Technique to inverse the planetary dipolar magnetic field based on the current loop model:

Sounding the interior via magnetic feld

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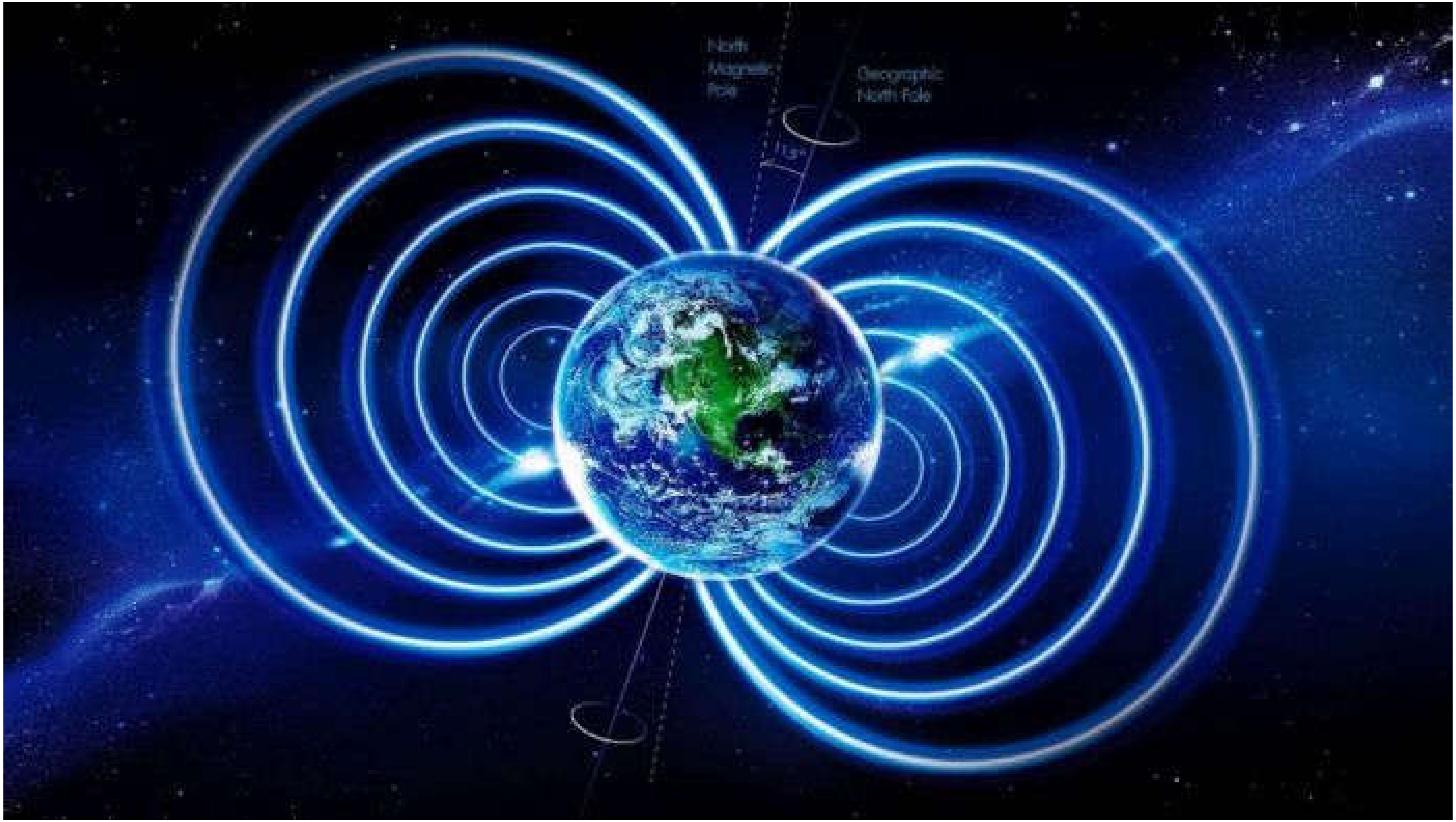
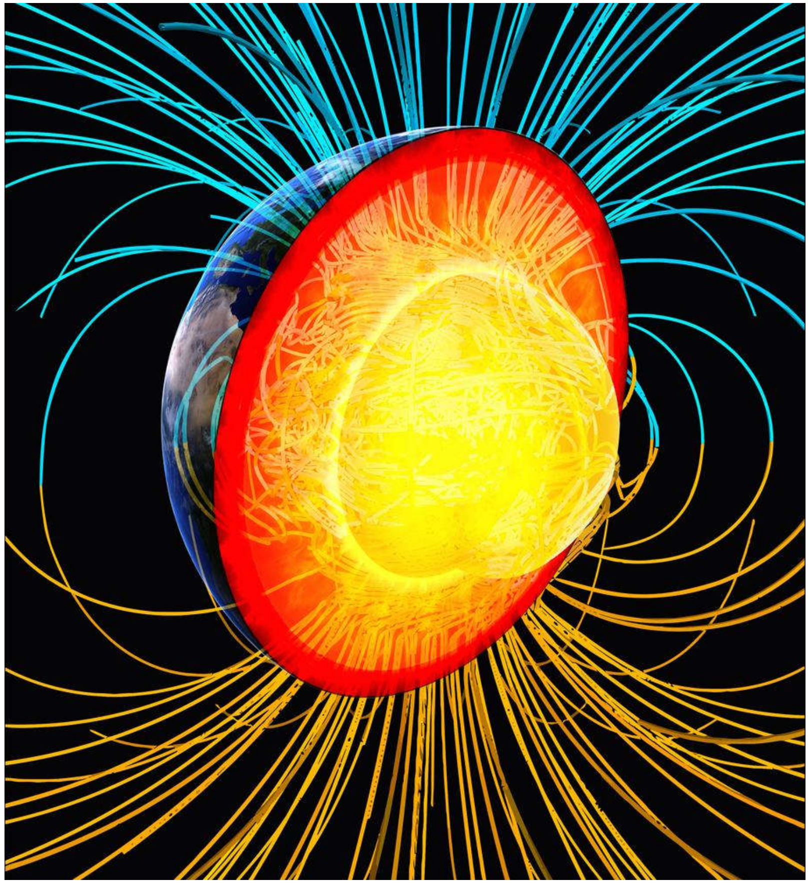
Earth has a dipolar magnetic field



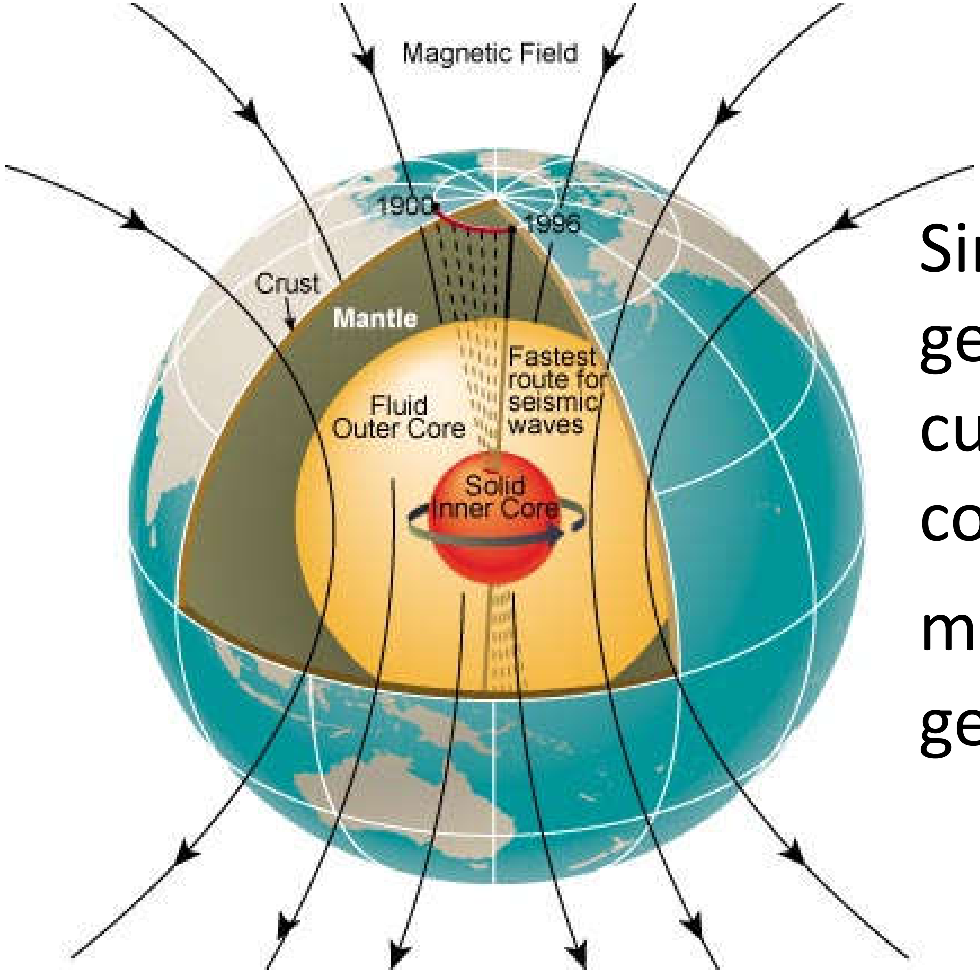
Geomagnetic field is generated by Geodynamo currents



- Where is the dynamo zone?
- How is the scale?
- Current orientation?
- Current strength?

