Pendulum MEMS Gravimeters for Semi-Absolute Gravimetry

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Introduction

By measuring tiny variations in the Earth's gravitational acceleration, g, one can infer density variations beneath the ground. Since magmatic systems contain rock of differing density, changes in gravity over time can tell us when/where magma is moving. Traditional gravity sensors (gravimeters) were costly and heavy, but with the advent of the technology used to make mobile phone accelerometers (MEMS – Microelectromechanical-systems), this is changing.

At Glasgow University we have already developed the first MEMS gravity sensor and we are now working with several other European institutions to make a network of gravity sensors around Mt Etna – NEWTON-g. It will be the first multi-pixel gravity imager – enabling unprecedented resolution of Etna's plumbing system.

While this work is ongoing, a second generation of MEMS gravity sensor is now under development. The first-generation sensor comprises a mass on a spring, which moves in response to changing values of g. This, however, can only ever be used to measure changes in gravity, which means it can be difficult to tell the difference between a geophysical signal and instrumental drift. If we could measure absolute values of gravity, then instrumental drift would become less of a concern, and we could remove the need to calibrate the sensors against commercial absolute gravimeters.

One way of making absolute measurements of gravity is to use a pendulum. This method was used for hundreds of years until the scientists and engineers essentially ran out of fabrication tolerance about 100 years ago. But now nanofabrication is at our disposal, so pendulums are a valid approach to gravimetry again. Such a gravimeter is now being designed and fabricated at the University of Glasgow. It consists of a pair of coupled pendulums, who's oscillation period is monitored to measure gravity. Here we present the preliminary design considerations of this new tool.

Background

The absolute gravimeters that are currently available on the market cost over £200k predominantly utilise a free-fall design. Their use is limited by their large size and high power consumption. Free-fall gravimeters measure gravity by observing a mass dropped over a set distance and can therefore make absolute measurements of gravitational acceleration.

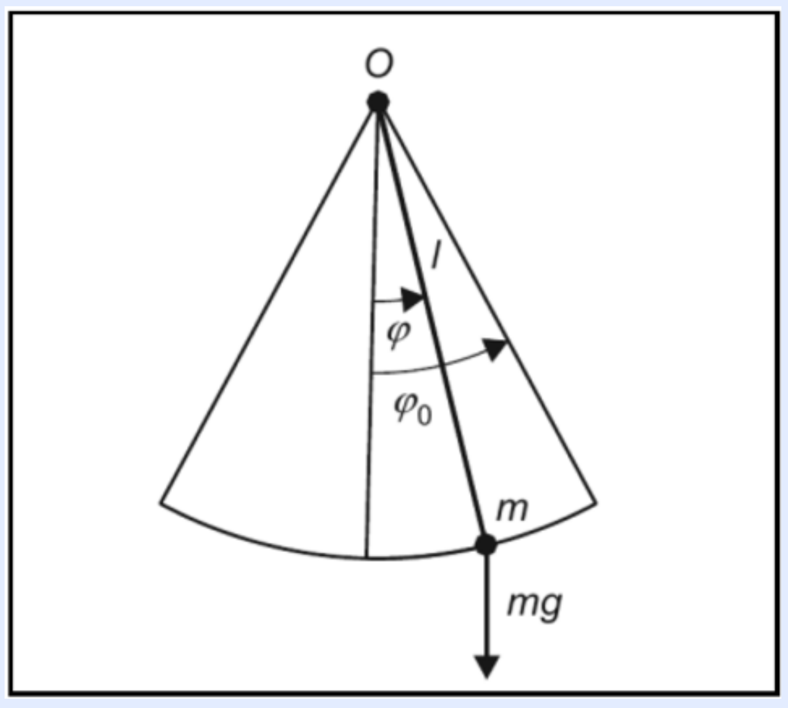

Figure 1: A perfect mathematical pendulum has an invariant length, a frictionless pivot, a point-source bob, and only oscillates in one plane.

A perfect mathematical pendulum (or canonical pendulum) could also be used as an absolute gravimeter. A mathematical pendulum is a perfect system in which a pendulum bob oscillates with zero friction in a single plane (figure 1). The mass of the bob is concentrated at a single point, and the wire holding the mass is of invariant length. For a system such as this, and for an oscillation of infinitely small amplitude, the local gravitational acceleration can be calculated by:

$$g = 4\pi \frac{l}{T^2} \qquad (eq.1)$$

where l is the length of the pendulum and T is the period of oscillation. Physical pendula do not behave in such a perfect manner.

Pendulum Design

Any MEMS pendulum will need to constructed monolithically, so a microscopic silicon flexure will be needed to create the pivot instead of a knife-edge. Such a pivot would have a spring constant be sensitive to temperature variation via changes to its Young's modulus. Such a pendulum pendulum gravimeter can therefore only be considered *semi-absolute*, since environmental and material factors can limit the number of decimal places to which they can measure g absolutely.

A double pendulum will be used to minimise recoil losses to the frame, therefore minimising energy transport to the ground which would damp the pendulum devices and increase thermal noise [1]. Seismic motion is also suppressed for double pendula since noise associated with inertial acceleration can be cancelled via a common mode rejection ratio (CMRR). This reduces the need for expensive seismic isolation systems.

