A Hybrid-Precision Numerical Orbit Integration Technique for Next Generation Gravity Missions

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Introduction and Motivation

The precision of numerical orbit integration is crucial in the precise orbit determination and gravity field recovery, especially in the context of future satellite gravimetry missions where nanometer or even sub-nanometer-level inter-satellite measurements are provided by Laser Ranging Interferometer (LRI). To fully exploit the ultra-high precision of LRI observations by the GRACE Follow-On (GRACE-FO) and Next Generation Gravity Missions (NGGM), the numerical integration errors should be controlled at least on the same level of the LRI noises', and had better be one-order magnitude smaller than that to fully avoid their contaminations on the solved-for parameters in the gravity field recovery. For one-day arc length of orbit integration, however, the currently widely-used double-precision arithmetic cannot meet above requirements given the nanometer or even sub-nanometer inter-satellite *range* measurement precision while is adequate for the *range rate* data. The primary cause to this incapability is the round-off errors accumulation with the increase of integration arc length, which can be greatly and efficiently tackled by the hybrid-precision orbit integration technique proposed by this contribution.

Methodology and Implementation

The equation of motion for the satellite in the inertial frame can be expressed in either the *Cowell's formulation* as Eq. (1) or the *Encke's formulation* Eq. (2),

$$\ddot{\boldsymbol{r}} = -\frac{\mu}{|\boldsymbol{r}|^3}\boldsymbol{r} + \boldsymbol{a}_p(t;\ \boldsymbol{r}, \dot{\boldsymbol{r}}) \tag{1}$$

$$\delta\ddot{\boldsymbol{r}} = \ddot{\boldsymbol{r}} - \ddot{\boldsymbol{r}}^{REF} = -\frac{\mu}{|\boldsymbol{r}|^3}\boldsymbol{r} + \frac{\mu}{|\boldsymbol{r}^{REF}|^3}\boldsymbol{r}^{REF} + \boldsymbol{a}_p(t;\ \boldsymbol{r}, \dot{\boldsymbol{r}}) \tag{2}$$

where $\boldsymbol{r}$, $\dot{\boldsymbol{r}}$ and $\ddot{\boldsymbol{r}}$ are the vectors of satellite position, velocity and acceleration, μ is the product of the Earth mass and gravitational constant, and $\boldsymbol{a}_p(t;\ \boldsymbol{r}, \dot{\boldsymbol{r}})$ is the perturbing forces. The Eq. (2) is obtained by subtracting the central gravitational force term, evaluated at the un-perturbed reference orbit $\boldsymbol{r}^{REF}$, from the Eq. (1), resulting the differential acceleration $\delta\ddot{\boldsymbol{r}}$ between the total true acceleration $\ddot{\boldsymbol{r}}$ and reference acceleration $\ddot{\boldsymbol{r}}^{REF}$.

Without loss of generality, the numerical orbit integration procedure can be divided into two steps, namely the increment calculation step as Eq. (3) and the orbit propagation step Eq. (4), for example, in the *Cowell's formulation*,

$$\boldsymbol{P}_{n,n+1} = \sum_{i=0}^{K-1} \alpha_i \ddot{\boldsymbol{r}}_{n-i}, \quad \boldsymbol{V}_{n,n+1} = \sum_{i=0}^{K-1} \beta_i \ddot{\boldsymbol{r}}_{n-i} \tag{3}$$

$$\boldsymbol{P}_{n+1} = \boldsymbol{P}_n + h^2 \boldsymbol{P}_{n,n+1}, \quad \boldsymbol{V}_{n+1} = \boldsymbol{V}_n + h\boldsymbol{V}_{n,n+1} \tag{4}$$

