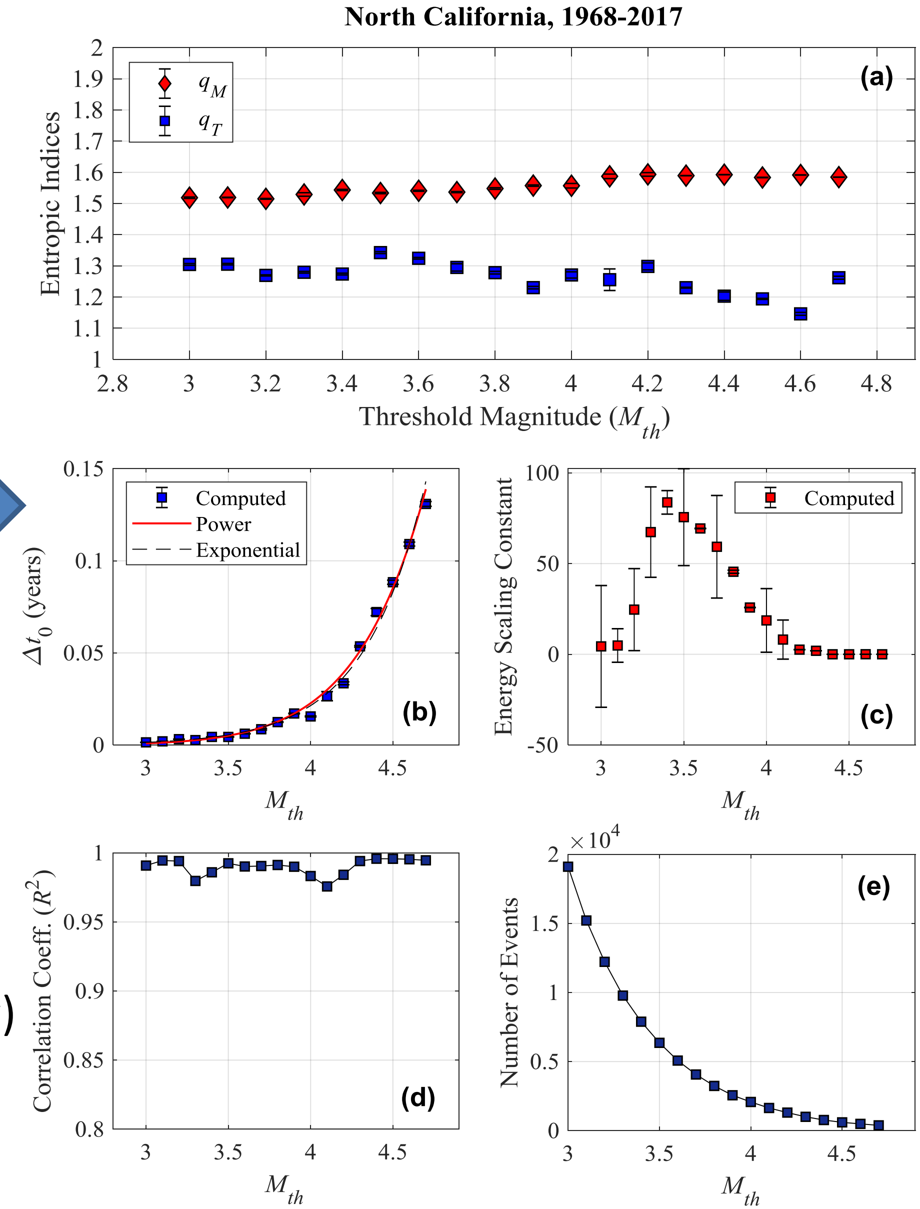
$$\log N\left(>\{M \geq M_{th},\ \Delta t : M \geq M_{th}\}\right) = a \quad + \left(\frac{2-q_M}{1-q_M}\right)\cdot \log\left(1 - \frac{1-q_M}{2-q_M}\cdot\frac{10^M}{\alpha^{2/3}}\right) \quad + \frac{1}{1-q_T}\log\left(1-(1-q_T)\frac{\Delta t}{\Delta t_0}\right)$$

Assumes	: Magnitudes and interevent times are statistically independent
M_{th}	: Threshold magnitude
α	: Energy scaling constant
q_M	: *Magnitude Entropic Index* : **indicates level of correlation in size-space**
q_T	: *Temporal Entropic Index* : **indicates level of correlation in time-space**
Δt	: *Interevent Time*
Δt_0	**: *q*-relaxation interval ≡ characteristic recurrence of $M \geq M_{th}$**

