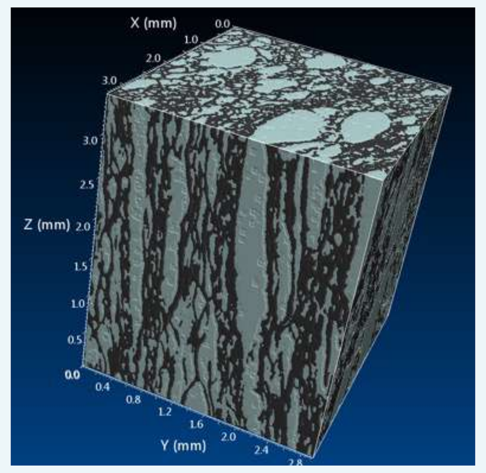
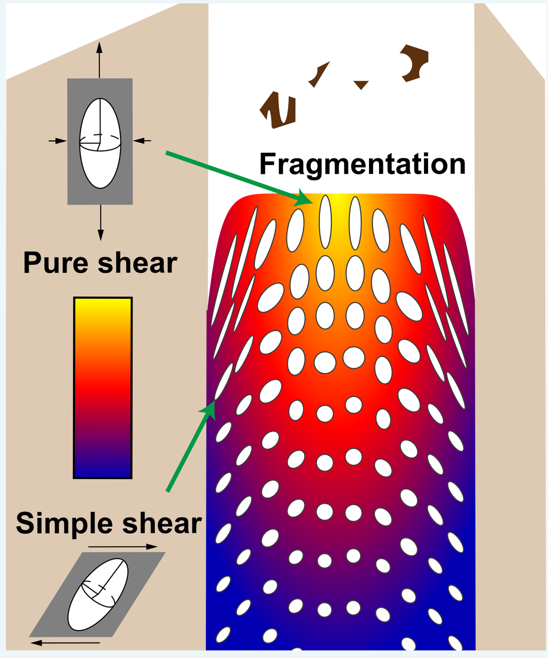
Numerical simulation of bubble deformation in various velocity profiles across a conduit

Masatoshi Ohashi[1] · Mie Ichihara[2] · Fukashi Maeno[2] · Darren Gravley[3] · Ben Kennedy[3] 1 Department of Earth and Planetary Sciences, Kyushu University 2 Earthquake Research Institute, the University of Tokyo 3 Geological Department, the University of Canterbury

Introduction



↑ 3D image of tube pumice
→ Schematic shear field in a conduit
(modified from Dingwell et al., 2016)