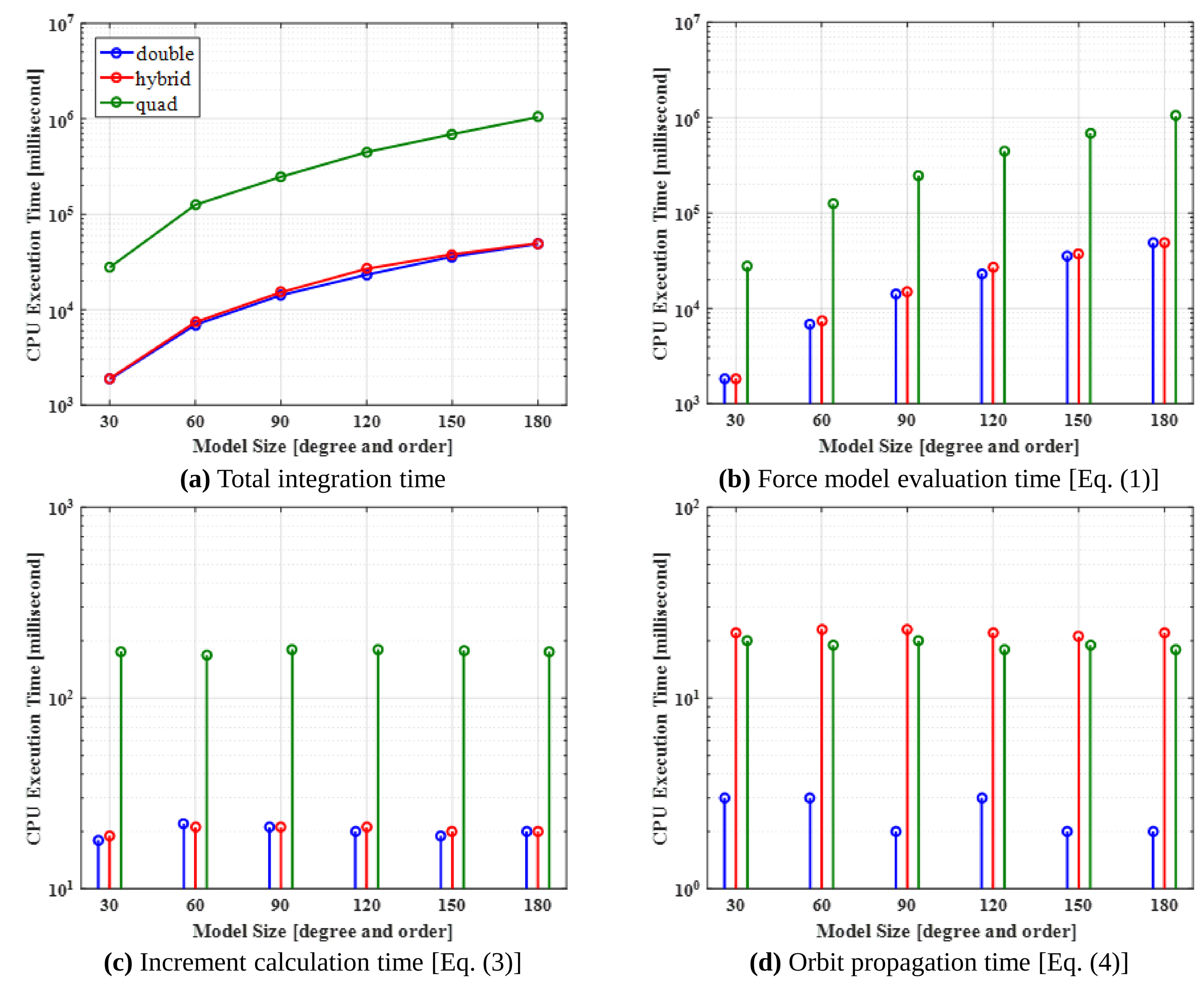
where α_i and β_i are integration coefficients of the K^{th}-order integrator, $\boldsymbol{P}_{n,n+1}$ and $\boldsymbol{V}_{n,n+1}$ are integration increments which propagate the state vectors $\boldsymbol{P}_n$ and $\boldsymbol{V}_n$ from epoch t_n to t_{n+1} with integration step size h. For the *Encke's formulation*, the increment calculation is the same as Eq. (3) except that $\ddot{\boldsymbol{r}}$ is replaced by $\delta\ddot{\boldsymbol{r}}$ and thus $\delta\boldsymbol{P}_{n,n+1}$ and $\delta\boldsymbol{V}_{n,n+1}$ are utilized, while the orbit propagation step reads as Eq. (5) given the analytical reference orbit at t_{n+1},