ESTIMATION PROCEDURE

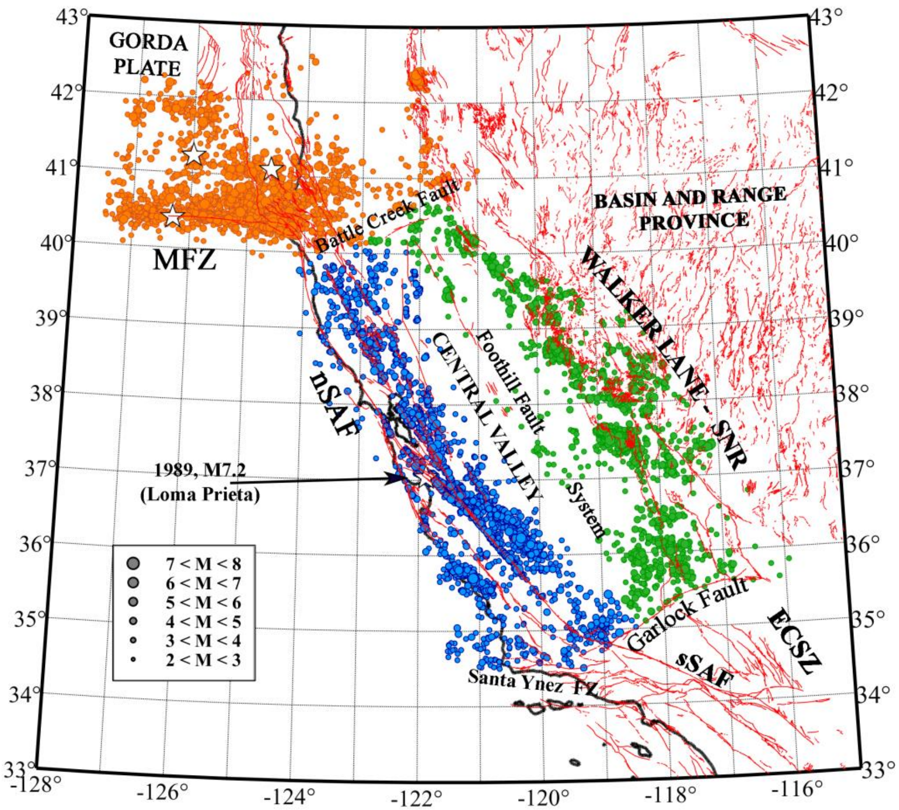






- ✓ All parameters positive (bounded from below)
- ✓ Entropic indices bounded from above (≤2)
- ➢ Problem solved with *trust-region reflective* algorithm + with *least absolute residual* (LAR) minimization to suppress outliers.

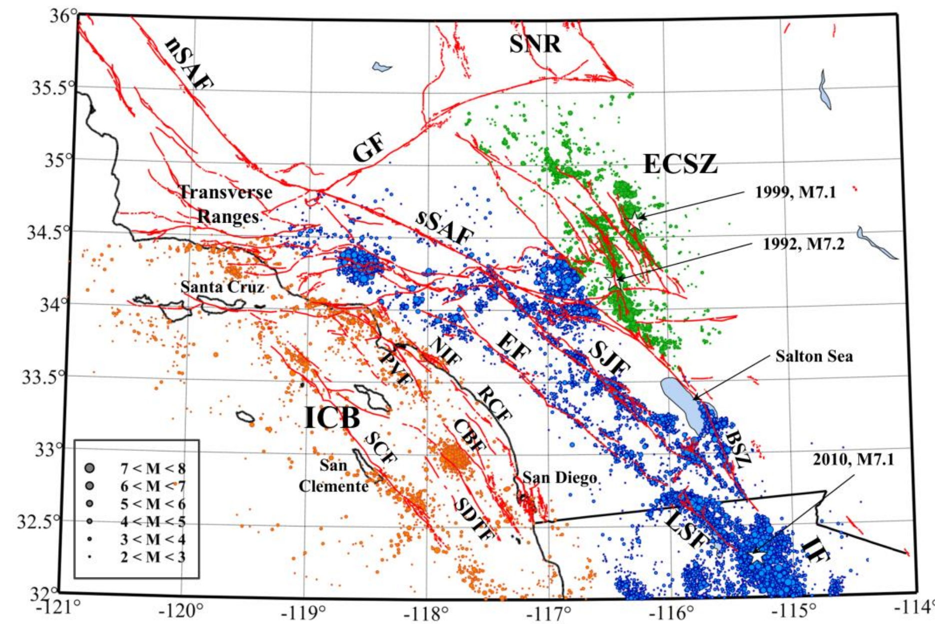
Example 1: Crustal (*schizospheric*) systems in *Transformational Plate Margins*: California, USA



Northern California Seismic Region

Data Source: North California Earthquake Data Centre @ http://www.ncedc.org.