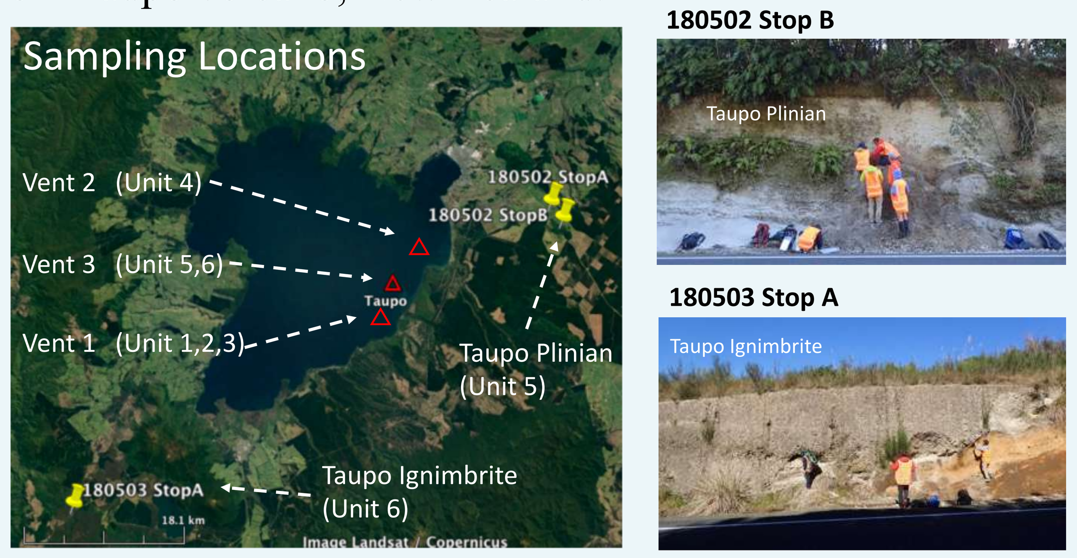


Tube pumice is characterized by aligned highly elongated bubbles and is a common product of explosive silicic eruptions. This characteristic pumice is thought to record the shear fields in a conduit. In previous works, it has been suggested that tube pumice can be generated either by pure or simple shear of a spherical vesicle (Dingwell et al., 2016). Pure shear may be dominant beneath a fragmentation surface where the ascent velocity is steeply accelerated. Alternatively, simple shear may be dominant near a conduit wall where strain is localized. Most previous studies interpret tube pumice forming from simple shear deformation, assuming a parabolic velocity profile across a conduit. However, simple shear cannot explain the observation that tube pumice is rare in plinian falls but frequent in ignimbrites (interpreted to have wider vents).

In this study, we calculate bubble deformation in a conduit flow of the 1.8 ka Taupo eruption, and compare them with bubble textures measured from Taupo pumice clasts.

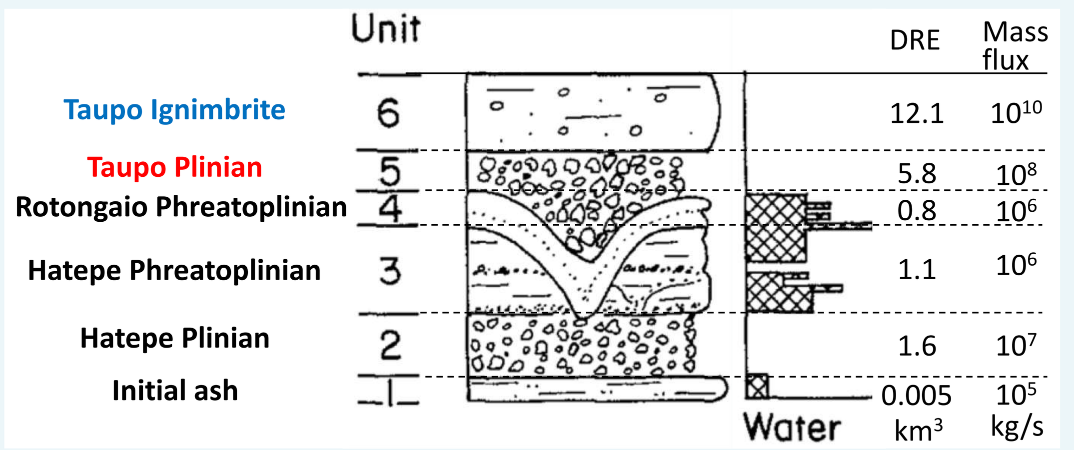
1.8 ka Taupo Eruption

The 1.8 ka eruption is the most recent large eruptive event from Taupo volcano, New Zealand.



Postulated vent locations are shown by red triangles after Smith and Houghton (1995).

- This eruption is characterized by various eruptive styles, including three phreatomagmatic fall depo sits (units 1, 3, 4), Hatepe plinian fall deposit (unit 2), **Taupo plinian fall deposit** (unit 5), and **Taupo Ignimbrite** (unit 6).
- We collected pumice clasts with the diameter of 16-32 mm. 100 clasts for unit 6, and 1000 clasts for unit 5.
- We measured the average bubble deformation degree (see top right section) for each bubble and compared them with the numerical simulations.