$$\boldsymbol{P}_{n+1} = \boldsymbol{P}_{n+1}^{REF} + \delta\boldsymbol{P}_n + h^2\delta\boldsymbol{P}_{n,n+1}, \quad \boldsymbol{V}_{n+1} = \boldsymbol{V}_{n+1}^{REF} + \delta\boldsymbol{V}_n + h\delta\boldsymbol{V}_{n,n+1} \tag{5}$$

Based on above two-step decomposition, we find that the round-off errors basically occur in the least time-consuming orbit propagation step while the more time-demanding increment calculation step is less sensitive to the round-off due to its small numerical magnitude. Therefore, in the hybrid-precision (*HB*) orbit integration technique, we perform the increment calculation Eq. (3) in the double-precision arithmetic (*DB*) while the whole orbit propagation step Eq. (4) or (5) in the quadruple-precision arithmetic (*QD*), which benefits from the more significant digits of the *QD* than the *DB* but not at the expanse of computational efficiency [1].

Numerical Results and Analysis

The computational efficiency and relative precision of the hybrid-precision orbit integration technique are evaluated in terms of the *Cowell's* (*COW*) and the *Encke's formulation*, where the reference orbit of the latter is chosen as either the *osculating* (*OSC*) or the *best-fit* (*BFT*) one as proposed by [2].



(a) Total integration time

(b) Force model evaluation time [Eq. (1)]

(c) Increment calculation time [Eq. (3)]

(d) Orbit propagation time [Eq. (4)]

Fig. 1. CPU execution times of double-, hybrid- and quadruple-precision arithmetic for the integration of one-day orbit arc in the *Cowell's formulation* by the 8[th]-order Adams-Cowell integrator with 1-second integration step size under different gravity model sizes.

Figure 1 gives the CPU execution time cost for the comparisons of total and individual parts of the orbit integration process by the *DB*, *HB* and *QD* arithmetic in the *Cowell's formulation* for one-day arc length under different gravity field model sizes, which indicates that the force model evaluation part Eq. (1) takes the most of the total time over 99% and should be performed in the *DB* considering that the *QD* is at least 5 to 10 times less efficient than the *DB* for current computer processors. Therefore, the proposed *HB* has the same efficiency as the *DB* since it only utilizes the *QD* at the least time-consuming orbit propagation step Eq. (4) or (5).

Relative Precisions are assessed for the *HB* and *DB* in both *Cowell's* (*DB-COW*, *HB-COW*) and *Encke's formulations* (*DB-OSC*, *DB-BFT*, *HB-OSC*, *HB-BFT*), where the integrated inter-satellite range and range rate are compared with the 'true' values generated by the *QD-COW* [1].

Table 1. The Root Mean Square Error (RMSE) and Maximum Absolute Error (MaxAE) of the integrated inter-satellite *range* and *range rate* by the double-precision (*DB*) and hybrid-precision (*HB*) arithmetic implemented in both the *Cowell's* (*COW*) and the *Encke's formulations* with the reference orbit chosen as the oscculating (*OSC*) or the *best-fit* (*BFT*) one. The 'true' values of *range* and *range rate* are generated by the quadruple-precision (*QD*) integration in the *Cowell's formulation*. The 8[th]-order Gauss-Jackson integrator with different step sizes *h* are used for the 6-hour and 24-hour arc integrations under the static gravity field model up to 120 degrees and orders. **(a)** presents the results for all three *DB* methods, and **(b)** for the *HB* methods.