- **$M_L \geq 3.0$; Period 1968-2017.5**

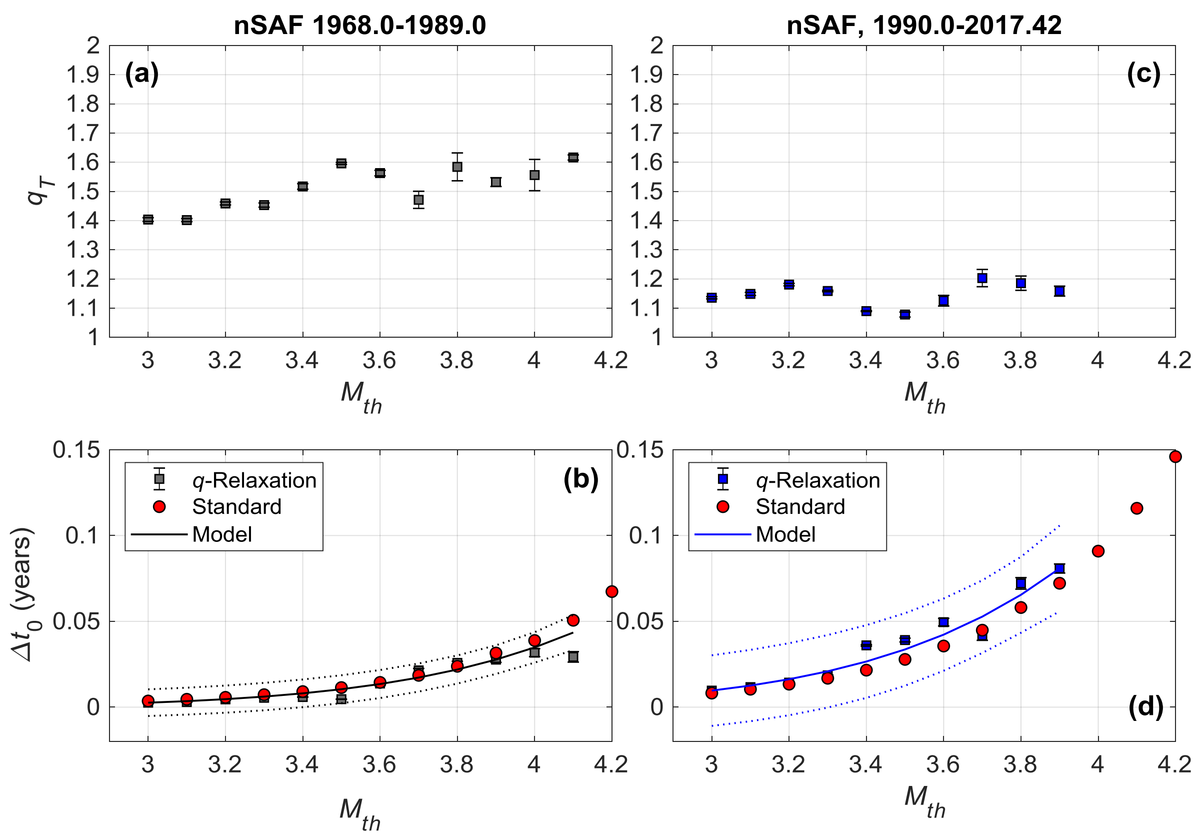


Southern California Seismic Region

Data Source: South California Earthquake Data Centre @(http://www.data.scec.org.

- **$M_L \geq 2.5$; Period 1968-2017.5**

San Andreas Fault – Northern Segment

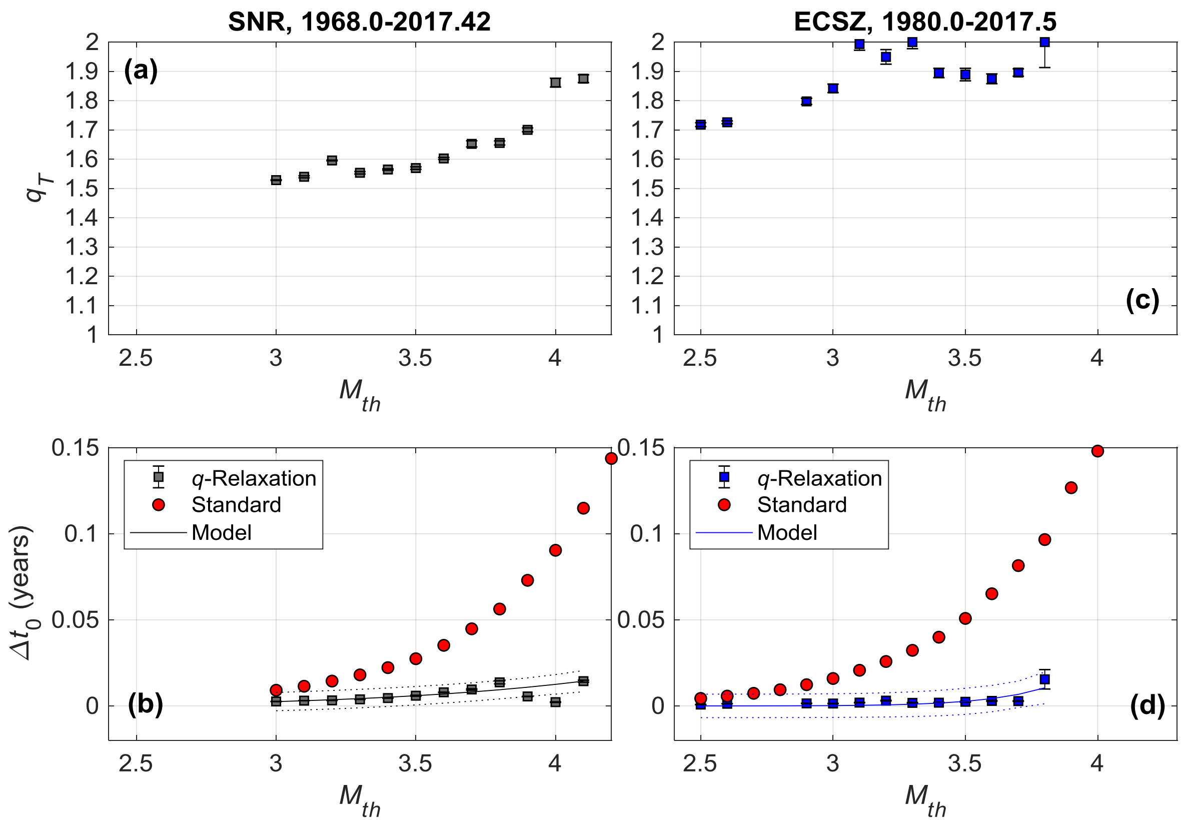


➢ Left column: *Prior* to the Loma Prieta event: **a)** The temporal entropic index is *significant* (*complexity/ moderate correlation*) and *increases with magnitude*; **b)** the *q-relaxation* interval resembles the "standard" and increases quasi-exponentially.