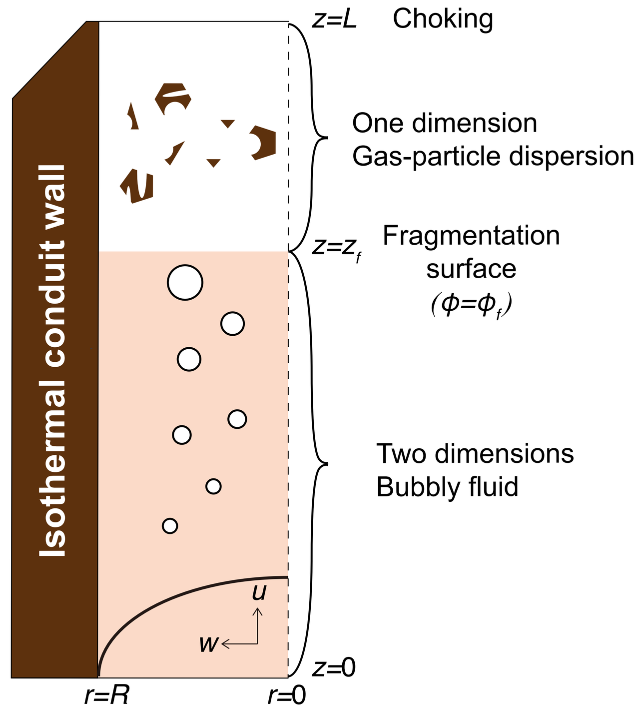
Eruptive products of the 1.8 ka Taupo eruptin (Smith and Houghton, 1995).
DRE data are form Wilson and Walker (1985). Mass flux data from Houghton et al. (2010)

Bubble deformation based on a quasi-2D conduit flow model

Numerical simulation is divided into two parts. First part is to get the velocity field around a bubble from a conduit flow model. Second part is to calculate bubble deformation in a obtained velocity field.

(1) Conduit flow model

- The conduit flow model is based on a viscous-heating model of Barmin et al. (2004).
- We add a new effect of compressibility in a bubbly fluid region below the fragmentation surface.
- Following this model, we also derived a Newtonian isothermal model and shear-thinning model due to bubble deformation.



Viscos-heating model (in the bubbly fluid region)

Viscous-heating causes the reduction in viscosity.

ρ : Density
κ : Thermal conductivity
c_V : Heat capacity
u : Vertical velocity
w : Radial velocity
p : Pressure
η : Viscosity
e : Energy density
ϕ : Gass fraction
T : Temperature
c : Water content
Q : Discharge rate

Continuity equation

$$\frac{1}{r}\frac{\partial}{\partial r}(r\rho w)+\frac{\partial}{\partial z}(\rho u)=0$$

Momentum equation

$$p(r,z)=p(z),\qquad \frac{dp}{dz}=\frac{1}{r}\frac{\partial}{\partial r}\left(r\eta\frac{\partial u}{\partial r}\right)-\rho g$$

Energy equation

$$\frac{1}{r}\frac{\partial}{\partial r}(r\rho we)+\frac{\partial}{\partial z}(\rho ue)=-p\,\mathrm{div}\,\vec{u}+\eta\left(\frac{\partial u}{\partial r}\right)^2+\frac{1}{r}\frac{\partial}{\partial r}\left(r\kappa\frac{\partial T}{\partial r}\right)$$

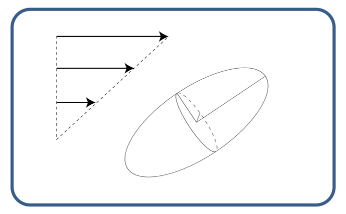
Melt viscosity (Hess and Dingwell ,1996)

$$\log\eta_m=-3.545+0.833\ln(c)+\frac{9601-2368\ln(c)}{T-(195.7+32.25\ln(c))}$$

Parameter	Value	Source	Parameter	Value	Source
Conduit length L	4000 m	Dunbar et al. (1989)	Melt density ρ_m	2500 kg/m³	Barmin et al. (2004)
Initial water content c	4.3 wt%		Thermal conductivity κ	0.8 J/(m s K)	
Initial magma temperature T_0	850℃	Dunbar et al. (1989)	Specific heat c_V	1200 J/(kg K)	

(2) Bubble deformation model

Bubble deformation is calculated along the streamline in a conduit flow. We used a bubble deformation model of Ohashi et al. (2018), which calculates the shape of a single bubble in an arbitrary shear field.



