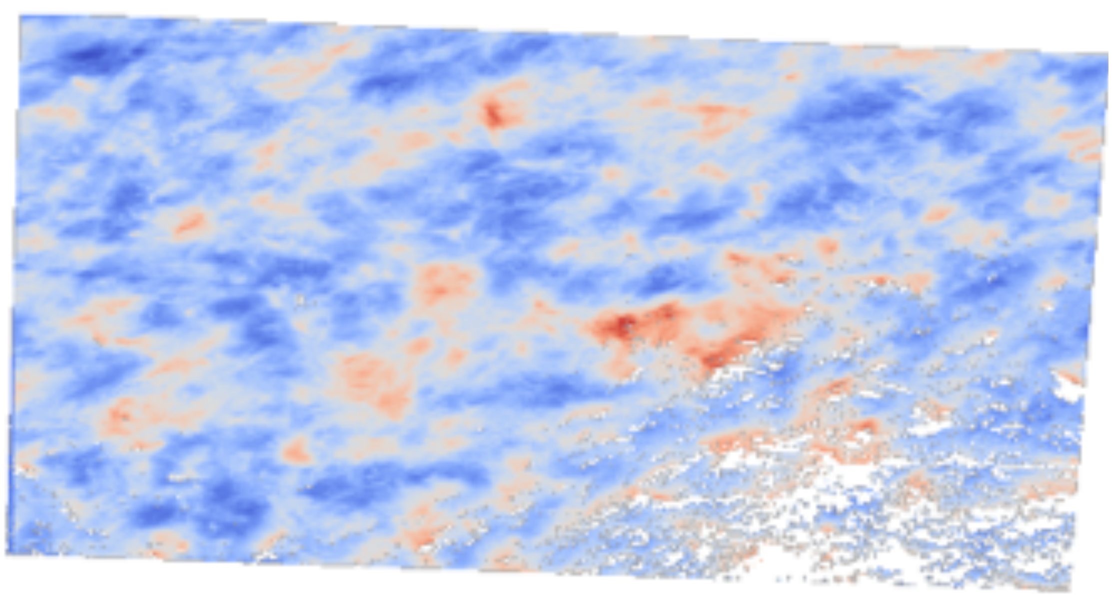
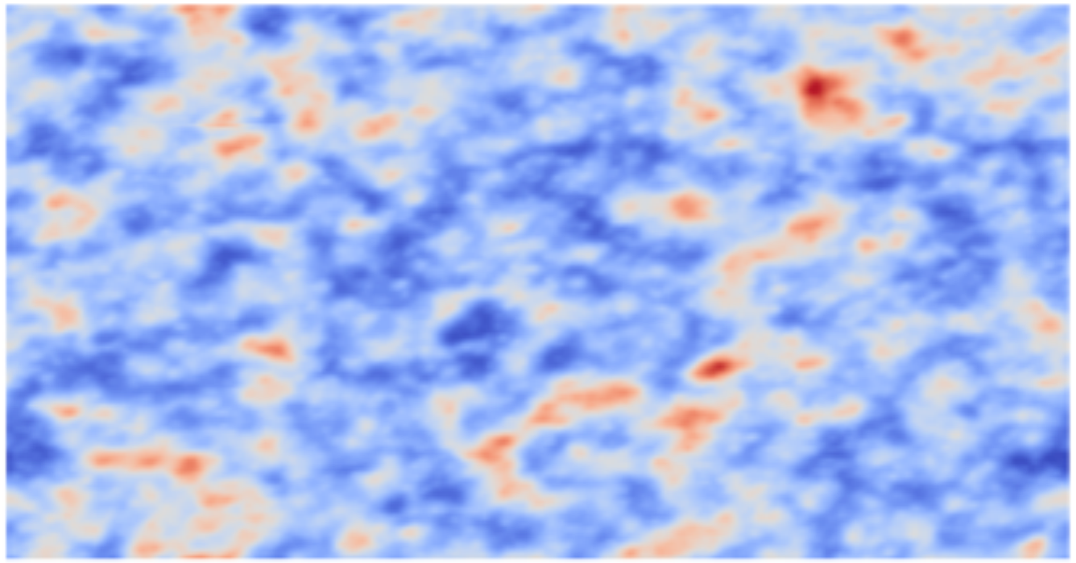
Sea ice snow topography captured by an idealized model

Predrag Popovic





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Summary

We develop a simple statistical model of snow topography on undeformed Arctic sea ice that accurately captures the statistics of 3d snow topography.

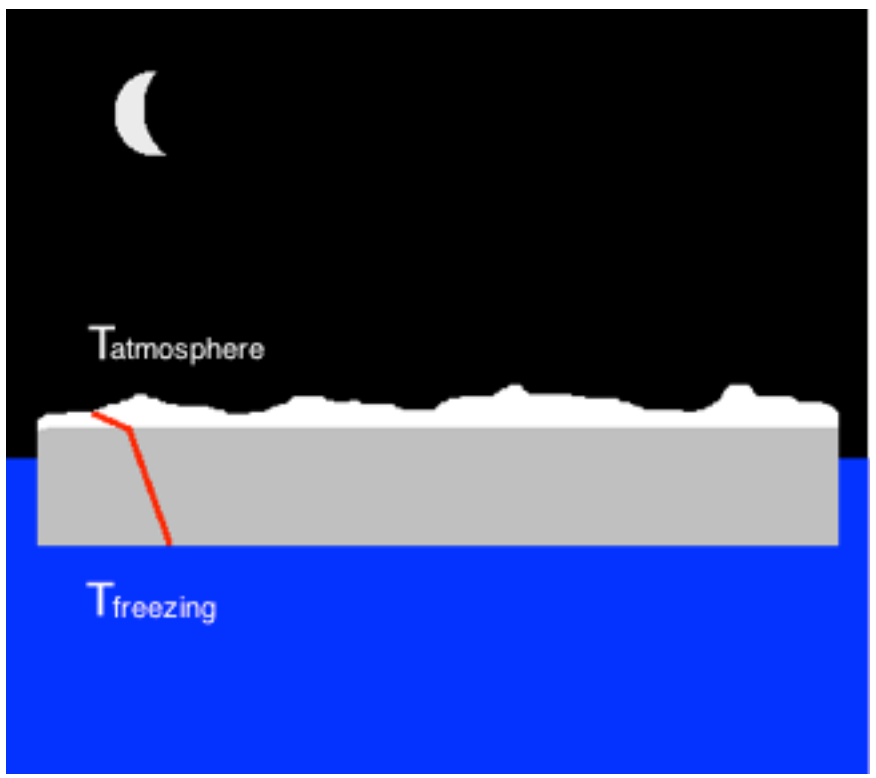
We find that

- The pre-melt surface on undeformed ice is fully characterized by 3 parameters - mean snow depth, roughness, and the correlation length.
- This allows us to understand the 3d heat transport and early melt pond evolution under arbitrary snow configuration on undeformed ice.