➢ Right column: *After* the Loma Prieta event: **c)** The temporal entropic index is *< 1.2* and indicates *weak correlation*; **d)** the *q-relaxation* interval resembles the "standard" and increases quasi-exponentially.

➢ Absence of stationarity in dynamic expression of the system is evidence against Self-Organized Criticality

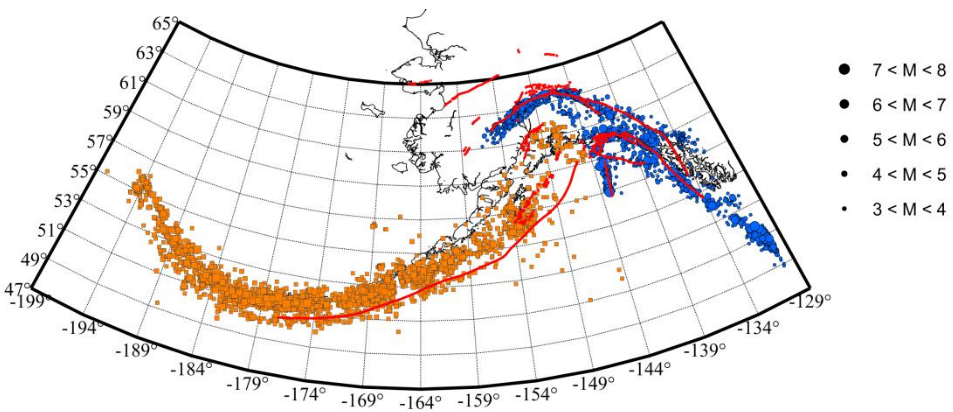
Walker Lane: Sierra Nevada Range (SNR) & Eastern California Shear zone (ECSZ)



- Both systems: In **(a)** *and* **(c)** the temporal entropic index is ***very significant*** (***very high correlation***) and ***increases*** with magnitude; in **(b)** and **(d)** *q-relaxation* interval ***does not*** increase.
- Both systems locked in the landward side of the primary plate boundary and experience strong long-range interaction.
- Shape of the *q-relaxation* curve indicates that upon occurrence of *any* event systems respond promptly and in a non-hierarchical manner; this is a hallmark of Self-Organized Criticality.

Example 2: Crustal (*schizospheric*) systems in *Convergent Plate Margins*

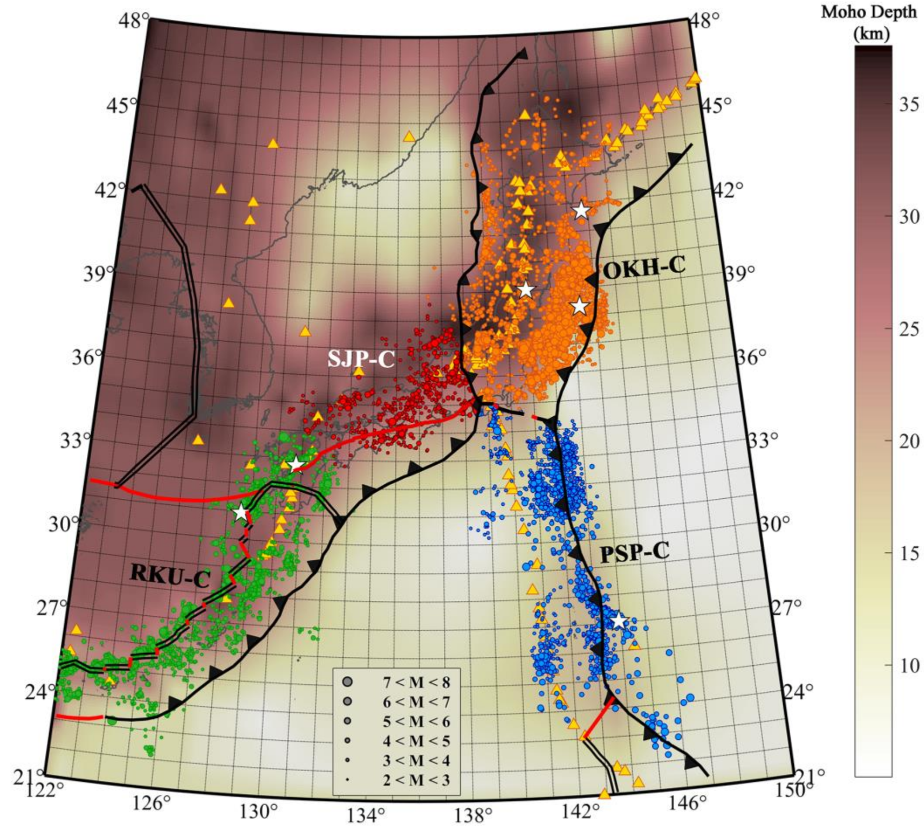
Alaskan – Aleutian Arc



Data Source: Regional earthquake database of the Alaska Earthquake Centre @ http://www.aeic.alaska.edu/html_docs/db2catalog.html