- Single bubble in an infinite fluid
- Arbitrary shear field
- Transient large deformation

Newtonian isothermal model

Temperature is isothermal throughout the conduit. The conservation equations of mass and momentum are the same as the viscous-heating model.

Shear-thinning model due to bubble deformation

The shear-thinning effect is also caused by bubble deformation. Instead of solving the energy equation, this model uses the bulk viscosity of bubbly fluid, η_b, of Llewellin et al. (2002).

$$\frac{\eta_b}{\eta_m}=\sqrt{\eta_{r,\infty}^2+\frac{\eta_{r,0}^2-\eta_{r,\infty}^2}{1+\left(\frac{6}{5}Ca\right)^2}}\qquad Ca=\frac{R_b\eta_m\dot{\gamma}}{\Gamma}$$

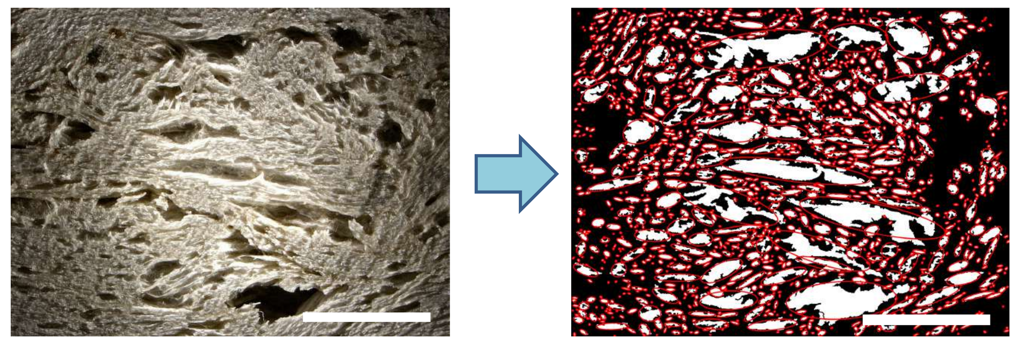
$$\eta_{r,0}=(1-\phi)^{-1}$$

$$\eta_{r,\infty}=(1-\phi)^{\frac{5}{3}}$$

R_b : Bubble radius
η_m : Melt viscosity
$\dot{\gamma}$: Shear rate of simple shear
Γ : Surface tension

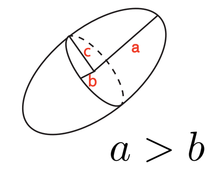
Methods of textural analysis

- Digital microscope (VHX-1000,Keyence)
- Magnification ×20
- **Adjustable illumination adapter**



The illumination adapter enables to illuminate the surface from the sides, clarifying the boundaries of bubbles. An ellipsoid is fitted to each bubble. Deformation degree, D, is defined by axes of the ellipsoid as follows.

$$D=\frac{a-c}{a+c}$$



$a>b>c$

D=1 Highly elongated
D=0 Spherical

Here, magma fragments that ascended in different paths are assumed to preserve the corresponding bubble textures in pumice clasts. In order to discuss the dynamics in a conduit, we define an average deformation degree, $\bar{D}$, for each pumice clast:

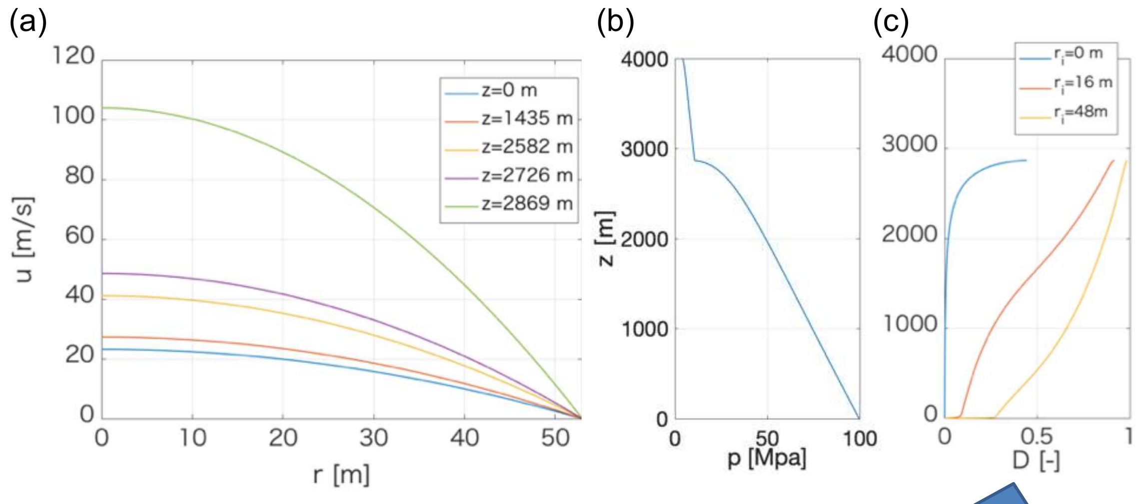
$$\bar{D}=\frac{\Sigma D_i A_i}{\Sigma A_i}$$

D : Deformation degree
A : Bubble area
Σ : Summation of bubbles larger than 0.15 mm in a pumice clast

Simulation results of Taupo Plinian fall (Discharge rate 2.5×10^8 kg/s)

Case 1: Newtonian isothermal model

- The velocity profile across the conduit becomes **parabolic**.
- Highly elongated bubbles were produced mainly by simple shear.

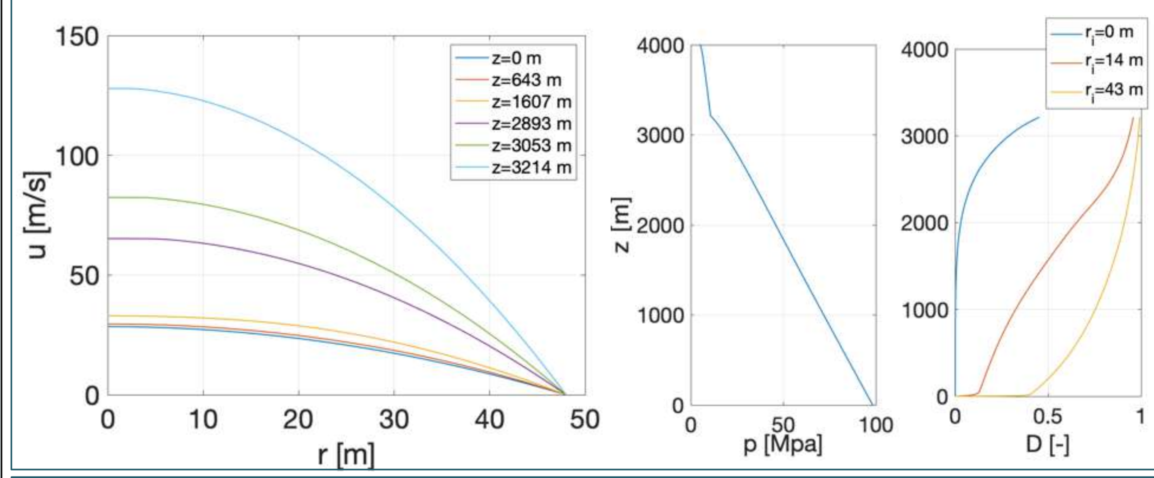


(a) The distribution of the vertical velocity across the conduit.
(b) The pressure change along the conduit.
(c) The vertical evolution of the bubble shape. The line color indicates the initial radial position at the conduit inlet.