Introduction

Snow on sea ice

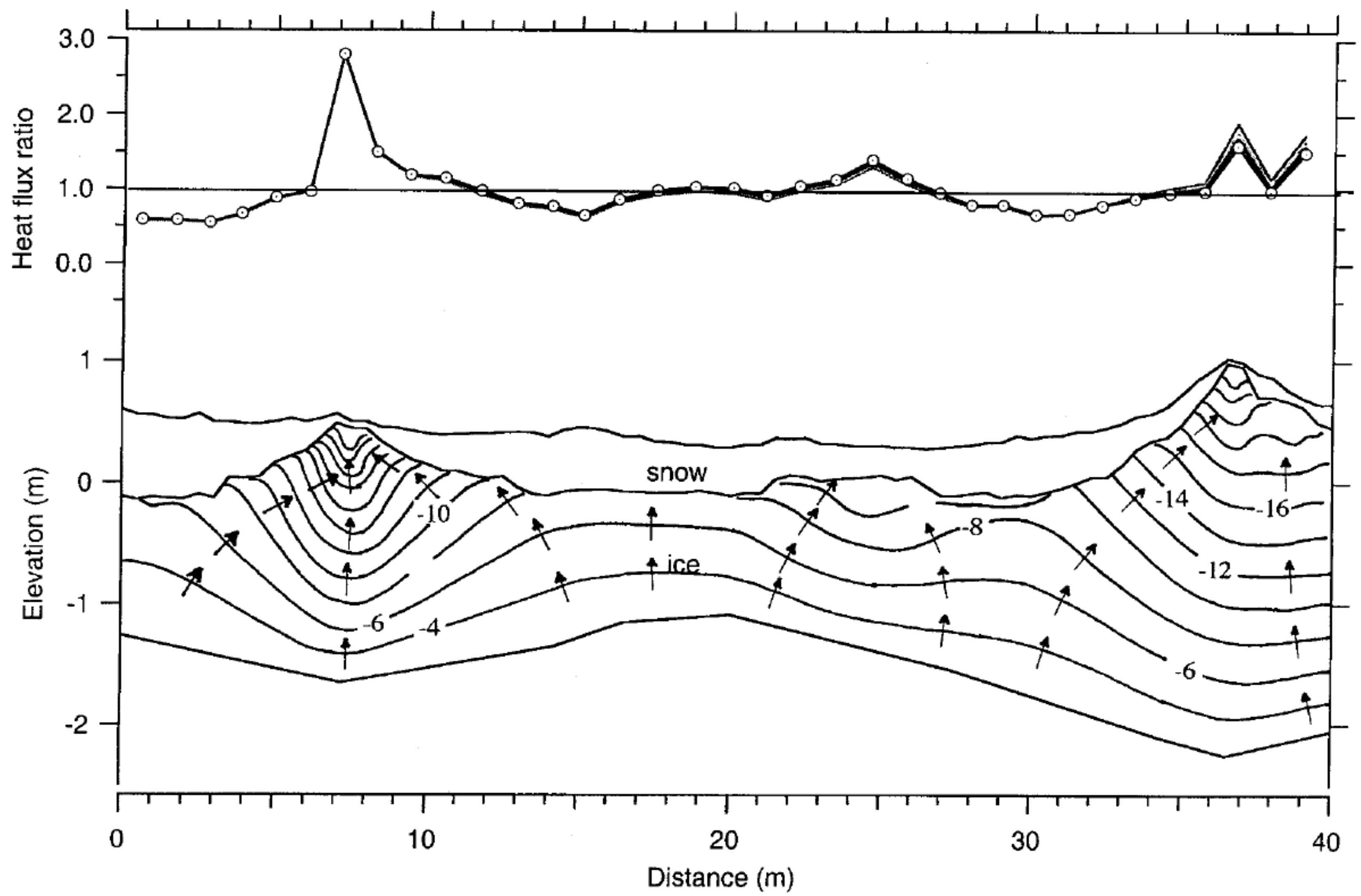
Snow heat conductivity is ~10 times less than ice conductivity



Snow albedo ~0.8
Bare ice albedo ~0.6
Melt pond albedo ~0.2

Snow significantly affects the heat budget of the ice. It thermally insulates the ice, slowing down growth during winter, it increases the ice albedo, slowing down ice melt during summer, and it provides a source of fresh meltwater for melt ponds that lower the albedo and accelerate the melt during summer.

Snow on sea ice

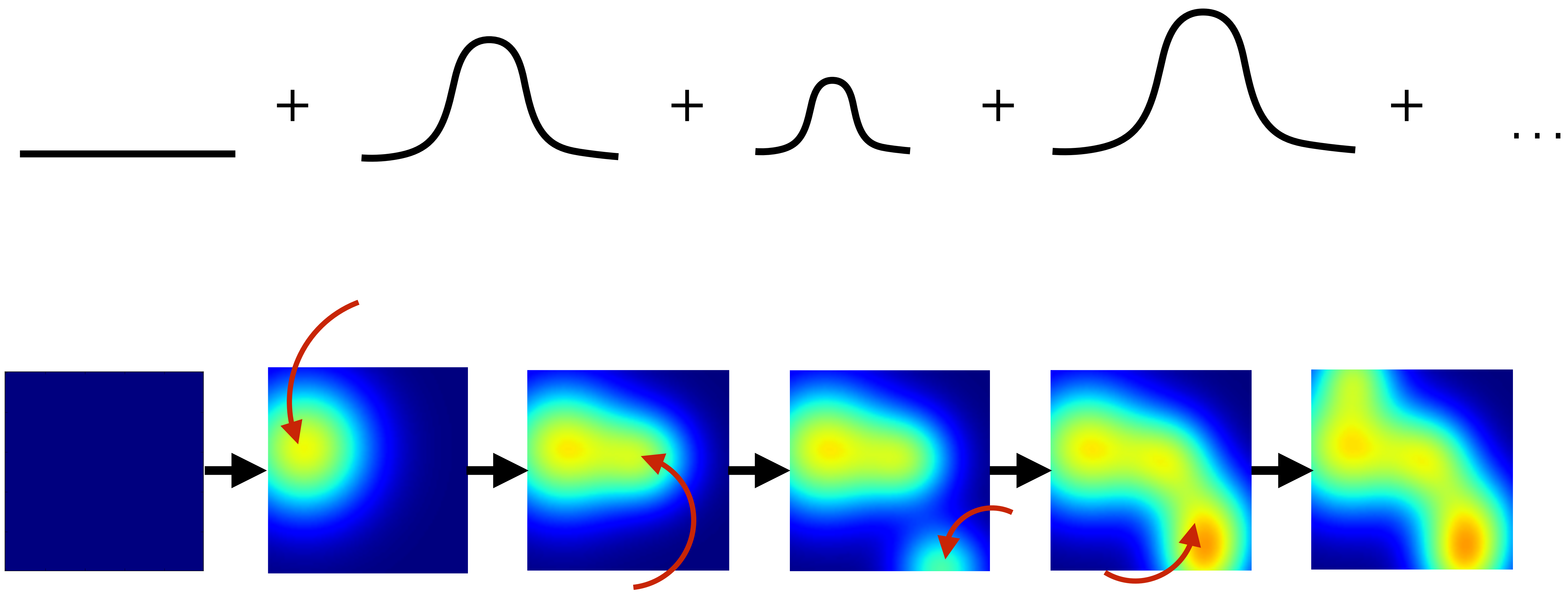


Spatial variability significantly alters the thermal properties of snow. Snow with more spatial variability will lead to more heat conduction, lower overall albedo, and sooner melt pond formation, thereby leading to both more winter growth and more summer melt.

* Sturm, M., D. K. Perovich, and J. Holmgren, "Thermal conductivity and heat transfer through the snow on the ice of the Beaufort Sea", J. Geophys. Res., 107(C21), 8043, doi:10.1029/2000JC000409, 2002.