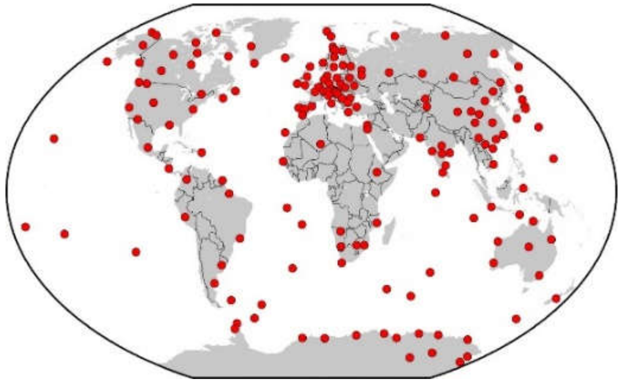
It's hard to probe the real dynamo currents since nobody can cut open Earth

Inversion of Geomagnetic field



Since geomagnetic field is generated by the dynamo current, geomagnetic field could be the most direct means to diagnose the real geodynamo current

Geomagnetic field model



Geomagnetic observatories

Spherical Harmonic Analysis

$$\nabla \times \mathbf{B} = 0$$

$$\mathbf{B} = -\nabla V$$

$$V = a \sum_{n=1}^{\infty} \sum_{m=0}^{n} P_n^m(\cos\theta)$$

= 0 for m > n

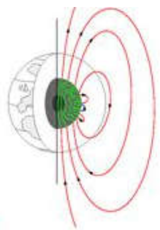
n = 1 --> dipole
n = 2 --> quadrapole

$$\times \left[\left(\frac{a}{r}\right)^{n+1} \left(g_n^m \cos m\lambda + h_n^m \sin m\lambda\right) \right.$$

internal sources

$$\left. + \left(\frac{a}{r}\right)^{-n} \left(A_n^m \cos m\lambda + B_n^m \sin m\lambda\right) \right]$$

external sources



$$X(r,\theta,\lambda,t) = \sum_{n=1}^{\infty} \sum_{m=0}^{m} [g_n^m(t) \cos m\lambda + h_n^m(t) \sin m\lambda] \frac{d P_n^m(\theta)}{d\theta} \left(\frac{a}{r}\right)^{n+2} \qquad (1)$$

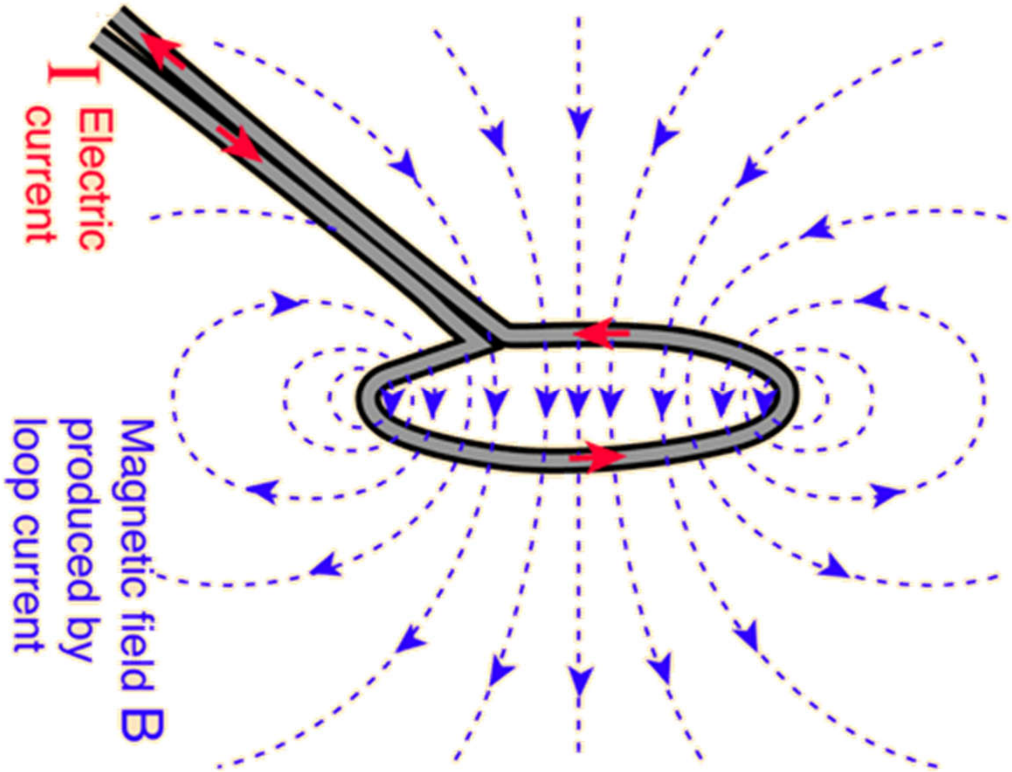
$$Y(r,\theta,\lambda,t) = \frac{1}{\sin\theta} \sum_{n=1}^{\infty} \sum_{m=0}^{m} [m g_n^m(t) \sin m\lambda + m h_n^m(t) \cos m\lambda] \, P_n^m(\theta) \left(\frac{a}{r}\right)^{n+2} \qquad (2)$$

$$Z(r,\theta,\lambda,t) = \sum_{n=1}^{\infty} \sum_{m=0}^{m} \{g_n^m(t) \cos m\lambda + h_n^m(t) \sin m\lambda\} \, P_n^m(\theta) \left(\frac{a}{r}\right)^{n+2} \qquad (3)$$

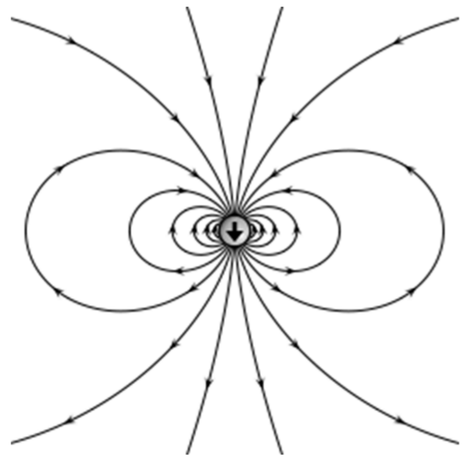
SHC tells few about the source of dynamo

- The potential field is required for the solution by SHC
- The dominant term of dipole component implies infinite value at Earth center
- An offset of a dipole center is mathematically indistinguishable from a dipole plus a series of higher orders multipole components [Merrill, McElhinny, & McFadden, 1996]

Models with "more physics" are required



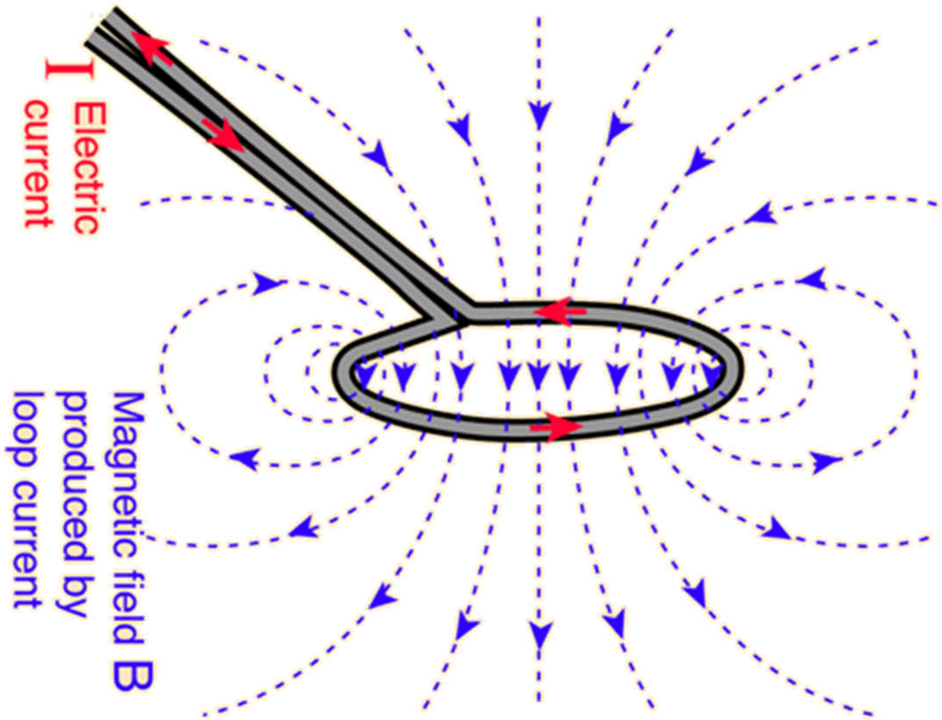
Current loop



Dipole field is the case with radius =0

Advantages of loop model

It can roughly indicate the geometric information of dynamo



- Loop center(x0,y0,z0):
  dynamo location
- Axis (ϑ_0, φ_0)
  orientation of dynamo current
- Radius (*a*)
  dynamo scale
- Carried current *(I)*
  dynamo strength

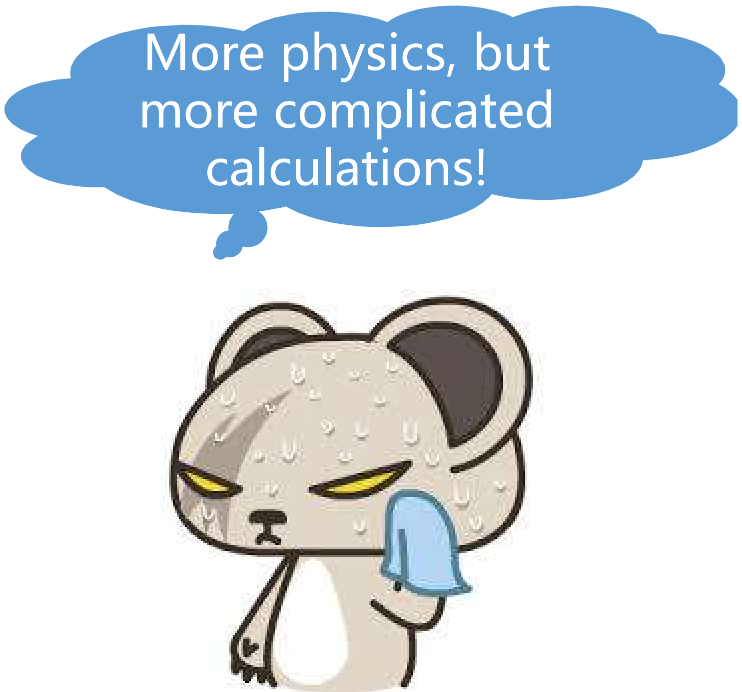
Loop models are tried in many studies [e.g. Zidarov and Petrova, 1974; Peddie, 1979; Zidarov, 1985; Demina and Farafonova, 2016]

Difficulties of loop model

Difficulties: the simultaneous fitting of these loop parameters would result in multiple error minima in the space of parameters. The fitting results are strongly dependent on the initial input values.