Deformation degree (see top right section)

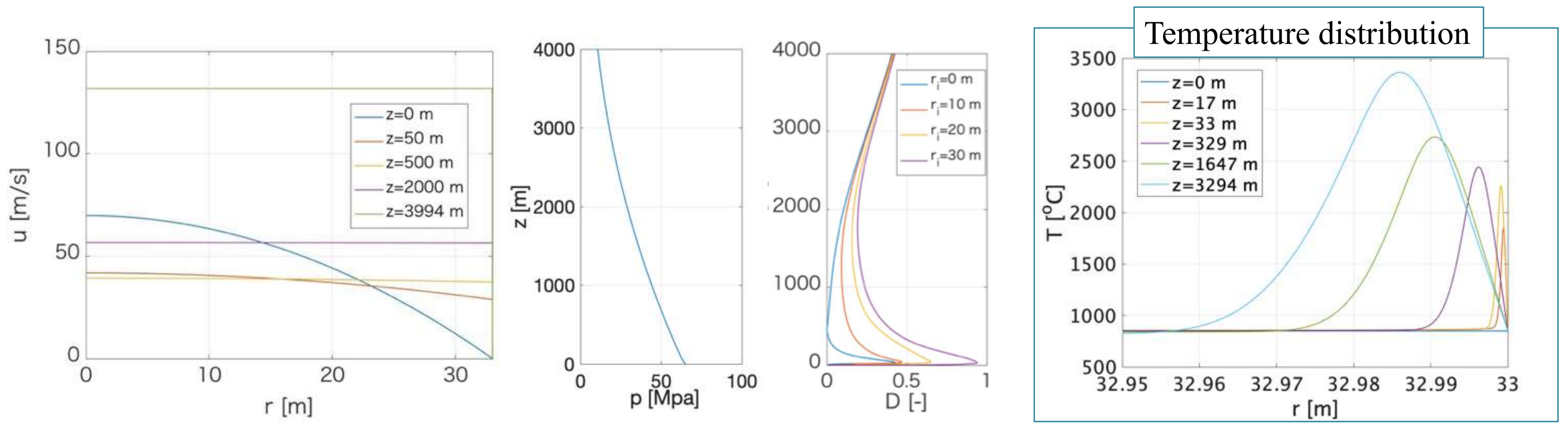
Case 2: Shear-thinning model due to bubble deformation

- As with the case 1, the velocity profile becomes **parabolic**.



Case3: Viscous-heating model

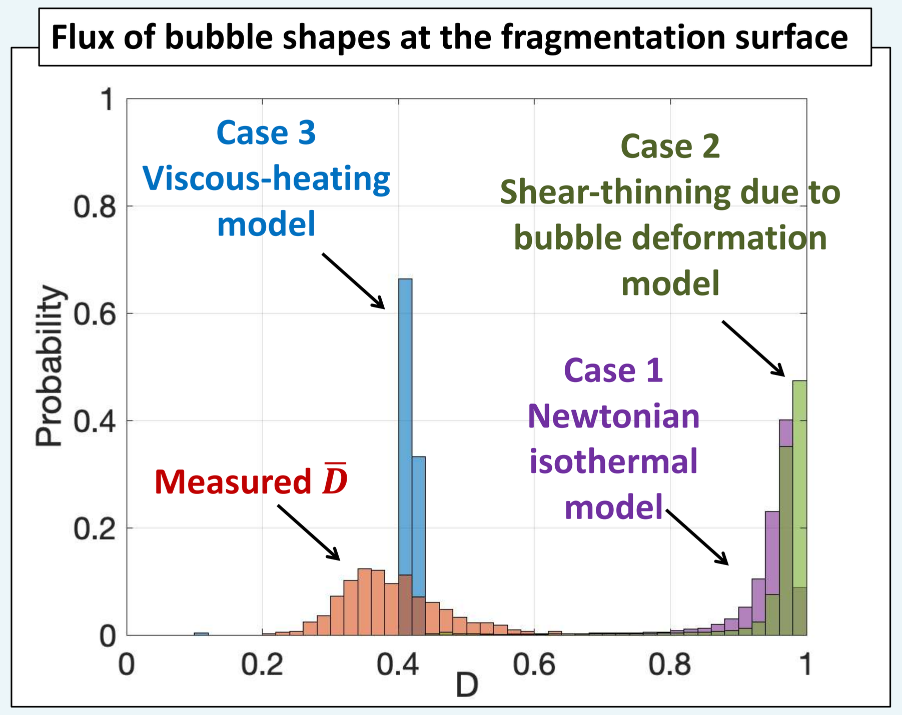
- The velocity profile becomes a **plug** shape due to the strong shear localization around the conduit wall.
- The plug-like velocity profile is dominated by pure shear and accumulates less strain to elongate bubbles.