- **Only seismicity shown in orange is considered**

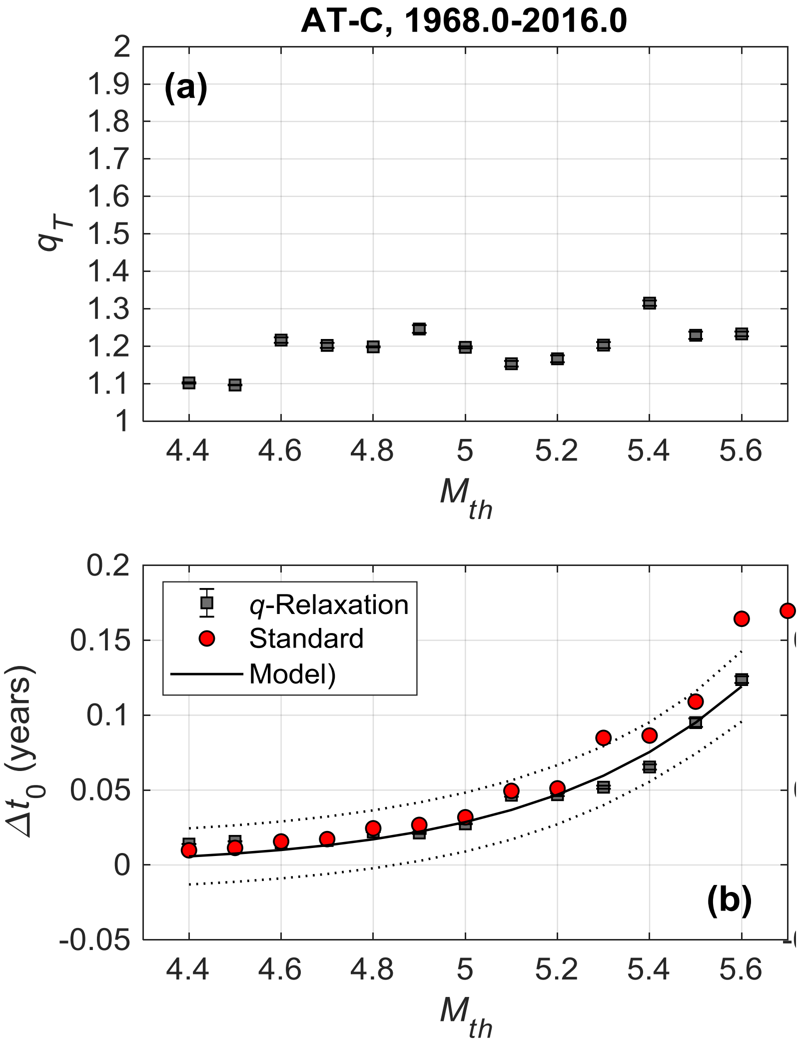
IZU – Bonin – Mariana Arc



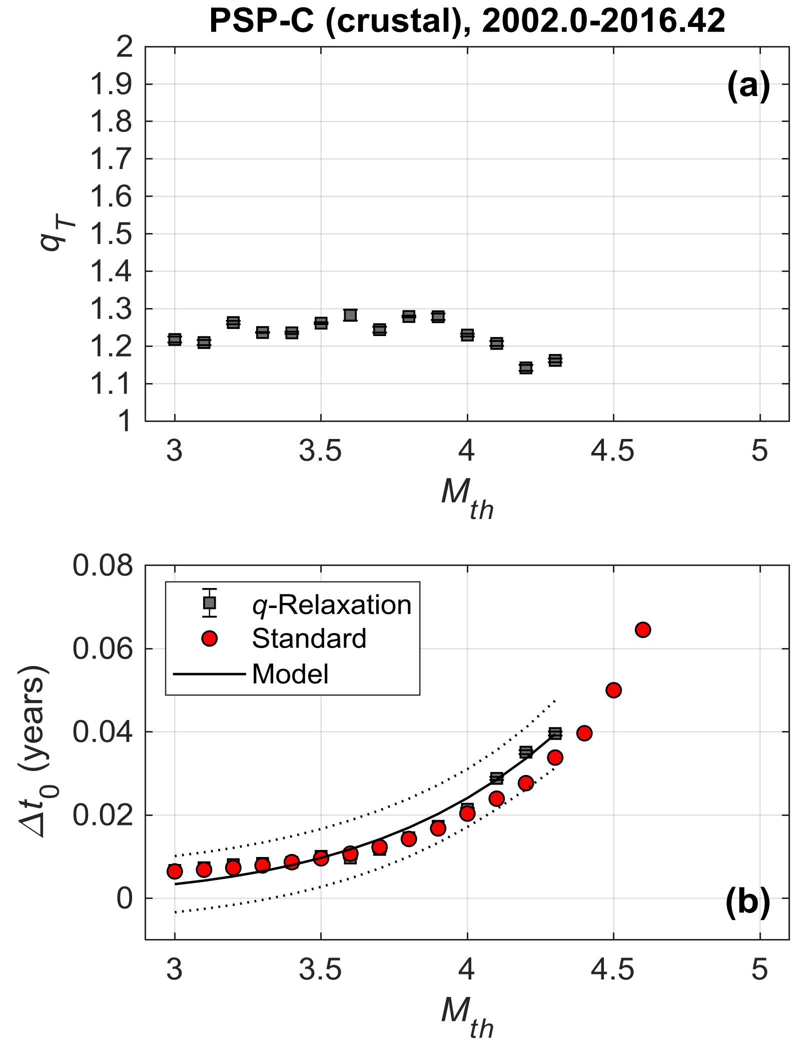
Data Source: National Research Institute for Earth Science and Disaster Resilience (NIED) of Japan, @ (http://www.hinet.bosai.go.jp)