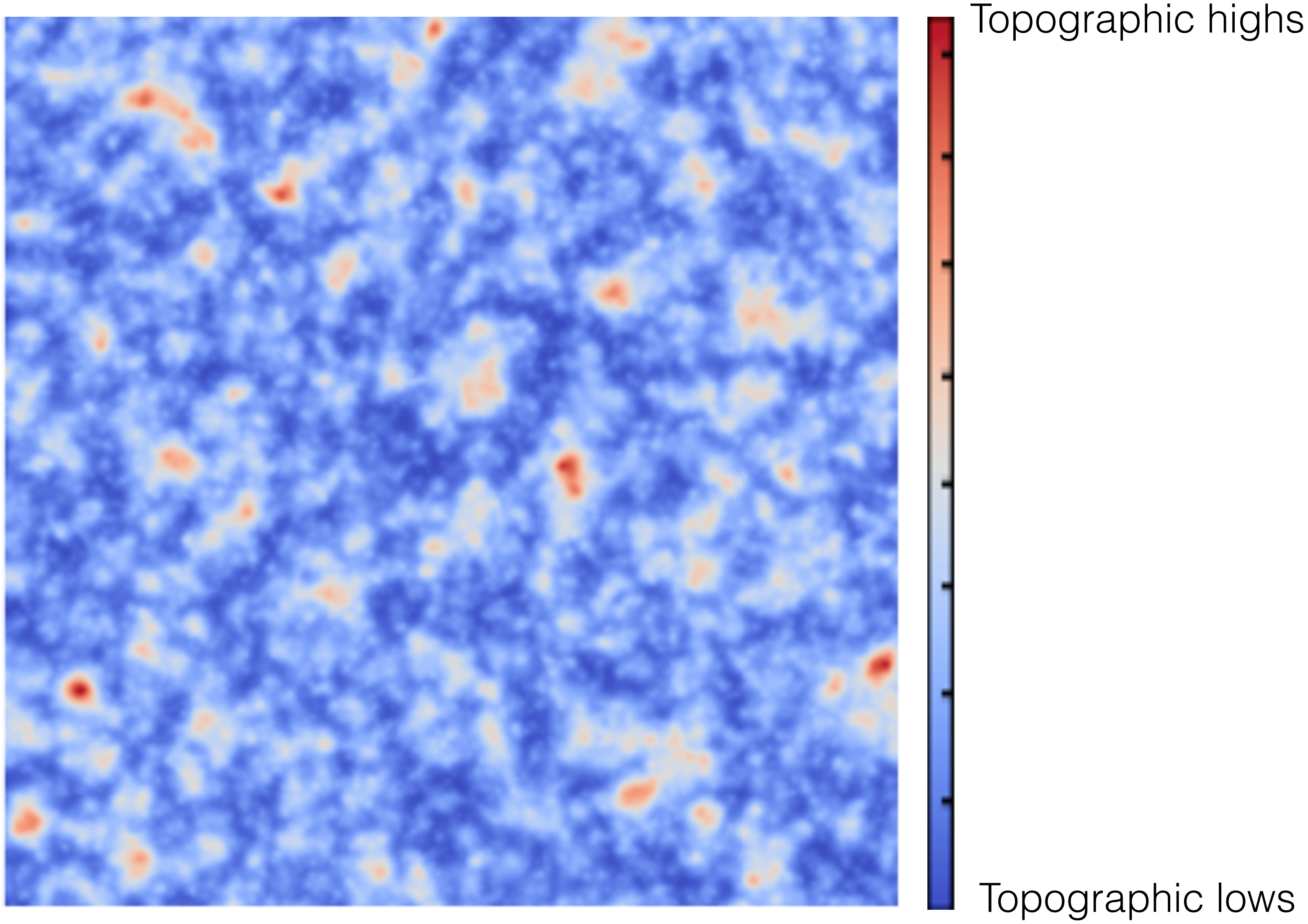
The “snow dune” model

The “snow dune” model



We represent the snow surface as a sum of many randomly placed Gaussian mounds on an initially flat surface. The mounds have variable width and height proportional to the width. The exact shape of the mounds or the distribution of widths do not matter.

The "snow dune" model



An example of a synthetic "snow dune" topography

The "snow dune" model

There are three free parameters
that fully specify the model:

$$\langle h \rangle,\ \sigma(h),\ \text{and}\ l_0$$

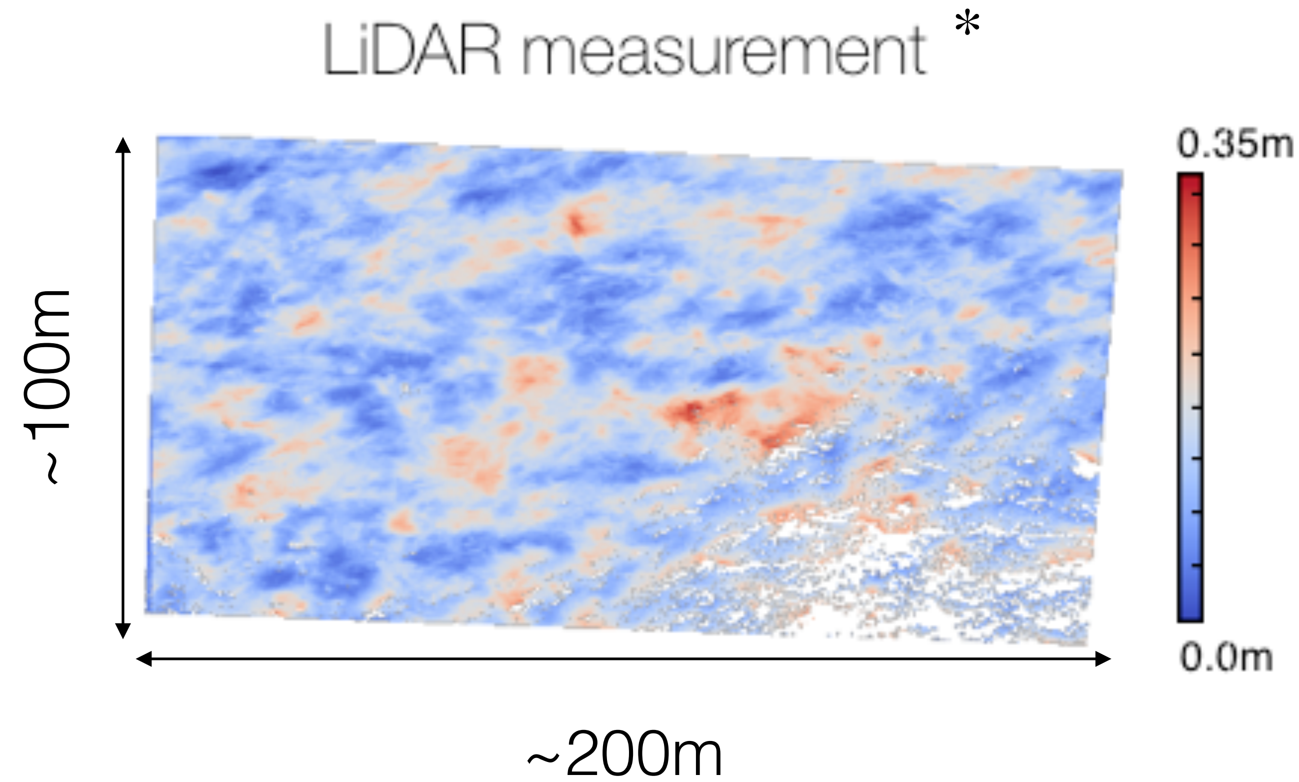
Mean
snow depth

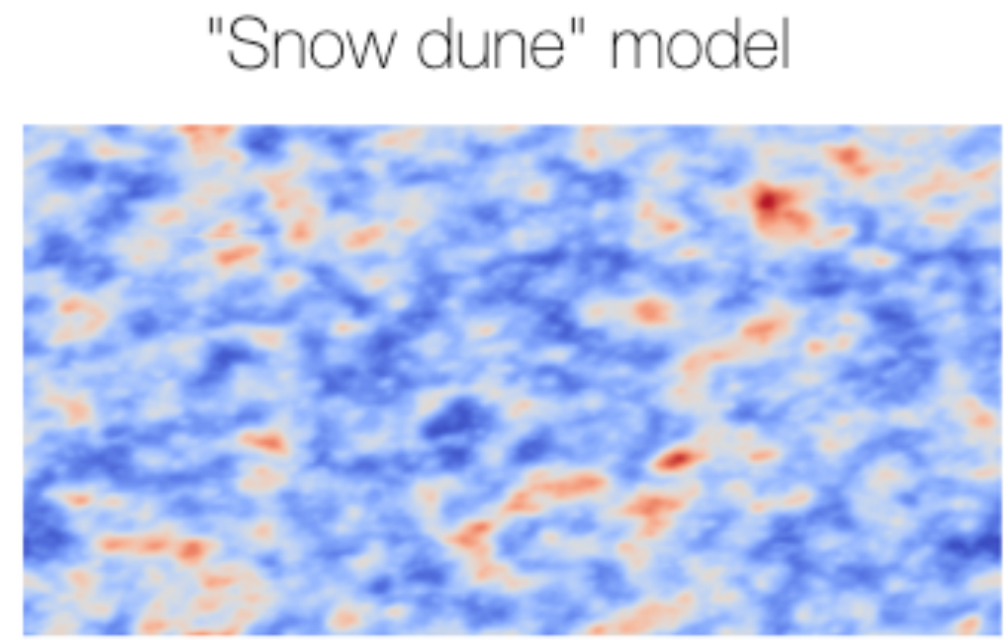
Roughness

Correlation
length

Comparing with observations

The "snow dune" model vs LiDAR observations

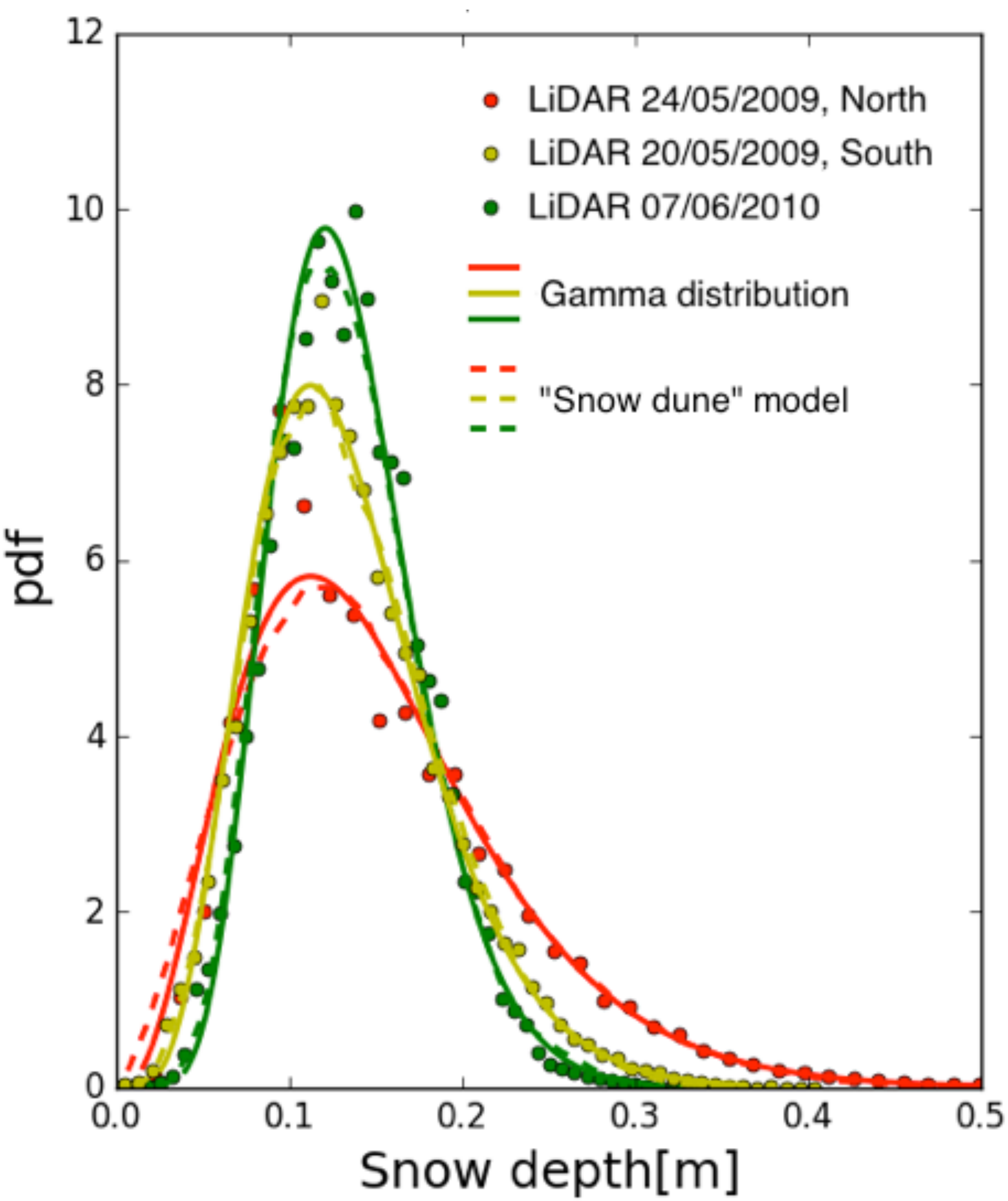




In 2009 and 2010, Polashenski et al. took detailed LiDAR measurements of the ice surface on undeformed first-year ice near Barrow, Alaska. We use these measurements to compare with our model. We can directly specify the mean, roughness, and the correlation length to our model from the measurements.