(a)

Error	Method	Arc	Range (μm) h = 1.0s	Range (μm) h = 2.0s	Range (μm) h = 5.0s	Range Rate (nm/s) h = 1.0s	Range Rate (nm/s) h = 2.0s	Range Rate (nm/s) h = 5.0s
RMSE	*DB-COW*	6-h	1.317	2.224	2.091	0.272	0.336	0.217
		24-h	**6.726**	**25.332**	**25.277**	**0.283**	**0.333**	**0.626**
	DB-OSC	6-h	0.026	0.096	0.119	0.009	0.013	0.008
		24-h	3.708	0.464	1.108	0.170	0.137	0.065
	DB-BFT	6-h	0.033	0.093	0.231	0.002	0.006	0.015
		24-h	0.159	0.320	0.998	0.003	0.007	0.016
MaxAE	*DB-COW*	6-h	1.939	3.129	2.685	0.345	0.489	0.316
		24-h	7.153	26.167	26.318	0.476	0.537	0.982
	DB-OSC	6-h	0.057	0.143	0.160	0.018	0.026	0.016
		24-h	4.253	0.801	1.382	0.235	0.230	0.101
	DB-BFT	6-h	0.065	0.123	0.278	0.008	0.014	0.027
		24-h	0.251	0.424	1.135	0.009	0.017	0.030

(b)

Error	Method	Arc	Range (nm) h = 1.0s	Range (nm) h = 2.0s	Range (nm) h = 5.0s	Range Rate (pm/s) h = 1.0s	Range Rate (pm/s) h = 2.0s	Range Rate (pm/s) h = 5.0s
RMSE	*HB-COW*	6-h	1.231	2.885	1.179	0.491	1.094	0.709
		24-h	12.027	18.579	40.319	1.572	1.396	2.612
	HB-OSC	6-h	0.505	0.225	0.391	0.049	0.025	0.270
		24-h	14.468	1.223	47.130	0.593	0.285	1.498
	HB-BFT	6-h	0.055	0.053	0.100	0.013	0.008	0.029
		24-h	0.016	1.126	0.033	0.017	0.034	0.052
MaxAE	*HB-COW*	6-h	1.945	4.319	2.679	0.773	1.757	1.416
		24-h	13.336	20.996	46.782	**2.827**	**2.990**	**4.341**
	HB-OSC	6-h	0.599	0.260	0.982	0.085	0.053	0.476
		24-h	15.351	1.655	51.068	1.035	0.532	1.858
	HB-BFT	6-h	0.086	0.069	0.165	0.024	0.014	0.039
		24-h	**0.028**	**1.164**	**0.109**	**0.035**	**0.056**	**0.102**

It is shown in the Table 1 that the *HB* methods significantly outperform their *DB* counterparts in both Root Mean Square Error (RMSE) and Maximum Absolute Error (MaxAE) [1]. With the *HB*, the integration errors of *range rate* can be controlled at the pm/s-level in terms of 24-hour arc MaxAE. Moreover, the sub-nanometer-level *range* integration precision can be achieved by the *Encke's formulation* with the best-fit reference orbit (*HB-BFT*), which guarantees enough numerical precision of orbit integrations for the GRACE-FO and NGGM, while no *DB* methods could produce such competitive precisions.

Summary and Discussion

- The hybrid-precision orbit integration technique can efficiently achieve the picometer/second-level inter-satellite *range rate* precision for one-day arc orbit integration in terms of the maximum absolute error, and the sub-nanometer-level *range* precision when implemented in the *Encke's formulation* with the *best-fit* reference orbit;
- Due to sensor noises and background models deficiencies, the contribution of improved orbit integration precision to the final gravity field recovery can be rather limited as for the GRACE-FO, but it is important for the NGGM where better satellite constellations and onboard sensors are provided.

Reference

[1] Nie, Y., Shen, Y., Chen, Q., Xiao, Y. (2020) Hybrid-Precision Arithmetic for Numerical Orbit Integration towards Future Satellite Gravimetry Missions. *Adv. Space Res.* (in press)

[2] Ellmer, M., Mayer-Gürr, T. (2017) High precision dynamic orbit integration for spaceborne gravimetry in view of GRACE Follow-on. *Adv. Space Res.* 60 (1), 1-13.

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