- **Only the seismicity shown in blue is considered (PSP-C)**

Alaskan – Aleutian Arc



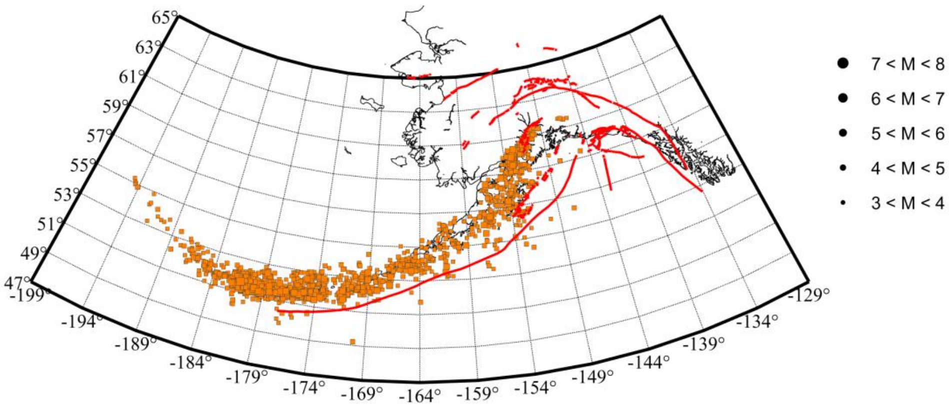
Izu – Bonin – Mariana Arc



- Both systems are *marginally correlated:* $q_T \leq 1.3$ indicates low level of long-range interaction and delayed feedback.
- Both systems: the *q-relaxation* interval and the "*standard*" recurrence interval are comparable and increase exponentially.

Example 3: Sub-Crustal (*below Moho*) systems in *Convergent Plate Margins*

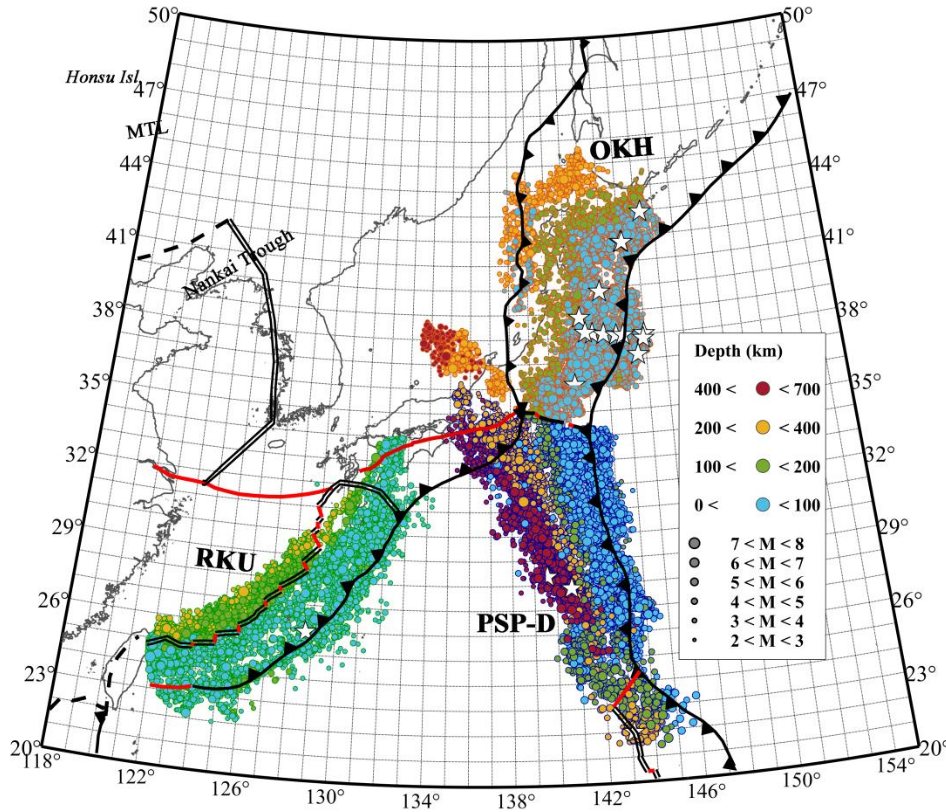
Alaskan – Aleutian Wadati-Benioff Zone



Data Source: Regional earthquake database of the Alaska Earthquake Centre @ http://www.aeic.alaska.edu/html_docs/db2catalog.html