The sum-of-squares function for eccentric current loops may have multiple minima in the parameter space. That is, in general, least squares solutions for current loops are not unique. Unfortunately, no general technique exists for determining whether a particular minimum is the desired universal minimum, that is, the best possible solution. To increase the likelihood of finding the universal minimum, a random search technique was used. For each model, 20 preliminary computer

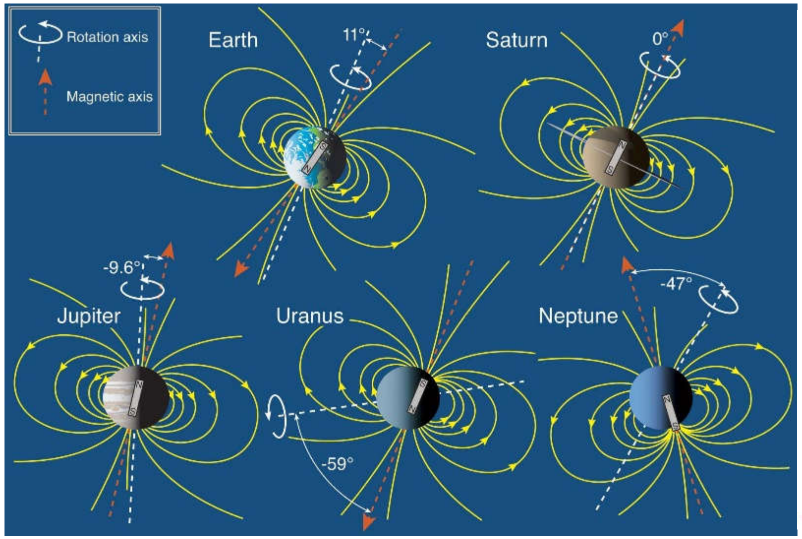
by N.W.Peddie [JGR,1979],
the generator of 3rd IGRF model

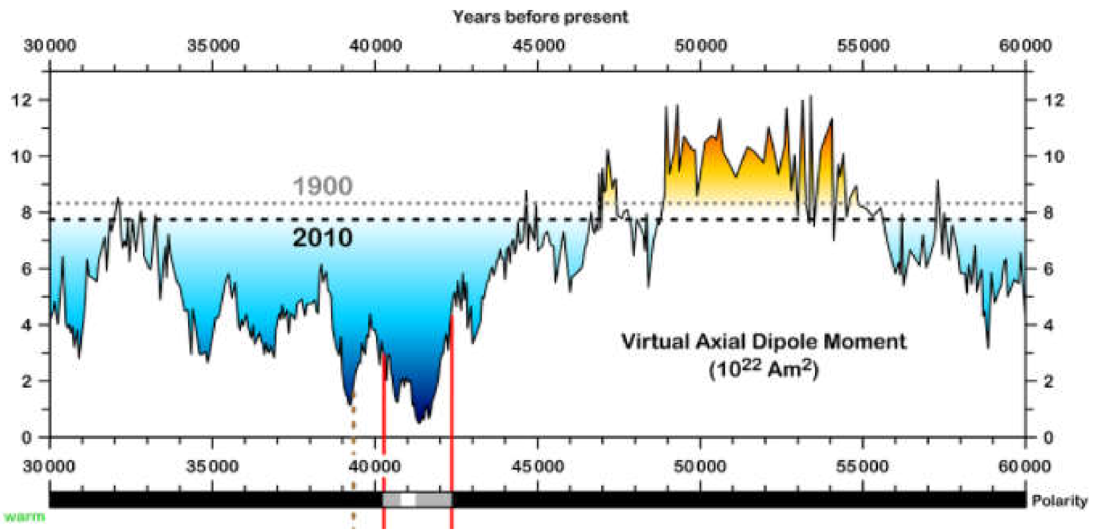


Motivation and Target

Motivation: Trying to develop a new technique to separate and solve the loop parameters successively based on the sampled magnetic field data.

Target: quick, effective, and applicable. It could be applied generally to planetary dipole field and paleomagnetic field



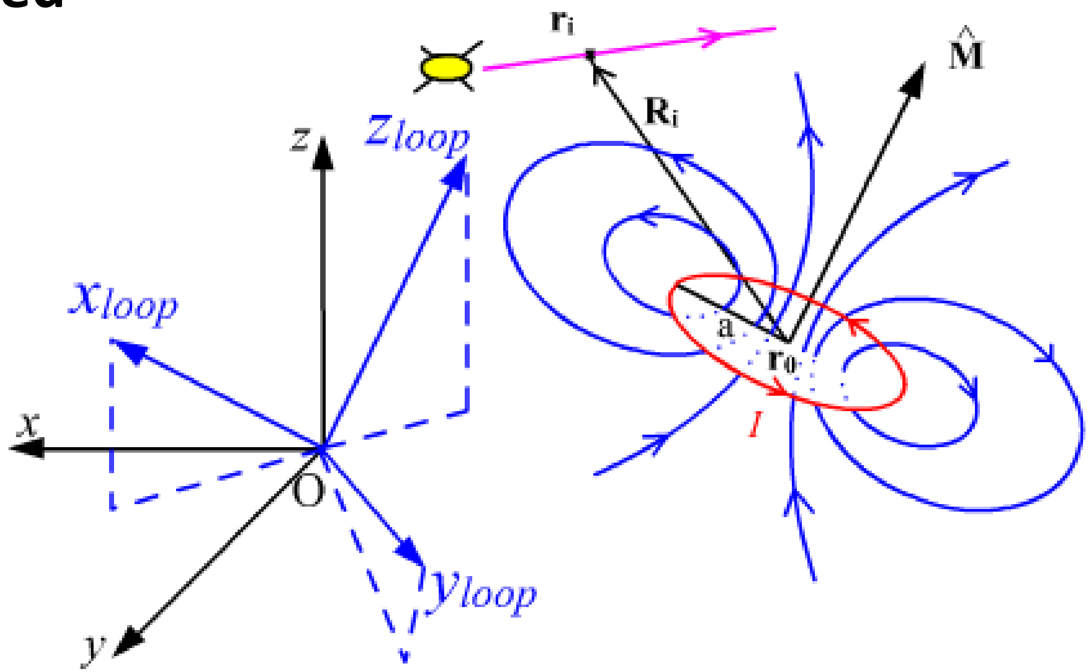


Method: Setup of loop coordinates

Loop coordinates expressed in GEO coordinates

$$\begin{cases} \hat{\mathbf{z}}_{loop} = \hat{\mathbf{M}} \\ \hat{\mathbf{y}}_{loop} = \dfrac{\hat{\mathbf{z}}_{loop} \times \hat{\mathbf{x}}}{\left|\hat{\mathbf{z}}_{loop} \times \hat{\mathbf{x}}\right|} \\ \hat{\mathbf{x}}_{loop} = \hat{\mathbf{y}}_{loop} \times \hat{\mathbf{z}}_{loop} \end{cases}$$

Origin is at planet center O

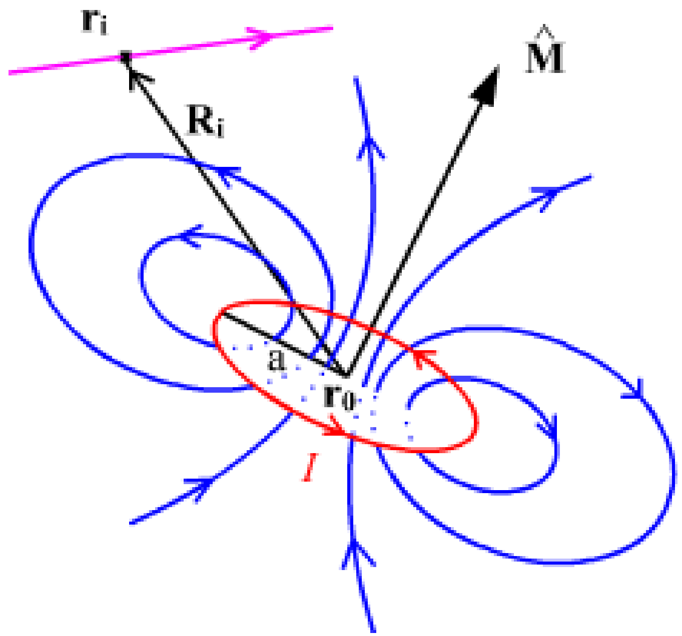


Current loop is in the Xloop-Yloop plane

Method: Parameter separation

The loop parameters can be separated according to the field geometry of loop current

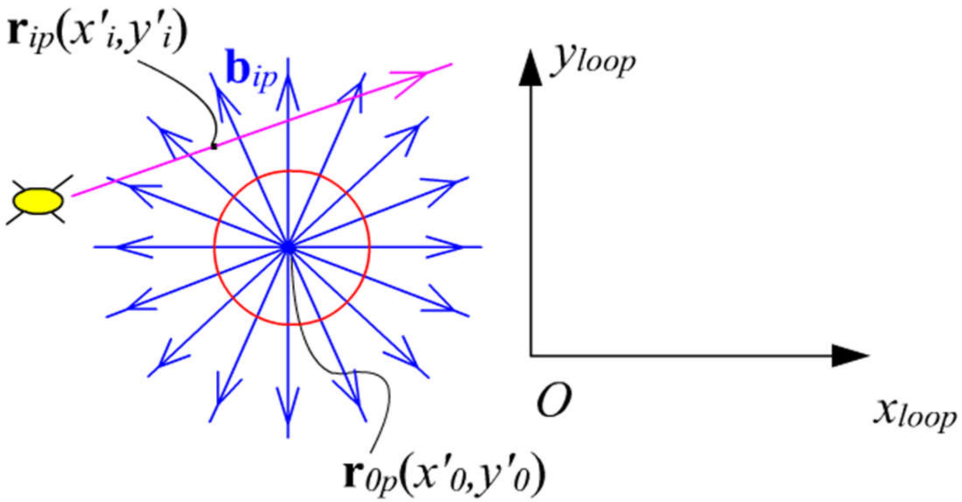
- Dipole center, r0 (x0,y0,z0), related with displacement of field structure.
- Axis orientation, M(ϑ_0, φ_0), related with the orientation of field structure;
- Loop radius, *a*, related with the geometric configuration of magnetic field;
- Current, *I*, only related with the field strength



Assumption: Sampled field should be purely internal field, or the external field can be evaluated and subtracted.

