Statistical Approaches for Modelling Ice Sheet Interconnectivity

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with

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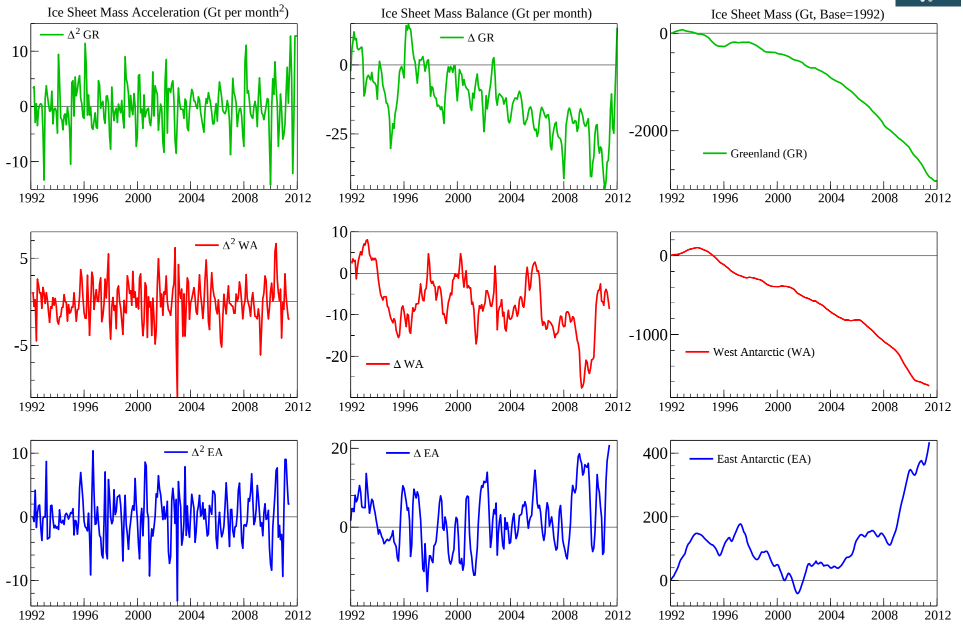
Overview

- Sea level rise uncertainty driven by Greenland and Antarctic (Church et al., 2013)
  - Typically modeled separately and without interactions
- Empirical linkages exist over very long time scales; Johnsen et al. (1972)
  - Broecker (1998); Stocker (1998): Bipolar seesaw model
- Bamber and Aspinall (2013) use expert surveys to derive correlations
  - Correlations hard to interpret when series are non-stationary
- We take an empirical approach:
  - Monthly data on ice sheet mass (IMBIE: Shepherd et al., 2012)
  - Econometric tools can test for non-stationarity and interdependence jointly
    - Cointegration captures the long-run relationships (Johansen, 1991)
    - Simulate how shocks propagate through the system

IMBIE (2012) Data



Bivariate Correlations

If data are non-stationary then empirical correlation coefficient does not capture the true correlation because average is not the expectation:

$$\mathbb{E}(x_{i,t}) \neq \frac{1}{T}\sum_{t=1}^{T} x_{i,t}$$

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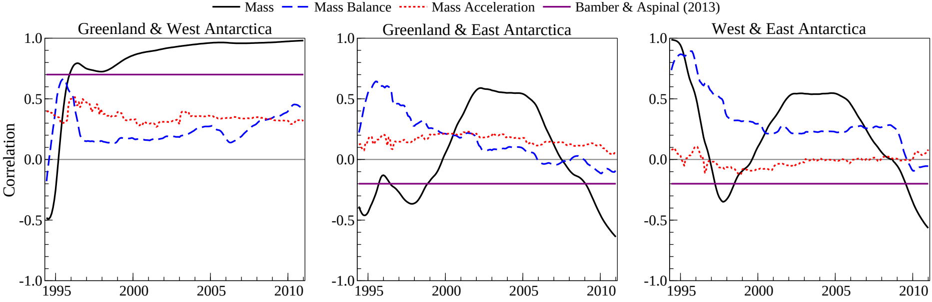
The recursively estimated correlation coefficients show instability:



→ Cointegration addresses non-stationarity and correlation simultaneously

I(2) Cointegrated VAR Model

Test for and estimate I(2) CVAR model from Juselius (2006):

$$\begin{bmatrix} \Delta^2 GR_t \\ \Delta^2 WA_t \\ \Delta^2 EA_t \end{bmatrix} = \begin{bmatrix} -0.007 & \underline{0} \\ -0.003 & -0.005 \\ 0.01 & -0.01 \end{bmatrix} \begin{bmatrix} \widetilde{LR1}_{t-1} \\ \widetilde{LR2}_{t-1} \end{bmatrix} + \begin{bmatrix} 0.07 & 0.02 \\ 0.007 & 0.21 \\ 0.18 & 0.30 \end{bmatrix} \begin{bmatrix} \widetilde{MR1}_{t-1} \\ \widetilde{MR2}_{t-1} \end{bmatrix}$$

$$+ \begin{bmatrix} 0.36 & -0.14 & -0.02 \\ 0.10 & 0.30 & -0.04 \\ 0.07 & 0.23 & 0.38 \end{bmatrix} \begin{bmatrix} \Delta^2 GR_{t-1} \\ \Delta^2 WA_{t-1} \\ \Delta^2 EA_{t-1} \end{bmatrix} + \text{Deterministic terms} + \begin{bmatrix} \epsilon_{GR,t} \\ \epsilon_{WA,t} \\ \epsilon_{EA,t} \end{bmatrix}$$

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Medium-run relationships:

Bi-polar Relation, $\widetilde{MR1}_t : \Delta WA_t = 0.4\Delta GR_t - 4$

Antarctic Relation, $\widetilde{MR2}_t : \Delta EA_t = -\Delta WA_t - 7$

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Climate Econometrics

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Long-run relationships:

Bi-polar Relation, $\widetilde{LR1}_t : WA_t = 0.4 GR_t - 4t - 15 \Delta WA_t - 8 \Delta GR_t + 3$

Antarctic Relation, $\widetilde{LR2}_t : EA_t = -WA_t - 7t - 52 \Delta WA_t - 43 \Delta EA_t + 14$

→ Trend likely a proxy for physical forcing: temperature / ocean warming
→ East Antarctica only affects Greenland indirectly in the long-run

I(2) Cointegrated VAR Model

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$$\begin{bmatrix} \Delta^2 GR_t \\ \Delta^2 WA_t \\ \Delta^2 EA_t \end{bmatrix} = \begin{bmatrix} \underline{0} & \underline{0} \\ -0.004 & \underline{0} \\ 0.01 & -0.01 \end{bmatrix} \begin{bmatrix} \widetilde{LR1}_{t-1} \\ \widetilde{LR2}_{t-1} \end{bmatrix} + \begin{bmatrix} \underline{0} & \underline{0} \\ \underline{0} & \underline{0} \\ 0.17 & 0.28 \end{bmatrix} \begin{bmatrix} \widetilde{MR1}_{t-1} \\ \widetilde{MR2}_{t-1} \end{bmatrix}$$

$$+ \begin{bmatrix} 0.30 & \underline{0} & \underline{0} \\ 0.12 & 0.30 & \underline{0} \\ \underline{0} & 0.23 & 0.36 \end{bmatrix} \begin{bmatrix} \Delta^2 GR_{t-1} \\ \Delta^2 WA_{t-1} \\ \Delta^2 EA_{t-1} \end{bmatrix} + \text{Deterministic terms} + \begin{bmatrix} \epsilon_{GR,t} \\ \epsilon_{WA,t} \\ \epsilon_{EA,t} \end{bmatrix}$$