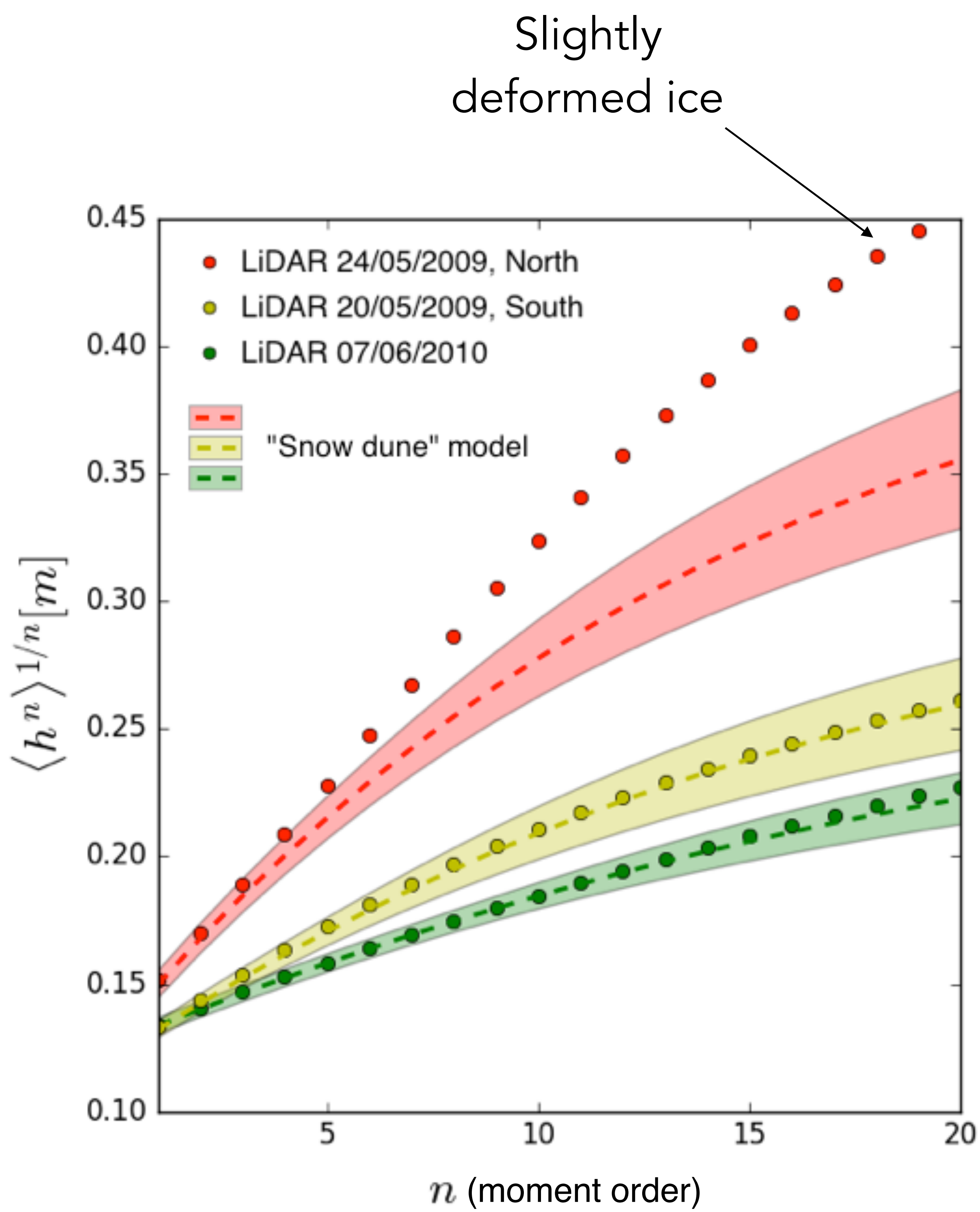
* Polashenski, C., D. Perovich, and Z. Courville (2012), "The mechanisms of sea ice melt pond formation and evolution", J. Geophys. Res., 117, C01001, doi:10.1029/2011JC007231.

Snow depth distribution



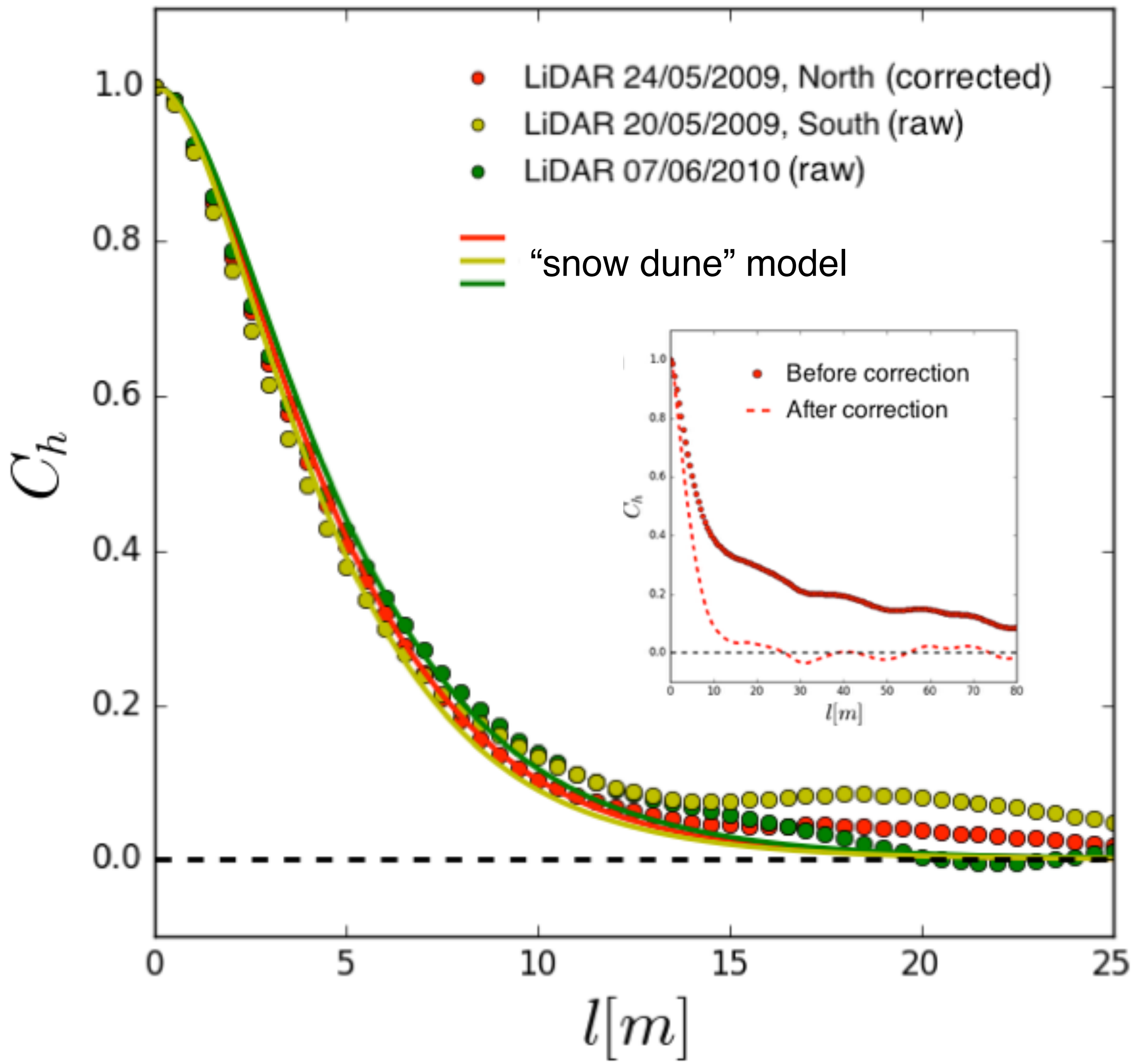
The model snow-depth distribution agrees with the measured snow-depth distribution according to the KS metric for all three available measurements. Our model is well-fit with a gamma distribution for all values of the model parameters, which suggests the same distribution should be applicable to real snow.