Method: 1. axis and loop center

The best axis orientation should make the projected field lines radially orientated for best



The projected position vector of S/C is

$$\mathbf{r}_{ip}\left(x_i^{'}\hat{\mathbf{x}}_{loop}, y_i^{'}\hat{\mathbf{y}}_{loop}\right)$$

The projected unit field vector is

$$\mathbf{b}_{ip}\left(b_{xi}^{'}\hat{\mathbf{x}}_{loop}, b_{yi}^{'}\hat{\mathbf{y}}_{loop}\right)$$

The projected loop center is

$$\mathbf{r}_{0p}\left(x_0^{'}\hat{\mathbf{x}}_{loop}, y_0^{'}\hat{\mathbf{y}}_{loop}\right)$$

$$\sin\alpha_i = \frac{\mathbf{r}_{ip}\times\mathbf{b}_{ip}}{|\mathbf{r}_{ip}||\mathbf{b}_{ip}|} = \frac{\left(x_i^{'}-x_0^{'}\right)b_{yi}^{'} - \left(y_i^{'}-y_0^{'}\right)b_{xi}^{'}}{\sqrt{b_{xi}^{'\,2}+b_{yi}^{'\,2}}\sqrt{\left(x_i^{'}-x_0^{'}\right)^2+\left(y_i^{'}-y_0^{'}\right)^2}}$$

$$\sigma\left(x_0^{'}, y_0^{'}, \theta_0, \varphi_0\right) = \frac{1}{N}\sum_i a\sin\left(|\sin\alpha_i|\right)$$

the optimum of $\mathbf{M}$ and $\mathbf{r}_{0p}$ should make σ reaches the global minimum

Method: 1. axis and loop center

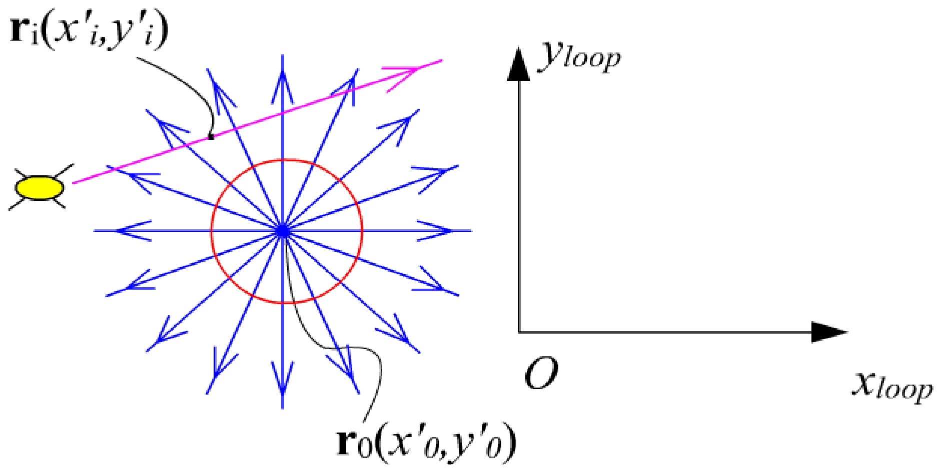
Each projected field line should cross the loop center

$$b'_{yi}\left(x'_0 - x'_i\right) - b'_{xi}\left(y'_0 - y'_i\right) = 0$$

$$A = \begin{pmatrix} b'_{y1} & -b'_{x1} \\ b'_{y2} & -b'_{x2} \\ \cdots & \cdots \\ b'_{ym} & -b'_{xm} \end{pmatrix}, X = \begin{pmatrix} x'_0 \\ y'_0 \end{pmatrix}, Y = \begin{pmatrix} x'_1 b'_{y1} - y'_1 b'_{x1} \\ x'_2 b'_{y2} - y'_2 b'_{x2} \\ \cdots \\ x'_m b'_{ym} - y'_m b'_{xm} \end{pmatrix}$$

The optimum solution of X is

$$X = \left(A^T A\right)^{-1} A^T Y$$



The optimum loop center is function of axis orientation (ϑ_0, φ_0).

$$\sigma\left(\theta_0, \varphi_0\right) = \frac{1}{N}\sum_i a\sin\left(\left|\sin\alpha_i\right|\right)$$

σ is only the function of axis orientation, the global minimum of σ can be quickly searched in the 2-D map constituted by ϑ_0 and φ_0 . Correspondingly, the optimum axis and loop center can be found.

Method: 2. loop radius

The ideal loop field component

$$\tilde{B}_{ir} = \frac{\mu_0 I}{4\pi} \frac{2\cos\theta_i}{\sin\theta_i \sqrt{a^2 + R_i^2 + 2aR_i \sin\theta_i}} \left[\frac{a^2 + R_i^2}{a^2 + R_i^2 - 2aR_i \sin\theta_i} E - K \right]$$

$$\tilde{B}_{iz} = \frac{\mu_0 I}{4\pi} \frac{2}{\sqrt{a^2 + R_i^2 + 2aR_i \sin\theta_i}} \left[\frac{a^2 - R_i^2}{a^2 + R_i^2 - 2aR_i \sin\theta_i} E + K \right]$$

$$\tilde{\mathbf{b}}_i = \frac{\tilde{B}_{ir}}{B} \hat{\mathbf{r}} + \frac{\tilde{B}_{ir}}{B} \hat{\mathbf{M}}$$

The ideal unit field vector is function of radius *a* and the shift component of loop center *z0'*.

$$\mathbf{b}_i = \frac{\mathbf{B}_i}{\left|\mathbf{B}_i\right|}$$

The sampled unit field vector

$$\gamma_i = a\cos\left(\mathbf{b}_i \cdot \tilde{\mathbf{b}}_i\right)$$

$$\varepsilon\left(z_0^{'}, a\right) = \frac{1}{N} \sum_i \gamma_i$$

The global minimum of ε can be quickly searched in the plane of (z_0', a). Correspondingly， the optimum radius a and the axis-shift component z_0' can be solved

Method: current and error

current

$$\tilde{B}_{ir} = IF_{ir} \qquad F_{ir} = \frac{\mu_0}{4\pi}\frac{2\cos\theta_i}{\sin\theta_i\sqrt{a^2+R_i^2+2aR_i\sin\theta_i}}\left[\frac{a^2+R_i^2}{a^2+R_i^2-2aR_i\sin\theta_i}E-K\right]$$

$$\tilde{B}_{iz}=IF_{iz} \qquad F_{iz}=\frac{\mu_0}{4\pi}\frac{2}{\sqrt{a^2+R_i^2+2aR_i\sin\theta_i}}\left[\frac{a^2-R_i^2}{a^2+R_i^2-2aR_i\sin\theta_i}E+K\right]$$

Since magnetic axis, loop radius and loop center have been solved, current can be derived as

$$I=\frac{1}{N}\sum_i\frac{B_i}{\sqrt{F_{ir}^2+F_{iz}^2}}$$

Method error

$$\delta=\frac{1}{N}\sum_i\frac{\left|\mathbf{B}_{it}\text{-}\mathbf{B}_{it0}\right|}{\left|\mathbf{B}_{it}\right|}$$

$\mathbf{B}_{it}$: sampled field

$\mathbf{B}_{it0}$: field of loop

Test

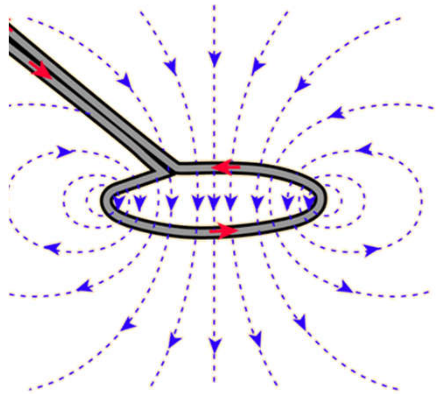
Input loop parameters

Loop center: （x0=0.1;y0=0.2;z0=0.5）m

Axis orientation: （θ_0=60° ,ϕ_0=40° ）

radius: a=0.5m

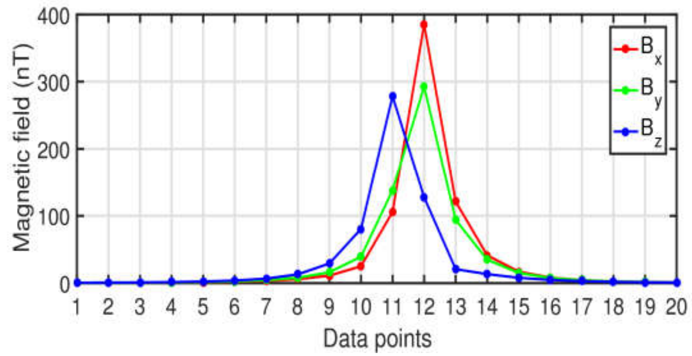
current: I=0.35A



along arbitrary trajectory

(x= −2, y= −2, z= −2) m to (x=2, y=2, z=2) m

Sampled 20 data points evenly.



Sampled field vector

Target: Applying our technique to the sampled data to inverse the input parameters