Bubble shape distribution

In the numerical simulations which showed the parabolic velocity profiles (case 1 and 2), the calculated bubble shapes are much more elongated than those measured in the Taupo plinian. On the other hand, the bubble shapes calculated by the viscous-heating model, which showed the plug-like velocity profile, are in the measured range.

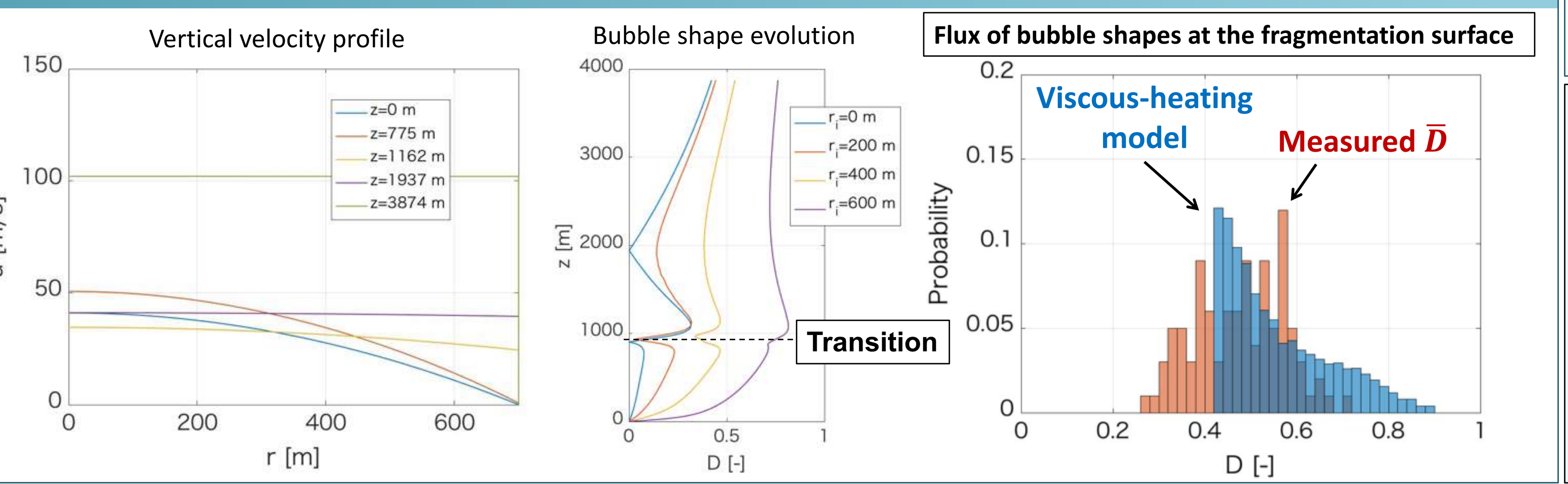
This result suggests that bubbles in the Taupo plinian event were deformed in the conduit flow with the **plug-like velocity profile,** which was likely to be caused by intense **viscous-heating** around the conduit walls.



Simulation results of Taupo ignimbrite (Discharge rate 5×10^{10} kg/s)

We also calculate bubble deformation for the Taupo ignimbrite eruption, using the viscous-heating model. The mass flux is increased from 2.5×10^8 to 5×10^{10} kg/s, and the conduit radius is increased from 33 to 700 m.

The numerical simulation shows that the **transition** from a parabolic to plug-like velocity profile is shallowed to $z=1000$ m. This change leads to a higher proportion of the conduit experiencing parabolic flow compared to the plinian phase, resulting in the large number of highly elongated bubbles (tube pumice) at the fragmentation surface.