Snow depth distribution - high order moments



We also looked at higher-order moments to compare the details of the snow-depth distribution. Here, dots represent data, while the dashed lines and the envelopes represent the mean and one standard deviation over an ensemble of random realizations of the model. For 2009 South and 2010 measurements, all higher order moments agree between data and the model implying that the real and model snow-depth distributions are statistically *indistinguishable* in these 2 cases. Discrepancy exists for 2009 North measurements for high-order moments. There, ice was slightly deformed, indicating that our model only applies to undeformed ice.

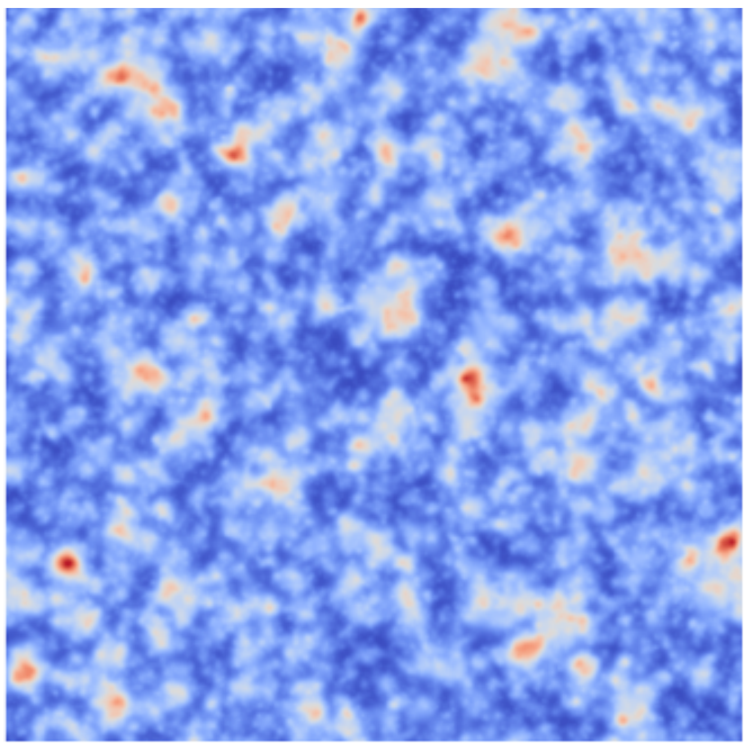
The correlation function



The model matches the 2010 and 2009 South spatial correlation function accurately without any manipulation of the data, while it matches the measured 2009 North correlation function only after removing the long-length scales from these data. This again suggests the model is accurate for undeformed ice, while there exist unaccounted factors on deformed ice.

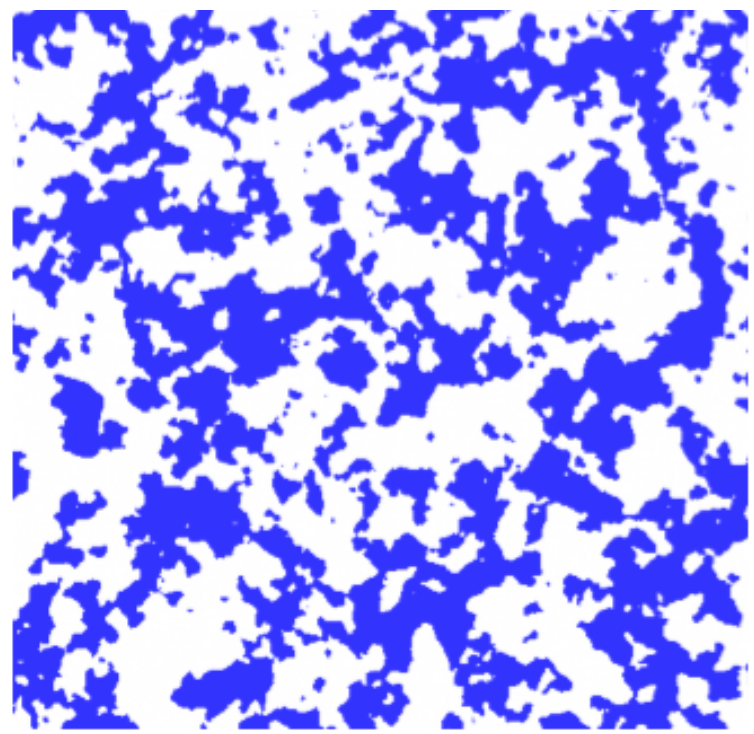
Comparing with melt pond geometry

“Snow dune” model



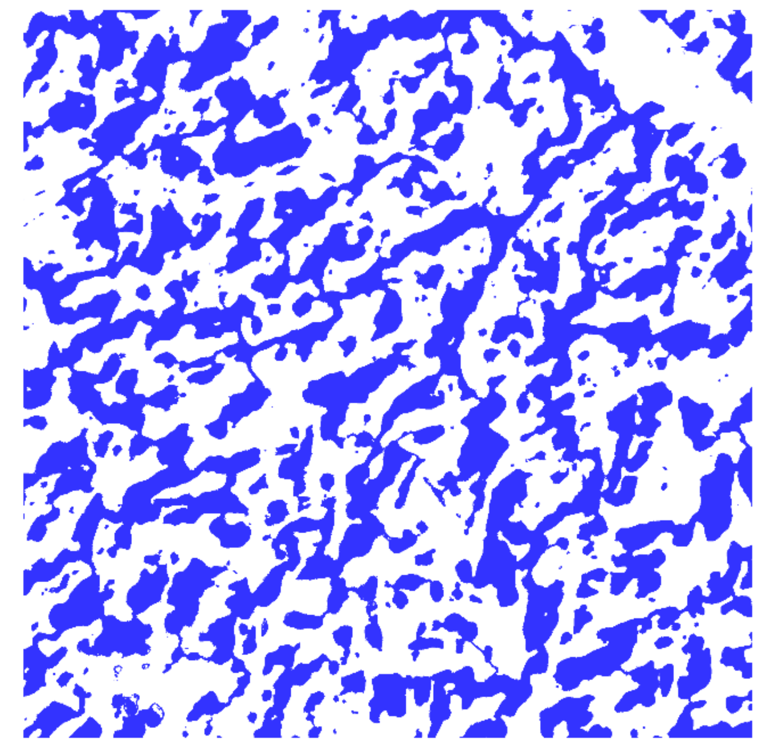
An example of a synthetic “snow dune” topography