Test result

Test results：

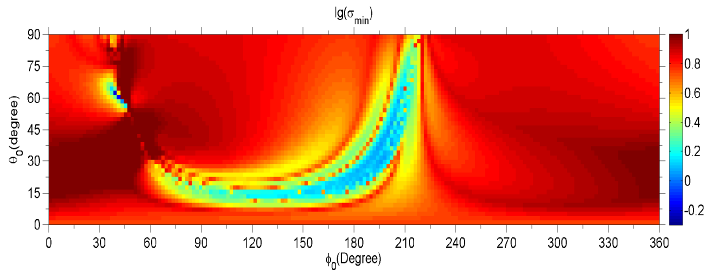
Loop center： （x0=0.1;y0=0.2;z0=0.5） m

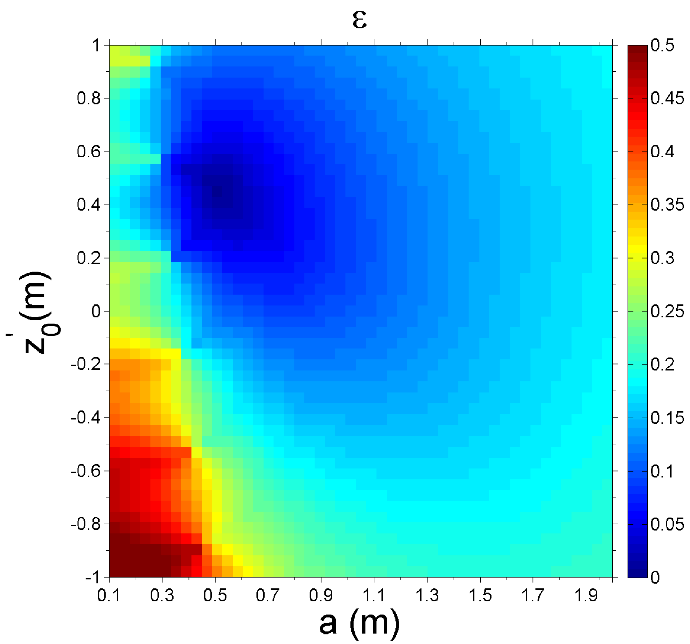
Axis orientation： （θ_0=60° ,ϕ_0=40° ）

radius： a=0.5m

current： I=0.35A

It is able to inverse parameters exactly

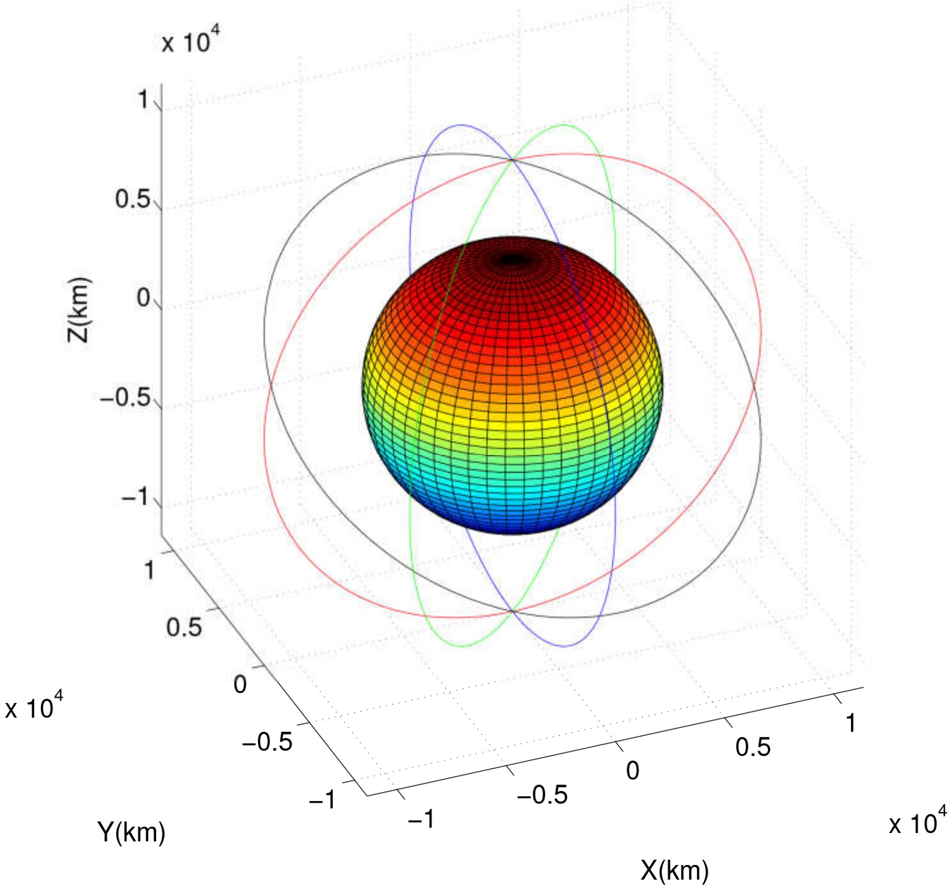




Result：

Yes! Quick, Stable, and Effective!

Application-IGRF



IGRF： 2015-01-01 00:00:00

4 orbits， variable altitude

Each orbit， 20 data points

In total， 80 data points

Application-IGRF

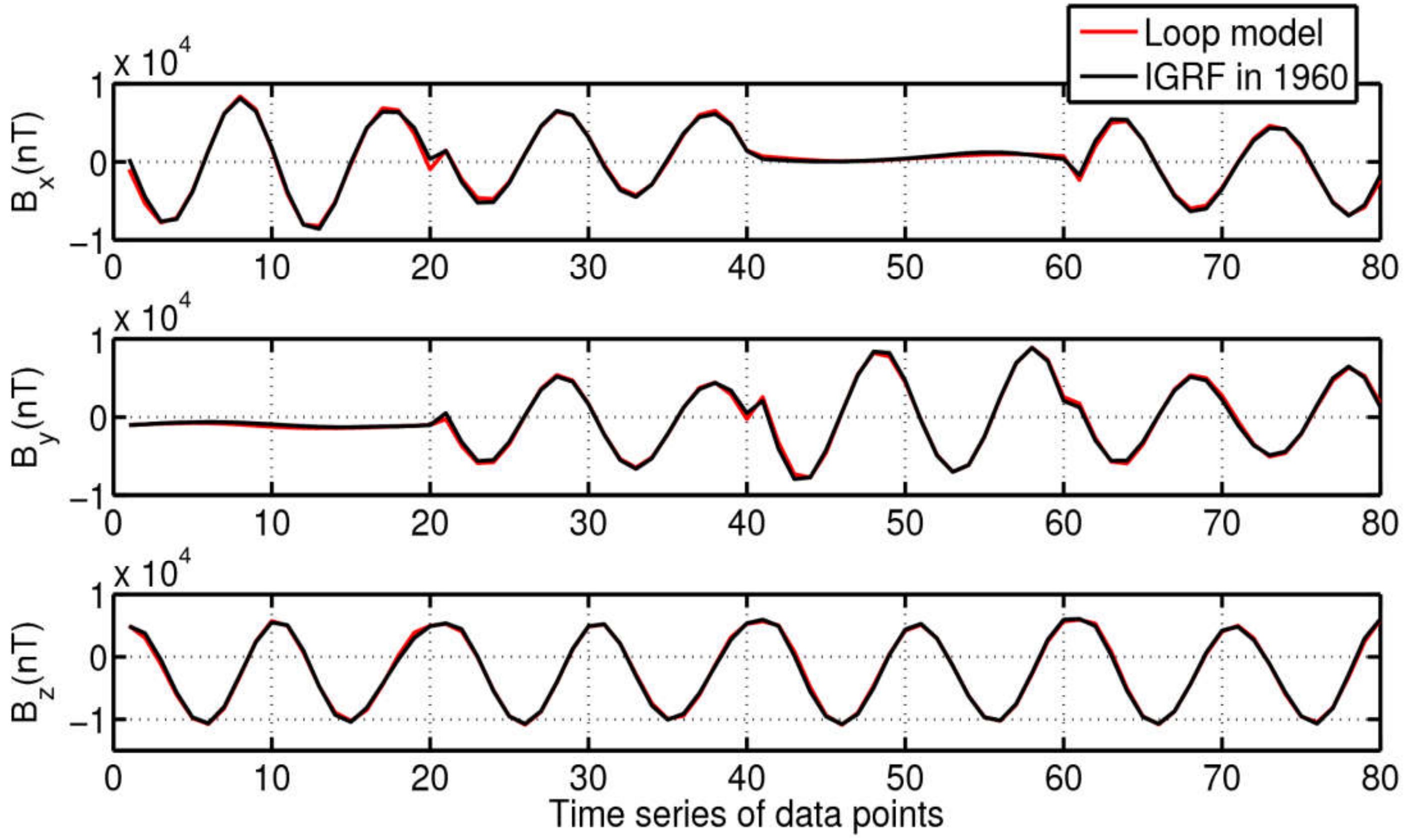
Table1. The inversed loop parameters for the IGRF model of the year 2015

Altitude (km)	x_0 (km)	y_0 (km)	z_0 (km)	a (km)	I ($*10^{10}$A)	M [a] ($*10^{22}$Am^2)	θ_0 (°)	φ_0 (°)	σ_{min} (deg)	ε (deg)	δ
0	-286	309	111	856	3.4	7.82	167.7	113.1	7.465	8.079	0.19
100	-274	320	96	817	3.7	7.81	167.8	118.0	7.261	7.907	0.18
500	-293	334	71	745	4.5	7.79	168.6	117.4	6.558	6.876	0.16
1000	-311	339	83	701	5.0	7.77	169.1	115.6	5.856	6.016	0.14
2000	-334	348	105	754	4.3	7.75	169.7	113.3	4.843	4.756	0.11
5000	-364	356	170	720	4.7	7.72	171.1	111.8	3.333	2.979	0.07
10000	-381	356	206	631	6.2	7.72	170.9	109.4	2.145	1.833	0.05
20000	-390	357	212	353	19.7	7.72	170.5	108.0	1.276	1.031	0.05
50000 [b]	~	~	~	~	~	~	170.4	107.5	0.581	~	~
IGRF [c]	-400	352	221	~	~	7.72	170.4	107.4	~	~	~

Inversed results quickly converged to the eccentric-dipole of IGRF as the increase of altitude.

Comparison with IGRF

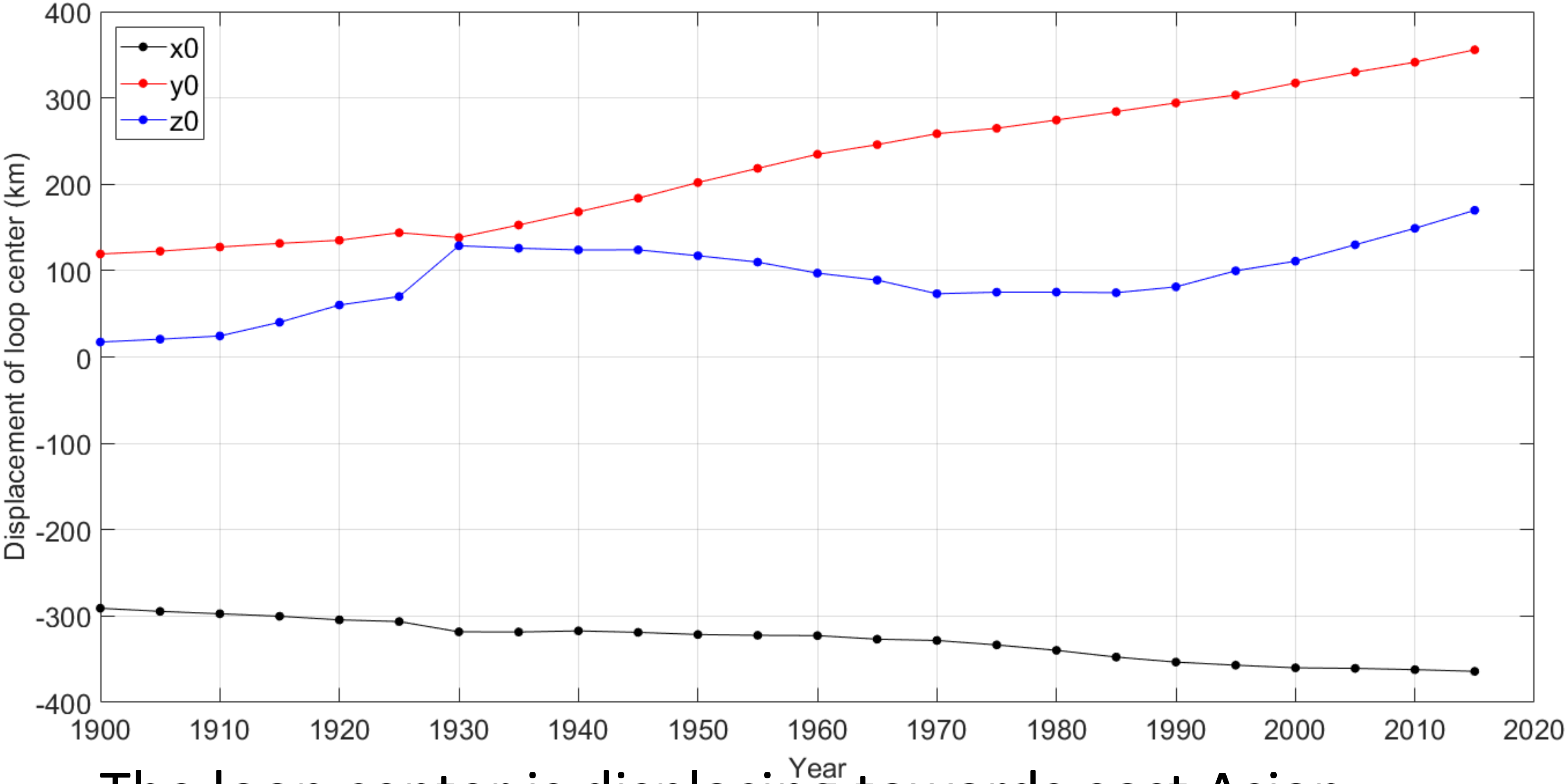
Four orbits with altitude 5000 km sample 80 data point at 1960:00:00:00