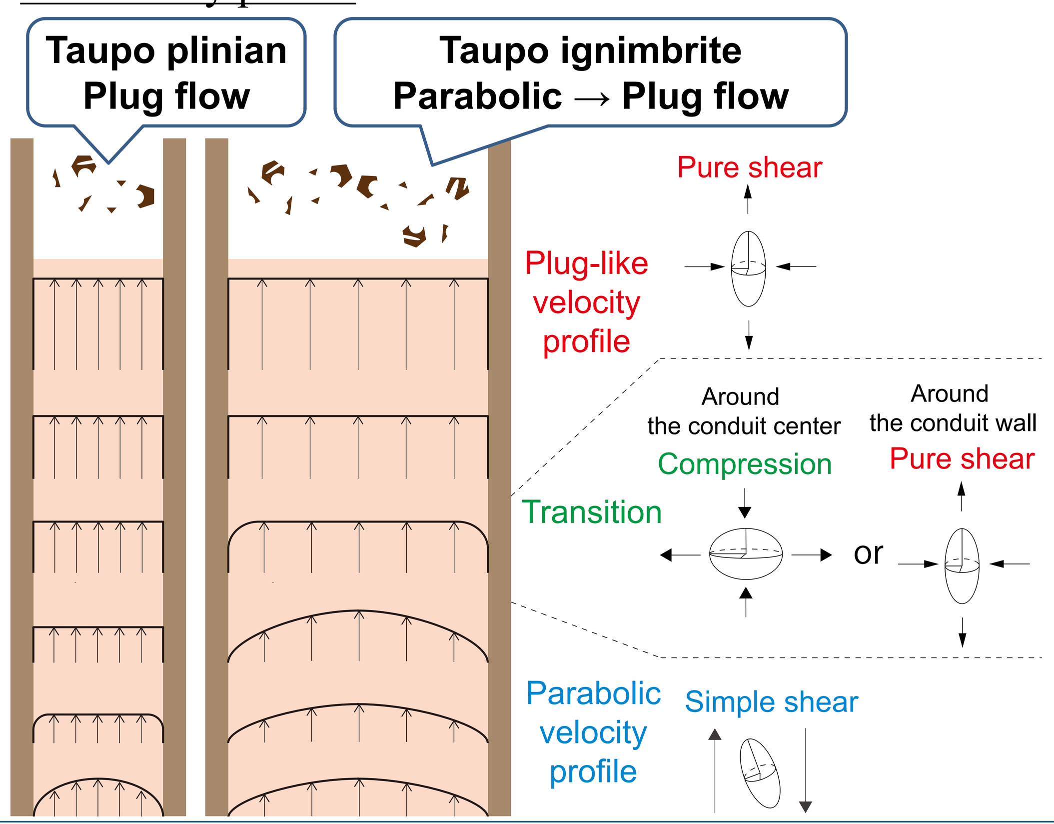
Discussion

The increase of tube pumice in the ignimbrite event can be ascribed to the shallowing of the transition from a parabolic to plug-like velocity profile. The shallowing is caused by the wider conduit radius in the ignimbrite event.

Parameter	Discharge rate [kg/s]	Conduit radius [m]	Shear rate $\left(\frac{\partial u}{\partial r}\right)$ * [1/s]
Taupo plinian	2.5×10^8	33	4.2
Taupo ignimbrite	5.0×10^{10}	700	0.12

* Shear rate next at the conduit wall just above the inlet

The shear rate in the Taupo ignimbrite eruption was smaller than the Taupo plinian eruption by an order of magnitude. Therefore, the conduit flow experienced a weak viscous-heating which suppressed the transition of the velocity profile.



Conclusions

- The comparison of the natural bubble textures with the simulation results suggested that the velocity profile of the plinian eruption was close to a plug-like shape.
- The reason why the ignimbrite eruption produced a number of tube pumice was explained by shallowing the transition depth at which the velocity profile changed from parabolic to plug-like.