- **Only seismicity shown in orange is considered**

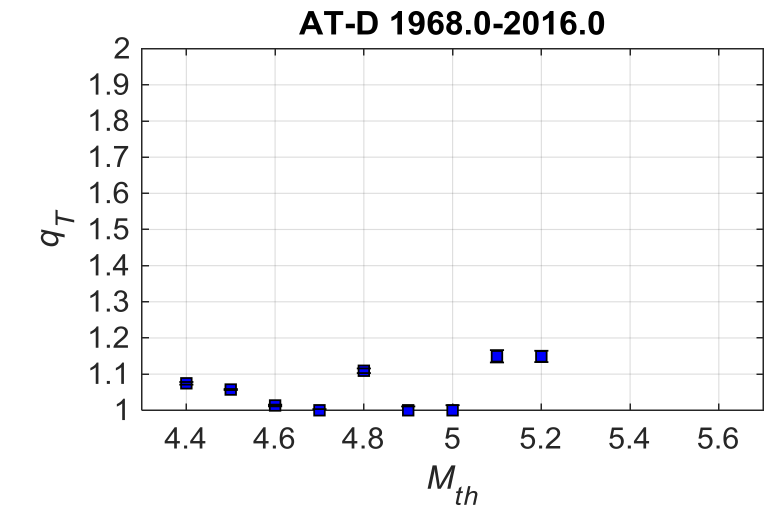
IZU – Bonin – Marianna Wadati-Benioff Zone

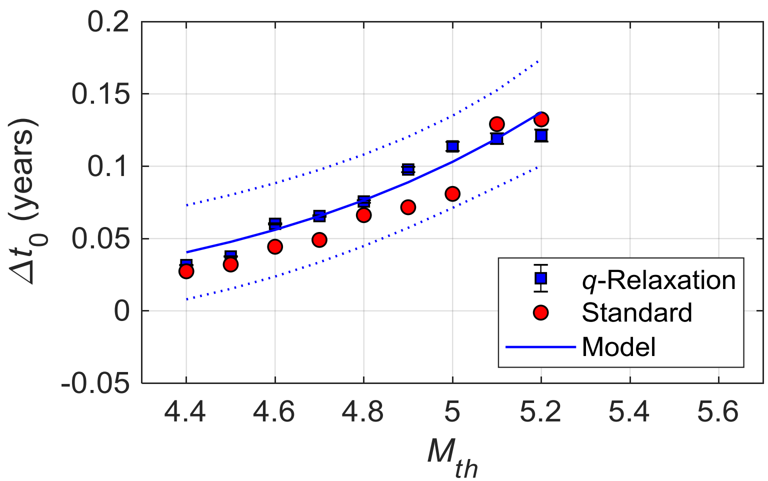


Data Source: National Research Institute for Earth Science and Disaster Resilience (NIED) of Japan, @ (http://www.hinet.bosai.go.jp)

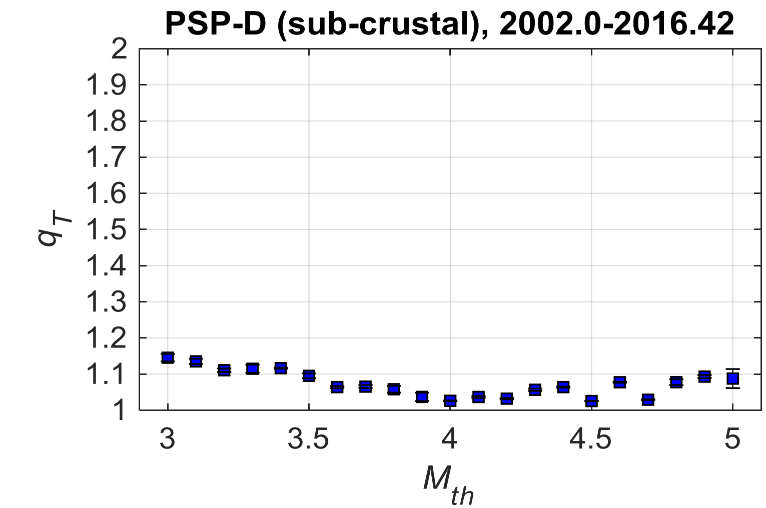
- **Only the seismicity designated as PSP-D blue is considered**