New technique is applicable and reasonable

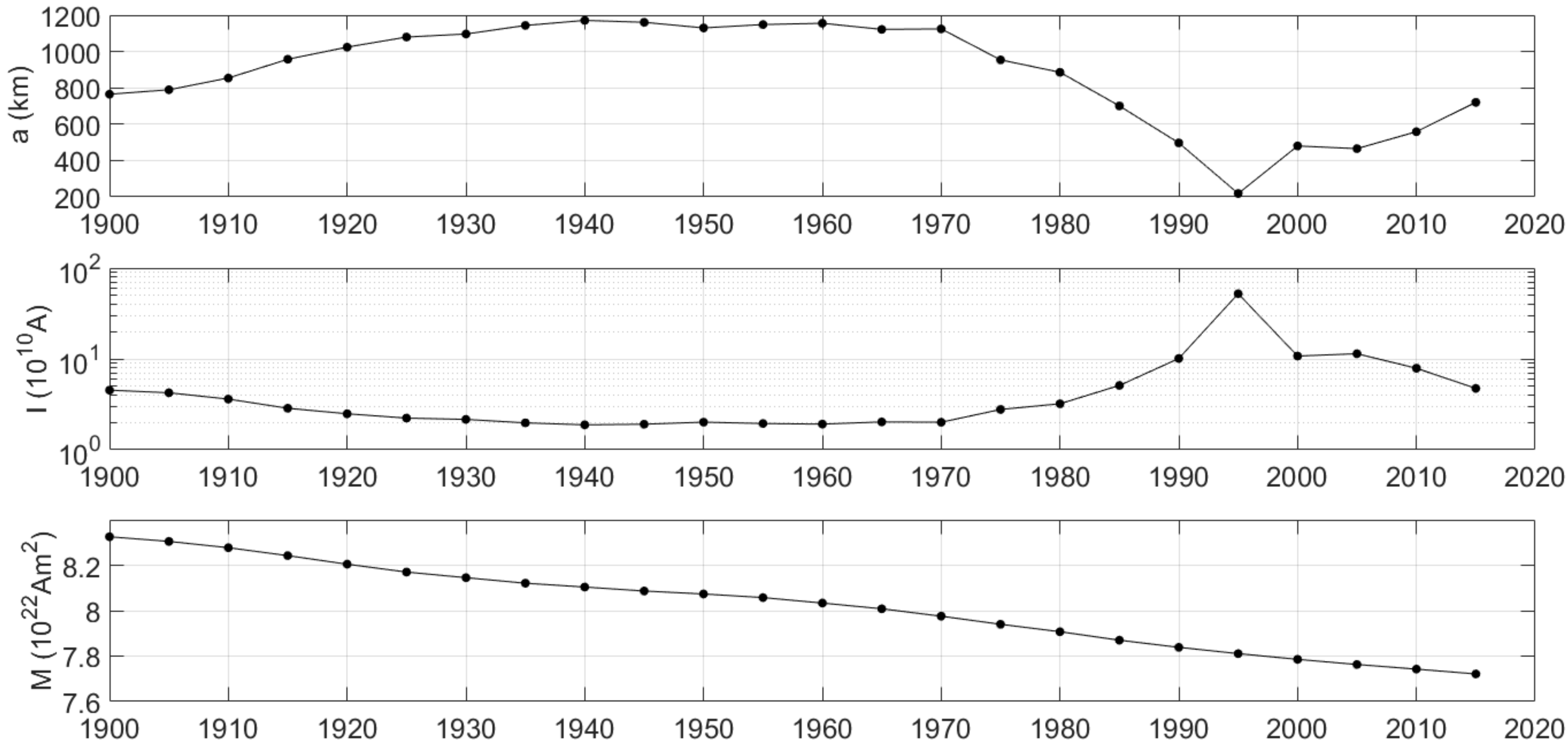
100 years variation of IGRF-loop model

Four orbits with altitude 5000 km sample 80 data point



The loop center is displacing towards east Asian

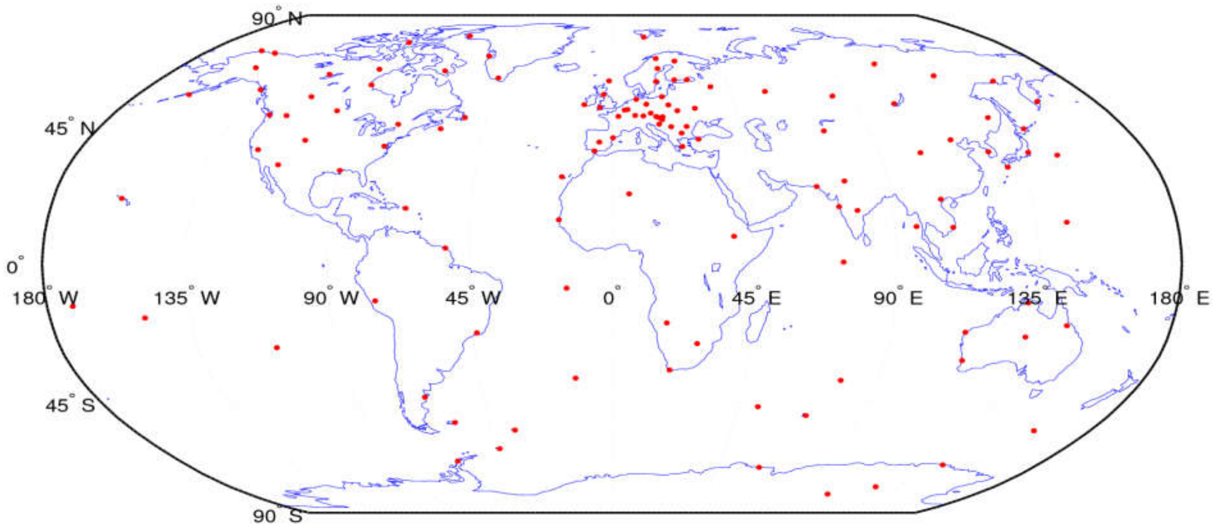
100 years variation of IGRF-loop model



Application-
Geomagnetic observatories

The moment of the sampled data is arbitrarily set at 2015-01-01 00:00:00, and the originally sampled data of 123 observatories are used.

The original recorded data set subjected to the disturbance of external space currents and local field anomaly



Model	x_0 (km)	y_0 (km)	z_0 (km)	a (km)	I (*10^{10}A)	M (*10^{22}Am^2)	θ_0 (°)	φ_0 (°)	σ_{min} (deg)	ε (deg)	δ
Loop model	-213	403	128	892	3.1	7.88	172.0	114.6	5.313	8.423	0.18
IGRF	-400	352	221	~	~	7.72	170.4	107.4	~	~	~

Further modifications

✓1. It can be reduced to a unconstrained dipole model

6 parameters are involved.

Loop center(3 parameters), axis orientaion (2 parameters),

and dipole moment(1 parameters)