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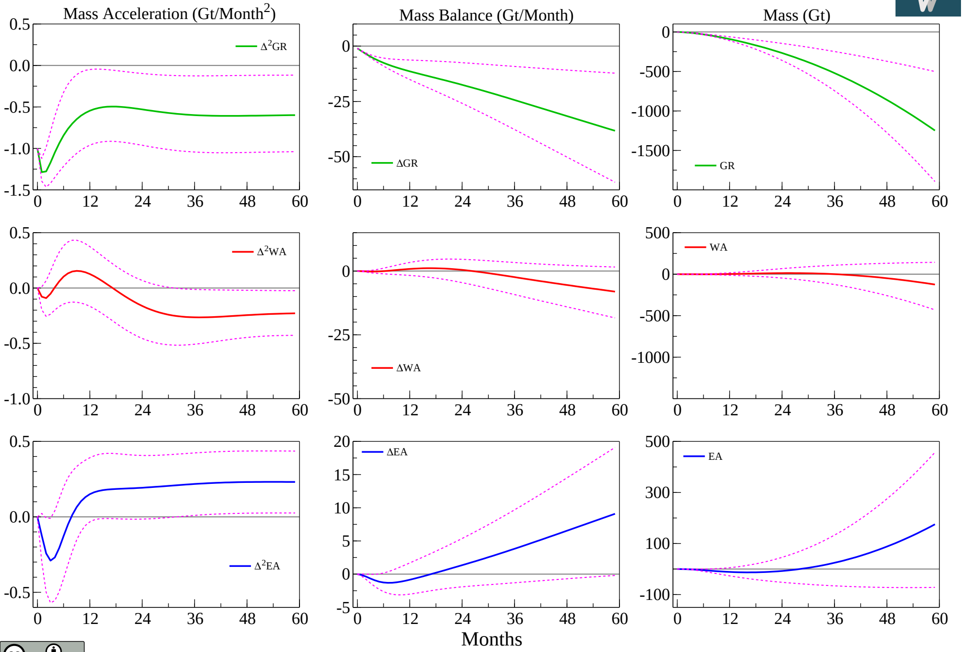
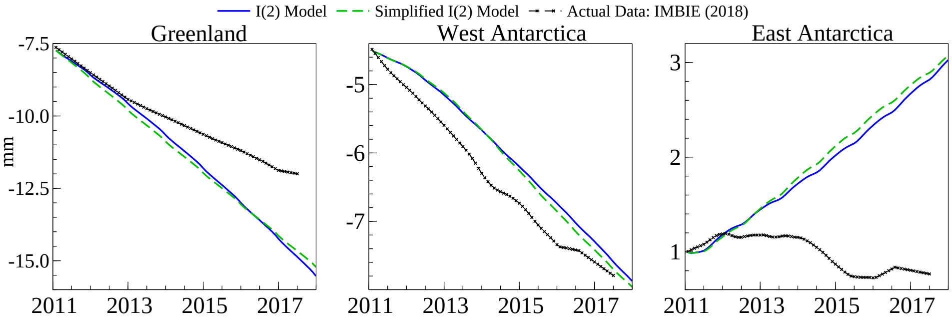
→ Can simplify using automatic model selection (Doornik, 2009)

Simulated impact of a sudden loss of ice in Greenland

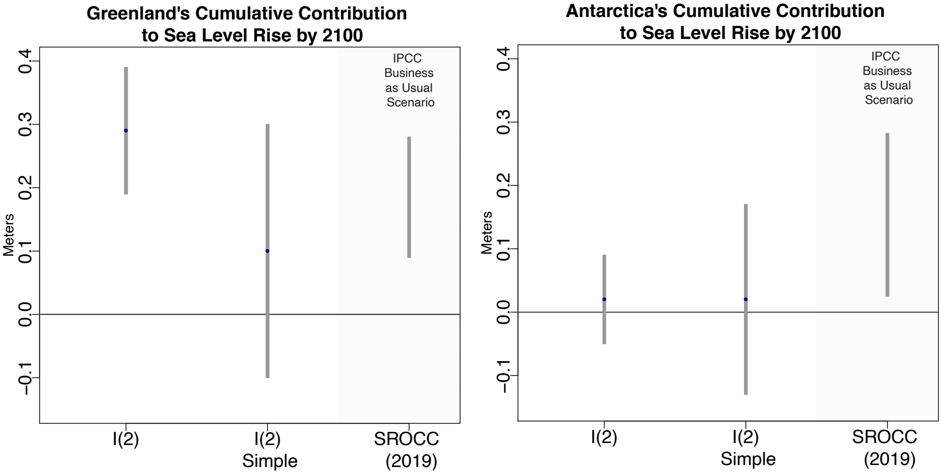


Out-of-Sample Forecasts



- Unconditional forecasts perform well for first 1-2 years
- Track West Antarctica up through 6-years-ahead
- Is East Antarctica still reacting to the long-run relationships?
- I(2) and simplified I(2) models are similar at these horizons

Predicted Sea Level Contributions (relative to 1995)



- **Greenland:** I(2) in upper tail and Simple I(2) in lower tail of IPCC range
- **Antarctica:** Both models in lower tail of IPCC range (East Antarctica?)

THANK YOU

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