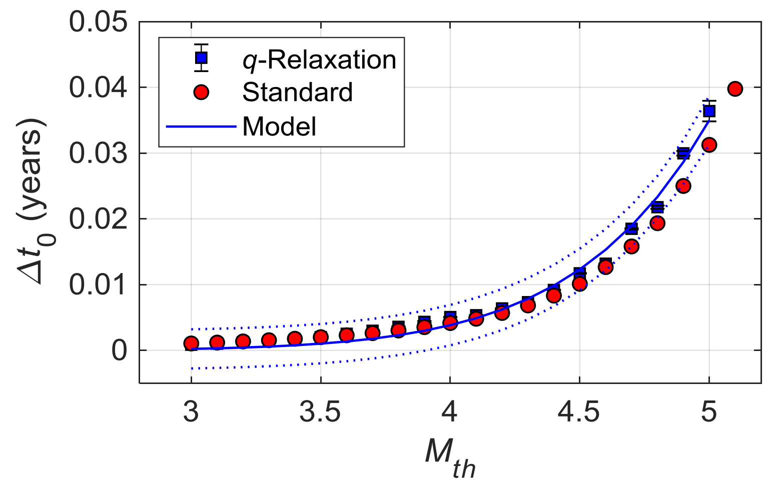
Alaskan – Aleutian Subduction





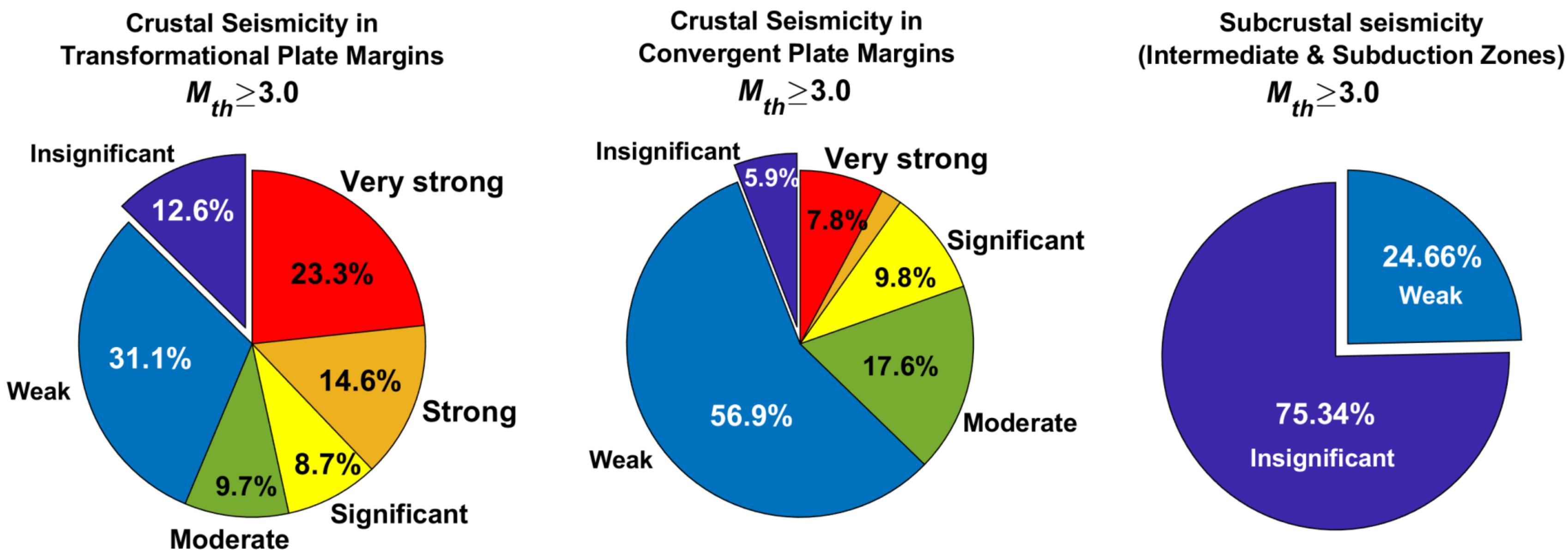
Izu – Bonin – Mariana Subduction





- Systems are *uncorrelated:* $q_T \leq 1.15$ indicates *practically non-existent long-range interaction* and delayed feedback; *systems are Poissonian*.
- Both systems: the *q-relaxation* interval and the "*standard*" recurrence interval are practically identical and increase exponentially as expected of Poissonian systems.

Summary of Correlation Properties (entropic states)



q_T Classification	$q_T < 1.15$	$1.15 \leq q_T < 1.30$	$1.30 \leq q_T < 1.4$	$1.4 \leq q_T < 1.5$	$1.5 \leq q_T < 1.6$	$1.6 \leq q_T$
	Insignificant	Weak	Moderate	Significant	Strong	Very Strong

- ✓ **Crustal** systems in transformational plate boundaries are generally ***correlated***
- ✓ **Crustal** systems in convergent and divergent plate margins are generally ***weakly–moderately correlated***
- ✓ **Sub-crustal/** Wadati-Benioff zone systems are definitely uncorrelated (***quasi-Poissonian***)

General Conclusions after analysis of 20 seismogenetic systems

- **The *q*-exponential distribution is a *universal descriptor* of Interevent Time statistics**.
- *The duration* of *q*-relaxation intervals is generally *reciprocal* to the level of correlation (q_T). The higher the correlation, the shorter the q-relaxation.
  - ✓ Both may change with time and across system boundaries.
- Crustal systems in transformational plate boundaries:
  - ✓ A few systems with *very strong correlation* and very short/ slowly increasing recurrence intervals exhibit attributes of Self-Organized Criticality.
  - ✓ Most other such systems are complex and with apparently significant long-range interaction but *most probably non-critical*!
- Crustal systems in convergent plate margins:
  - ✓ *q*-relaxation and standard recurrence intervals both increase exponentially, some at comparable and some at *different* rates.
  - ✓ Such fault networks exhibit moderate to strong correlation (complexity).
  - ✓ Attributes indicate that such systems are possibly *non-critical*.
- Sub-crustal and Wadati-Benioff zones:
  - ✓ *q*-relaxation and standard recurrence intervals increase exponentially with magnitude and are *congruent*.
  - ✓ Such systems are *generally uncorrelated* and appear to be *Poissonian in nature*.
- The blending of earthquake populations from adjacent but dynamically different systems *randomizes* the statistics of the mixed catalogue and over large seismogenetic provinces, *reduces the apparent level of Complexity*.

- ☞ Additional examples, documentation, discussion and a possible interpretation of the observations can be found in a pre-print available via this link

- ☞ Possible utility/utilization of the new information remains to be specified with future work.

Thank you
for your Patience