Axis orientation and dipole center

Same with loop model

Loop radius=0;
The shift component of dipole center is

$$\gamma_i = a\cos\left(\mathbf{b}_i \cdot \tilde{\mathbf{b}}_i\right)$$

$$\varepsilon\left(z_0^{'}\right) = \frac{1}{N}\sum_i \gamma_i$$

The dipole moment is

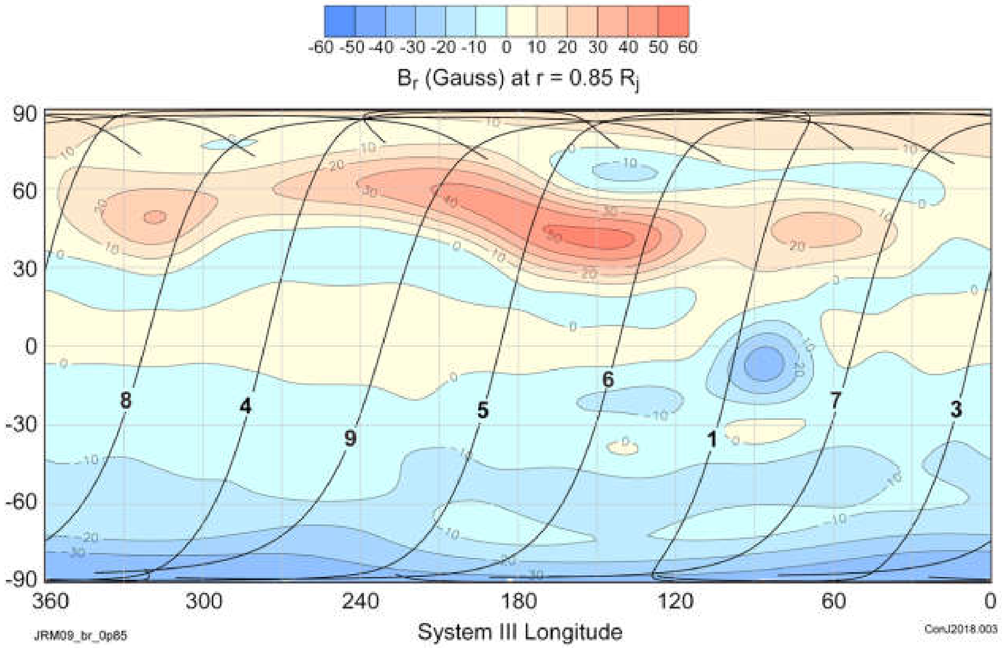
$$B_i = MF_i$$

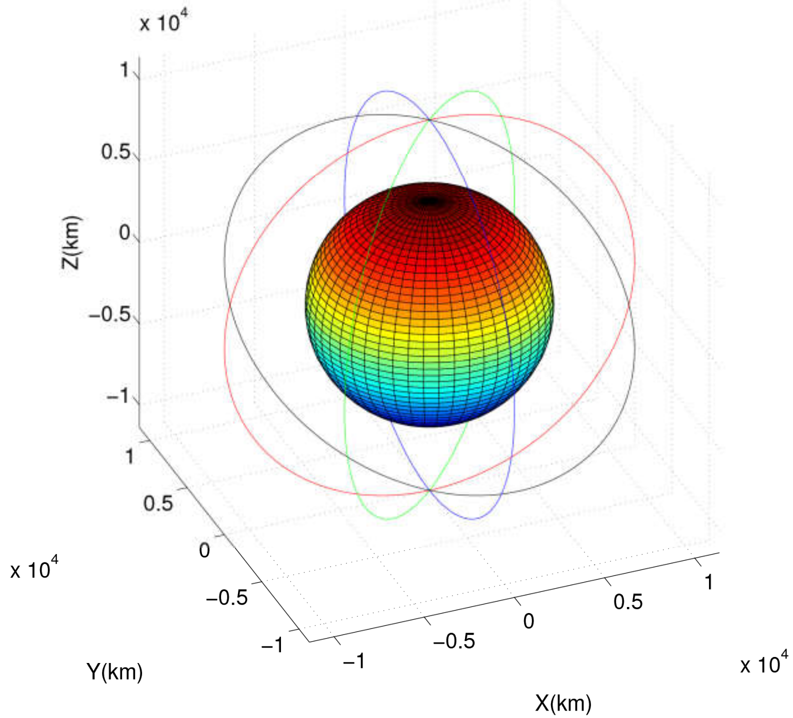
$$F_i = \frac{\mu_0}{4\pi R_i^5}\sqrt{1+3\left(\frac{\tilde{z}_i}{R_i}\right)^2}$$

$$M = \frac{1}{N}\sum_i B_i F_i^{-1}$$

Application to Jupiter

Jupiter magnetic field model





Connerney et al.,2018,GRL

4 circular orbits with variable altitude
Each orbit, sampled 20 data points
In total, 80 data points obtained

- Results of loop model

for the sampled data when r=3Rj,4j,and 5j,
code terminated, and returned a=0.

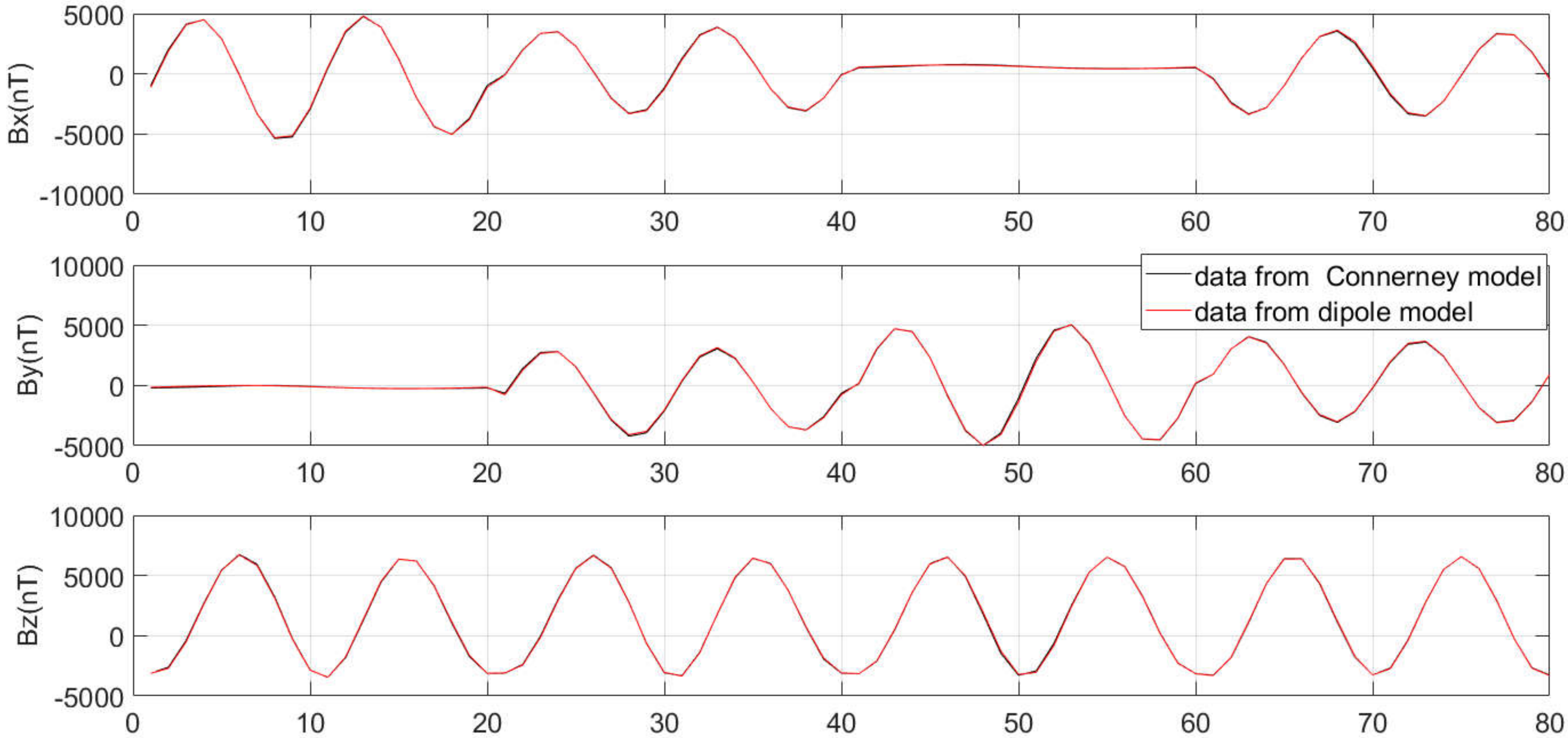
- Results of reduced dipole model

Jupiter dipole center is eccentric !

Orbit distance (RJ)	x_0 km	y_0 km	z_0 km	M (*10^{27}Am^2)	θ_0 (°)	ϕ_0 (°)	σ_{min} (deg)	ε (deg)	δ
3RJ	-5594	-4903	263	1.52	8.9	168.0	3.2	2.3	0.05
4RJ	-5629	-4809	560	1.52	9.9	167.9	2.6	1.5	0.03
5RJ	-5709	-4734	920	1.52	10.3	166.9	2.3	1.2	0.02
wiki	~	~	~	1.52	10	159			

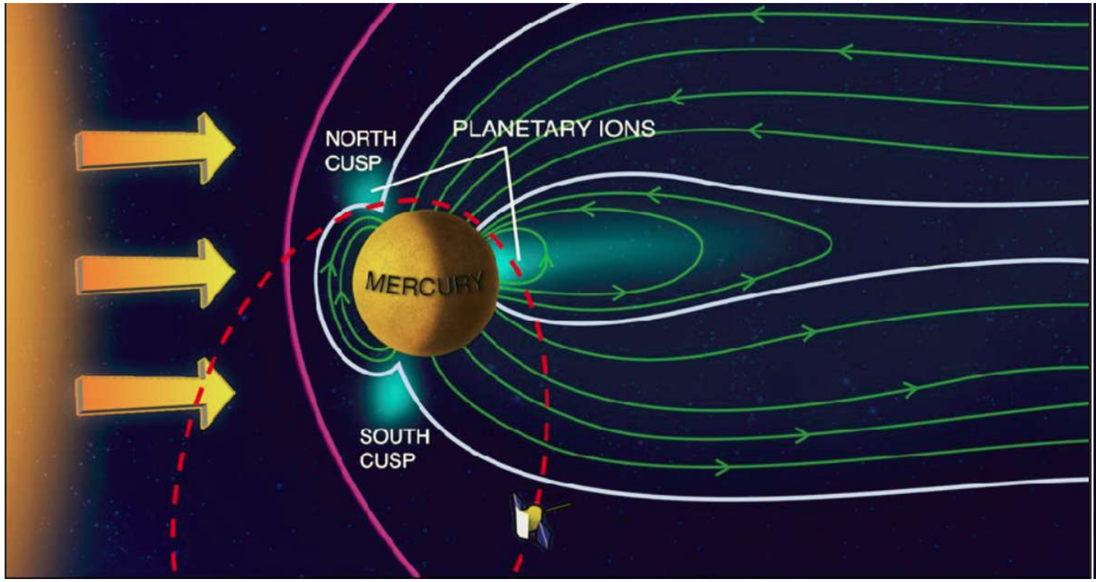
Comparison with sampled data

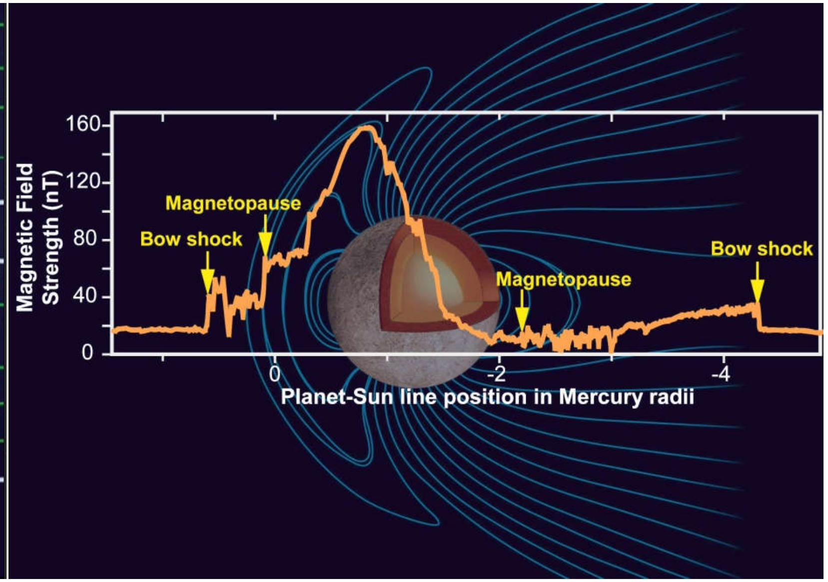
In this example, the orbit radius is 5 RJ