Research has recently been focussed on the design of a suitable pivot point. As mentioned above, a canonical pendulum is considered to have a pivot that is both completely free to move – i.e. has zero stiffness – and that rotates about an infinitesimal point. However, with a MEMS device the pivot must by necessity be substituted with a flexure, introducing the problems of rotational accuracy and stiffness. Therefore, one of the optimization parameters for the gravimeter that must be considered is the design of this flexure. This is not a new problem, and there exists considerable literature attempting to categorize flexure designs based on various metrics. For a gravimeter, the ratio of effective gravitational stiffness to flexure stiffness gives what is termed the dilution coefficient. This ratio gives a measure of the absoluteness of the device, and is why a pendulum-based MEMS gravimeter must be termed semi-absolute. The dilution coefficient is calculated as follows:

$$\frac{k_{gravitational}}{k_{elastic}} = \frac{g}{R\,\omega_{elastic}{}^2}\,\frac{\sin\theta}{\theta} \qquad (eq.2)$$

with stiffness k, gravitational acceleration g, radius of gyration R, angular resonant frequency ω and angular displacement θ.

During the preliminary stages of development, three types of flexure design have been identified as potential candidates. These three choices represent the extremes of rotational accuracy versus stiffness – two parameters that are generally in opposition.

A straight flexure design has the lowest stiffness, bending most easily, but sacrifices rotational accuracy due to the migration of the pivot point as the flexure bends. This type of flexure may be of use if the gravimeter is to be forced in the very small angle regime.

A hyperbolic flexure has the best rotational accuracy, but is considerably stiffer. The addition of parabolic notches relieves some of this stiffness, but this type of flexure still ranks as the least compliant. If the gravimeter is to be forced to a significant amplitude, and the pendulum bob mass is high enough to dilute the effects of the increased stiffness, then this flexure could be ideal.

The final flexure design is semi-circular. This has the advantage of being a satisfactory compromise between stiffness and rotational accuracy in most forcing regimes. It is also the design that distributes mechanical stress in the flexure most efficiently, which may or may not become an important factor depending upon the required bob mass.

The anticipated sensitivity of the device depends on the dilution coefficient, the various damping mechanisms inherent in the system, and material variations with temperature. Preliminary calculations – without experimental verification – indicate that with adequate temperature control techniques this material variation should result in no more than a 0.5 μGal error in gravity measurement.

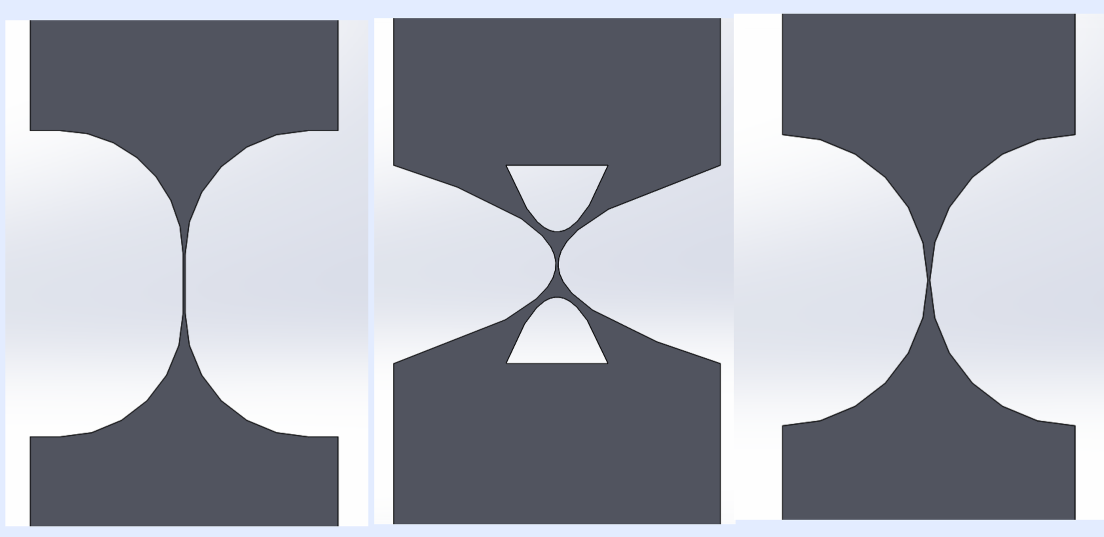

Figure 2: Three pivot designs under consideration. A) a straight flexure B) a hyperbolic flexure with notches and C) a semi-circular flexure.

Conclusion

Due to the mitigation of environmentally induced drift, the development of a low-cost means of conducting semi-absolute gravimetry could provide a significant benefit to those conducting long-period time-lapse gravimetry.

Acknowledgements

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[1] Dynamics of quartz tuning fork force sensors used in scanning probe microscopy, A Castellanos-Gomez, N Agraït and G Rubio-Bollinger, Nanotechnology, Volume 20, Number 21, 2009.