“Snow dune” model ponds



Cutting the “snow dune” surface with a horizontal plane

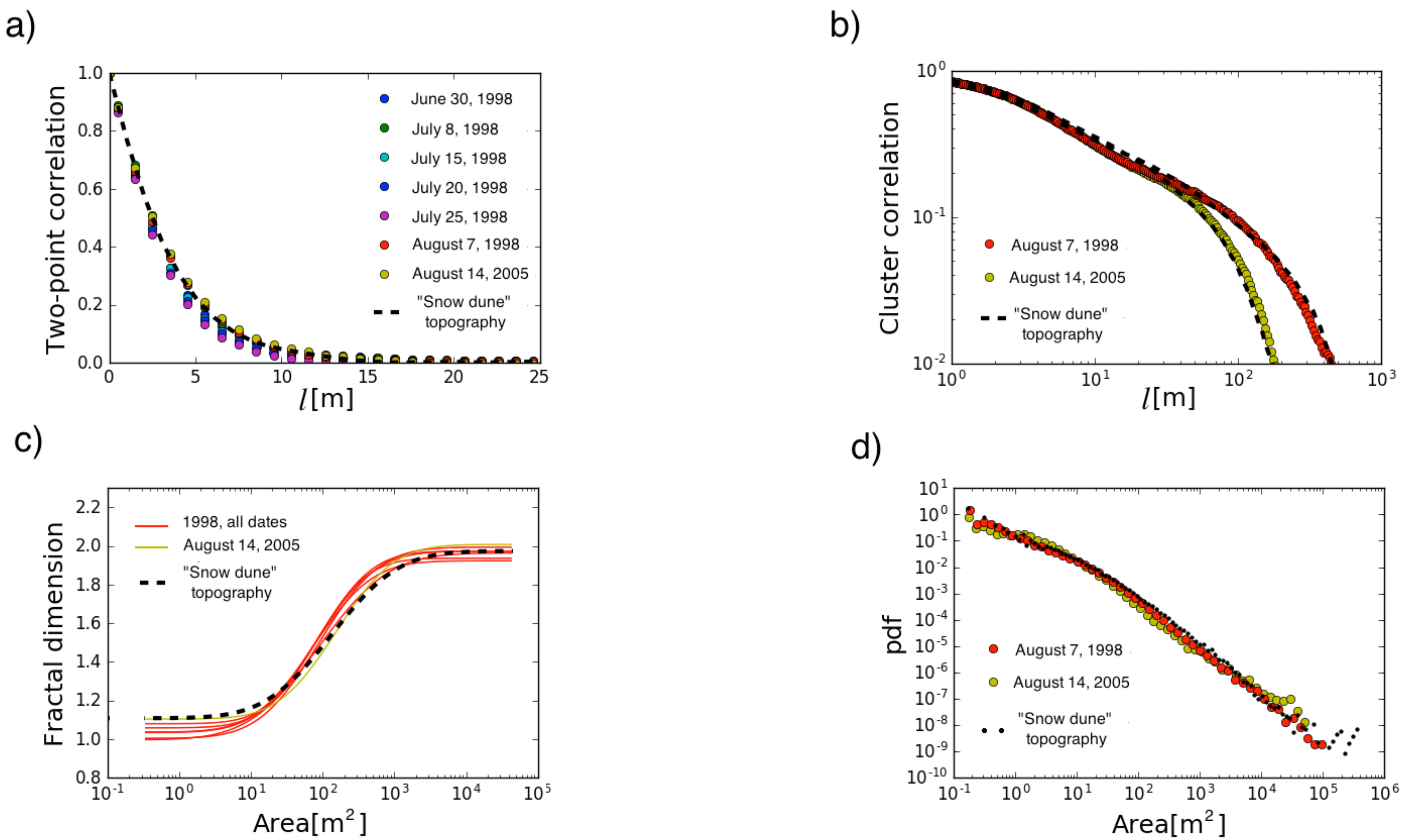
Melt pond image (HOTRAX mission)



Actual melt pond photograph

We create “melt ponds” in our model by cutting the synthetic surface with a plane and assuming melt ponds lie below this plane. We then compare the geometry of model melt ponds with actual melt pond images taken during the 1998 SHEBA and 2005 HOTRAX missions.

Comparing with melt pond geometry



The model accurately matches melt pond statistics describing (a) the pond length-scale, (b) pond connectedness, (c) pond fractal dimension, and (d) pond size distribution over the entire observation range. This suggests our model is applicable to much larger scales than the small-scale LiDAR measurements. The correlation length we used to match the pond data was approximately the same as the one we used to match the LiDAR data, confirming the close relationship between melt-ponds and pre-melt snow.

Applications of our model

Heat transport through the ice

3 non-dimensional parameters

$$\eta \equiv \frac{k_i}{k_s}\frac{\langle h \rangle}{H} \quad , \quad \Sigma \equiv \frac{\sigma(h)}{\langle h \rangle} \quad , \quad \Lambda \equiv \frac{l_0}{H}$$

Snow depth — Roughness — Correlation length

$\langle h \rangle$, $\sigma(h)^2$ - snow depth mean and variance

l_0 - topographic correlation length

H - ice thickness

k_i , k_s - ice and snow conductivity

T_f , T_a - freezing point and atmospheric temperature

Conductive heat flux

$$F_c = k_i \frac{T_f - T_a}{H} \Phi(\eta, \Sigma, \Lambda)$$

Φ - non-dimensional heat flux

Φ_v - purely vertical flux

Φ_h - purely horizontal flux

$c \approx 0.83$

$\tilde{f}_\Gamma$ - Gamma distribution

$$\Phi(\eta, \Sigma, \Lambda) \approx \Phi_v + \frac{\Phi_h - \Phi_v}{(1 + c\Lambda)^2}$$

$$\Phi_v \equiv \int_0^\infty \frac{\tilde{f}_\Gamma(z, \Sigma)}{1 + z\eta} \mathrm{d}z \qquad \Phi_h \equiv \frac{1}{1 + \eta(1 - \Sigma^2)}$$

If our model holds, the effect of snow on full 3d heat flux through undeformed ice is captured with only 3 non-dimensional parameters that correspond to mean, variance, and correlation length of snow depth. We derive analytic formulas to compute this heat flux.