Time sequence of sampled data

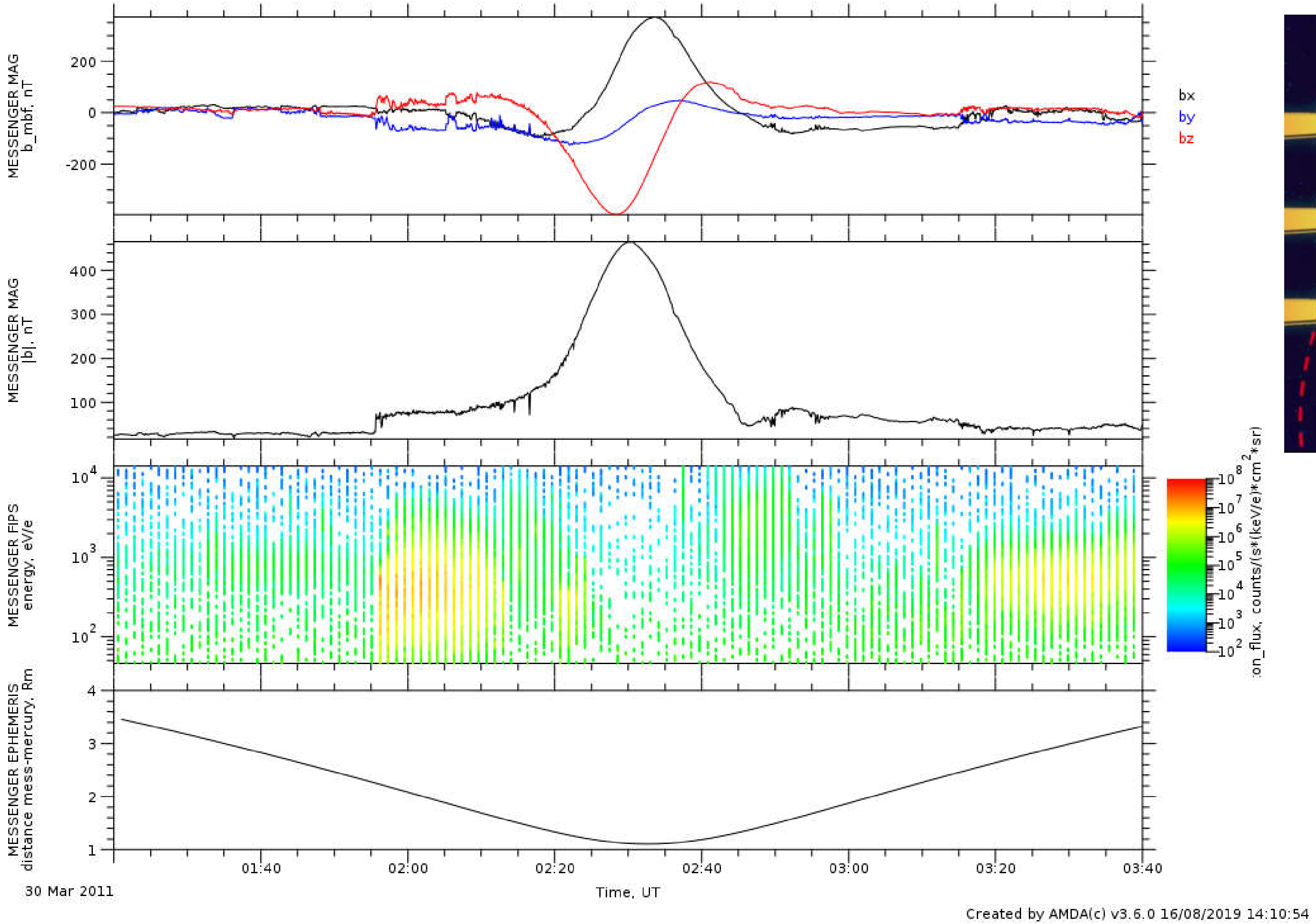
Application of the reduced dipole model -Mercury

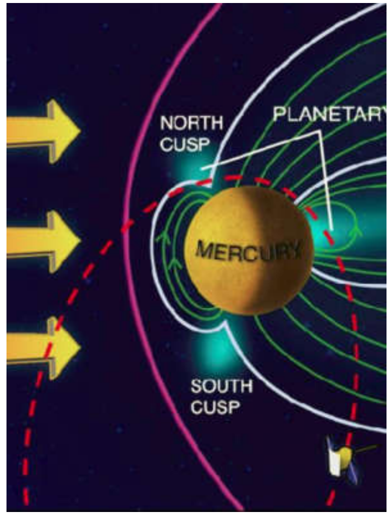




Paper	M (Gaussian unit)	North Shift of dipole center	Dipole tilt angle
Anderson, et al.2011,science	195±10nT*RM^3	484 km	<3 degree
Anderson, et al.2012,JGR	190±10nT*RM^3	479 km	<0.8 degree
Johnson et al.,2012,JGR	190 nT*RM^3	478 km	0

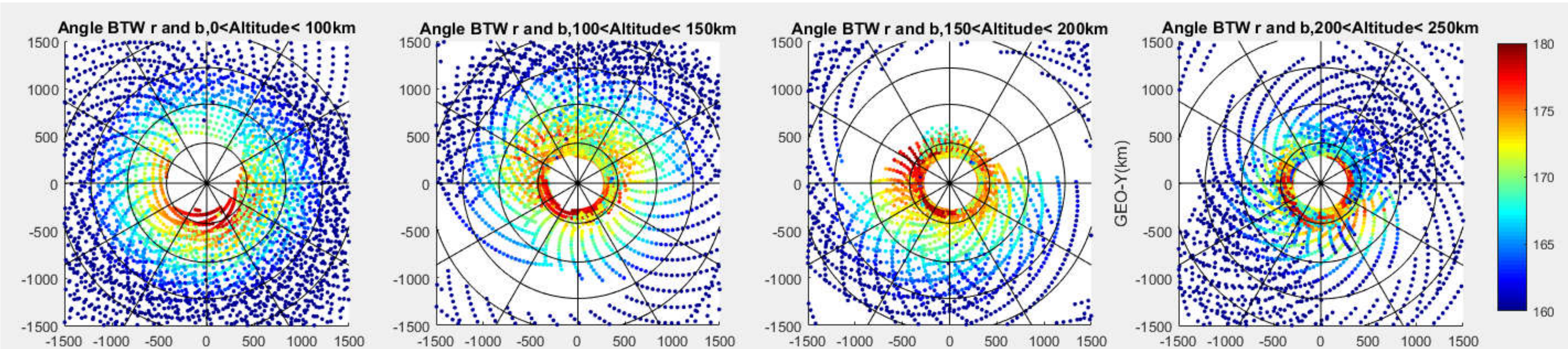
Application of the reduced dipole model -Mercury





Application to Mercury

Scattering plot of the angle a=acos (**b**·**r**)



H<100 km 100<H<150 km 150<H<200 km 200<H<200 km

1. Night side data set is used to reduce the disturbance of space current
2. Check the inversion errors for different ranges of altitude
3. Check the inversion errors for different ranges of latitude

Application to Mercury

To reach the minimum inversion error δ:

- Altitude: <100km
- night side data: 20:00< local time<04:00
- Latitude range:40<lat<50 degree

Paper	M (Gaussian unit)	North Shift of dipole center	Dipole tilt angle
Anderson, et al.2011,science	195±10nT*RM^3	484 km	<3 degree
Anderson, et al.2012,JGR	190±10nT*RM^3	479 km	<0.8 degree
Johnson et al.,2012,JGR	190 nT*RM^3	478 km	0
Our study	215nT*RM^3	480 km	3.7 degree

Further modifications

✓**It could be modified to a model of spherical surface current**

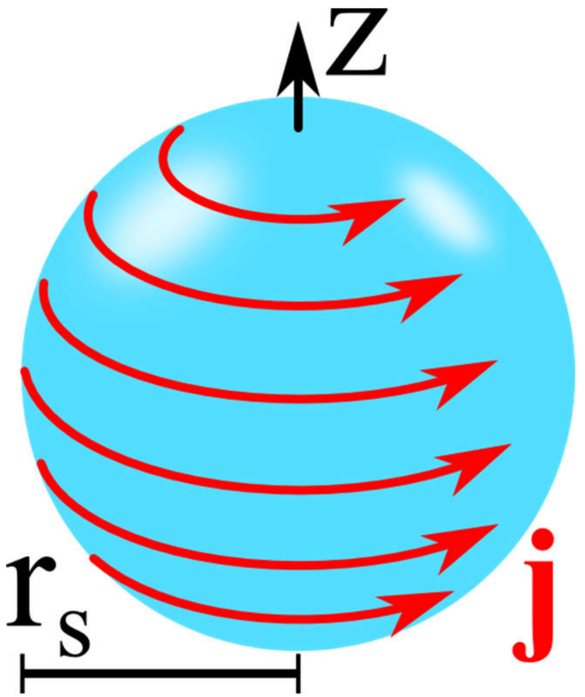
(7parameters are involved. 2 for the axis orientation, 3 for the spherical center, 1 for the spherical radius, and 1 for the surface current density).

This model could be closer to the real dynamo current pattern.

J=j0; J=j0*sin(ϑ); J=j0*sin(ϑ)^2;

It will be our Next Study

Further modifications



$$\mathbf{j}(\mathbf{r}) = j_0 \delta(r - r_s) \sin\theta \hat{\boldsymbol{\varphi}} \qquad r > rs,$$

$$\mathbf{B}(\mathbf{r}) = -\mu(\mathbf{r}) \nabla \psi_{out}(\mathbf{r}) = \frac{-\mu(\mathbf{r}) \mu_r r_s^{\ 3} j_0}{(\mu_r + 2)} \left(-2r^{-3} \cos\theta \hat{r} - r^{-3} \sin\theta \hat{\theta}\right)$$

It is ideal dipole field

We have to keep in mind that "The inversion is non-unique", we have to interpret the inversion results very carefully.

Summary

- 1. A new technique to diagnose the source of planetary dipolar field is developed based on a single circular current loop model.
- 2. The technique is able to separate the loop parameters successively.
- 3. This technique can be reduced to a dipole model or modified to a spherical surface model.
- 4. Tests and application to spacecraft's data demonstrate the effectiveness of this technique
- 5. It could be applied to study the local field anomaly after subtraction of main dipole/loop component.

Thank You

谢 谢！