Early melt pond evolution

Pond coverage evolution ODEs

$$\dot{w} \approx -\frac{r_i r_s (1-p)\dot{h}_s + Q}{1 - r_s(1-p)}$$

$$\dot{p} = f_\Gamma(w - \dot{h}_s t)\left(\dot{w} - \dot{h}_s\right)$$

$\dot{x} \equiv \mathrm{d}x/\mathrm{d}t$

w - water table

p - pond coverage fraction

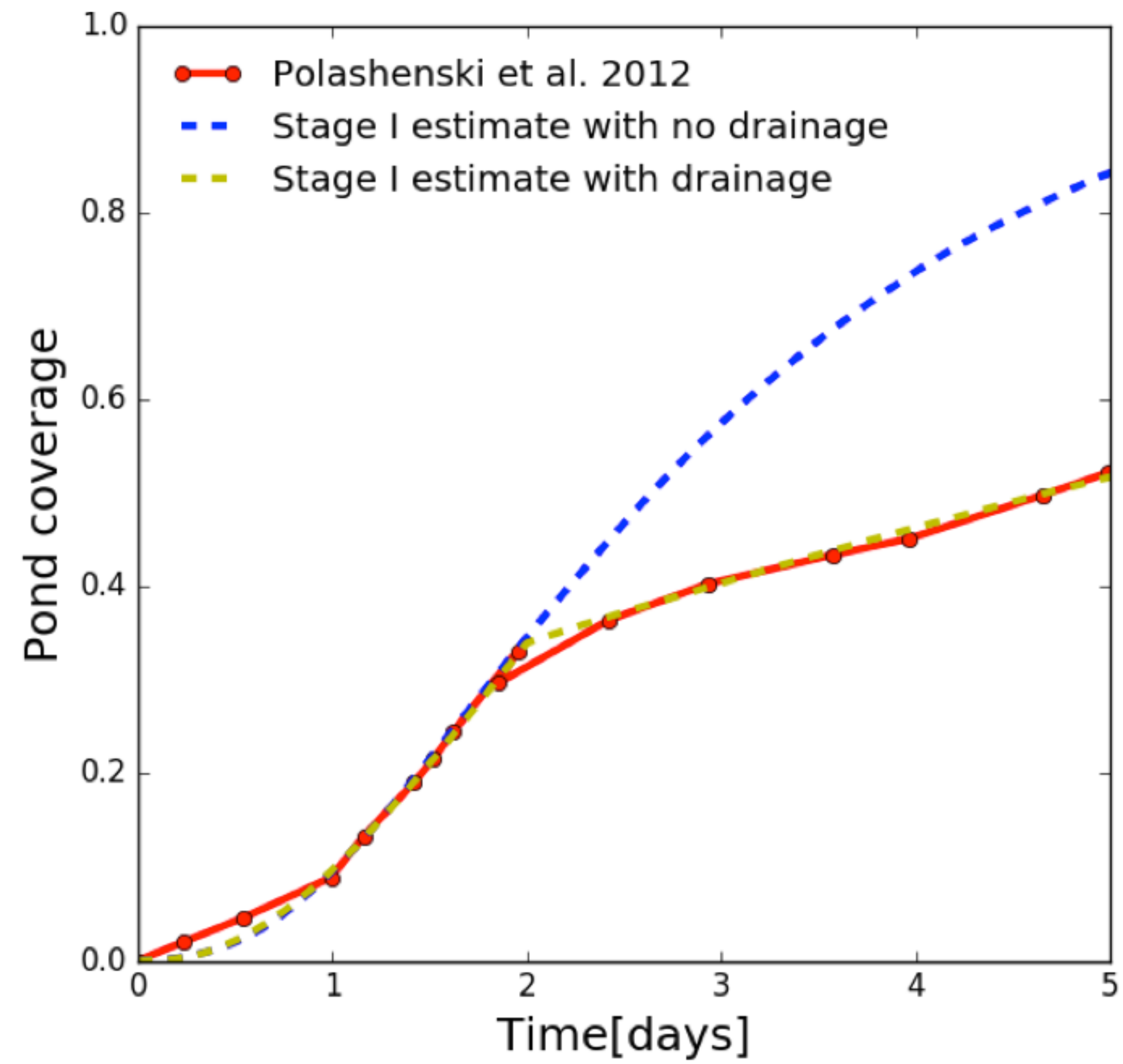
$\dot{h}_s$ - snow melt rate

Q - meltwater drainage rate

r_i - ratio of ice to water density

r_s - ratio of snow to water density

f_Γ - Gamma distribution



Using our “snow dune” model, we develop simple ordinary differential equations for evolution of melt pond coverage during the early summer during which ice is impermeable. We can match the measurements using this model with realistic parameters

Condition for pond development

Non-dimensional water level

$$\omega = -\frac{1 - r_s(1 - r_i)}{1 - r_s} \frac{\dot{h}_s T}{\langle h \rangle}$$

Non-dimensional snow roughness

$$\Sigma = \sigma(h)/\langle h \rangle$$

Condition for pond development

$$\omega > F_{\Gamma}^{-1}(\Sigma)$$

$\langle h \rangle$, $\sigma(h)^2$ - snow depth mean and variance

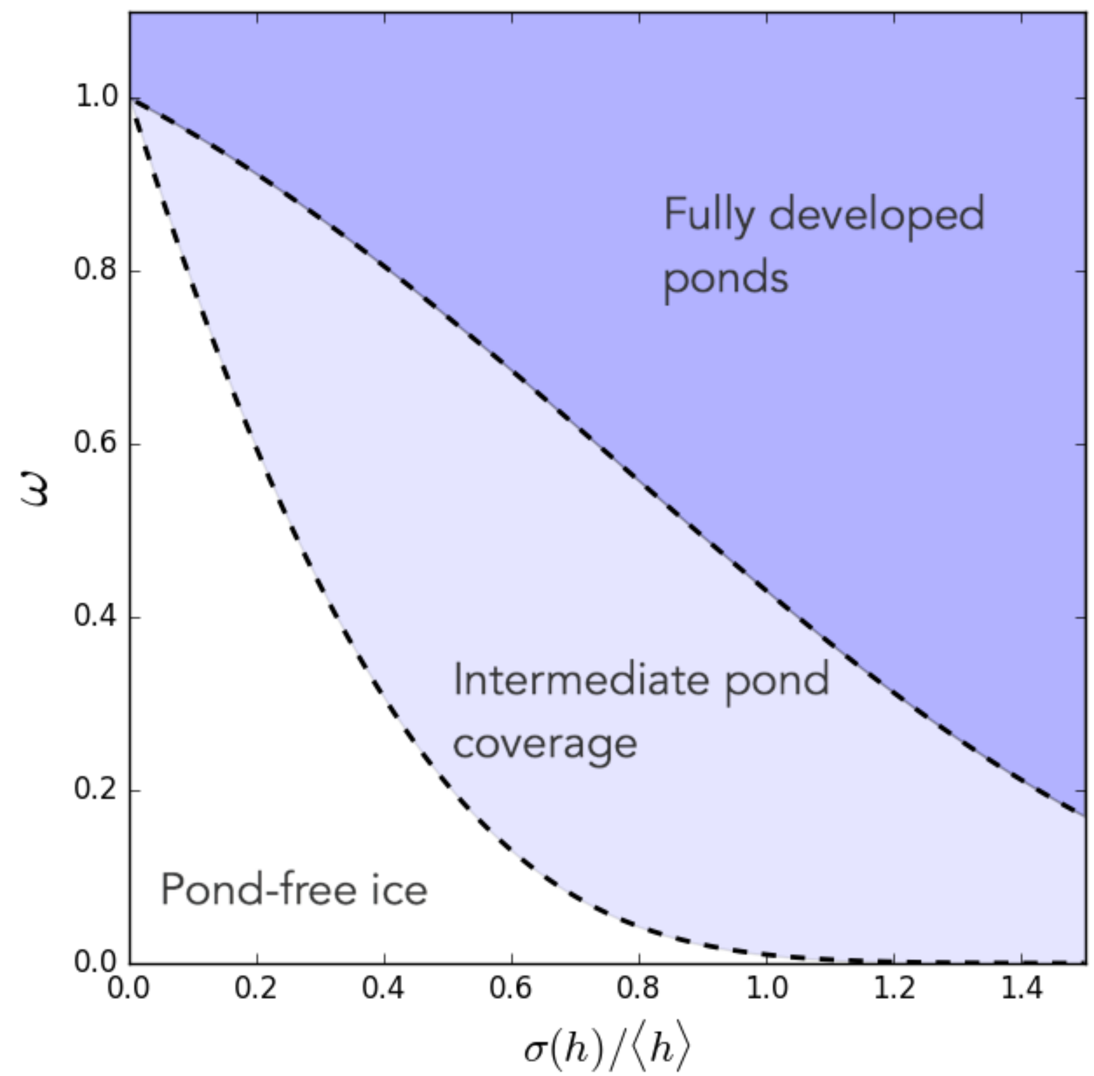
$\dot{h}_s$ - snow melt rate

T - duration of impermeable ice stage

r_i - ratio of ice to water density

r_s - ratio of snow to water density

F_{Γ}^{-1} - Gamma percentile function



We also develop an analytic condition for ponds to develop during summer. Ponds develop if the non-dimensional water level by the end of the impermeable stage exceeds a threshold that is a function of the non-dimensional snow depth roughness.

Conclusions

- The pre-melt surface on undeformed ice is fully characterized by 3 parameters - mean snow depth, roughness, and the correlation length.

- This allows us to understand the heat transport and early melt pond evolution under arbitrary snow configuration on undeformed ice.