Morphological controls on groundwater residence times of shallow aquifers and implications on streamwater transit time distributions

Alexandre Gauvain[1,*], Sarah Leray[2], Jean Marçais[3], Camille Vautier[1], Luc Aquilina[1], Jean-Raynald de Dreuzy[1]
[1] University of Rennes 1, CNRS, Géosciences Rennes – UMR 6118, F-35000 Rennes, France.
[2] Departamento de Ingenierìa de hidràulica y Ambiental, Pontificia Universidad Caòlica de Chile, Santiago, RM, Chile.
[3] INRAE, UR RiverLy, F-69625 Villeurbanne, France.
* Corresponding author : alexandre.gauvain@univ-rennes1.fr

ABSTRACT

In shallow aquifers, including weathered zones characteristic of crystalline geologic basements, subsurface flows strongly depend on the geomorphological evolution of landscapes as well as on the geological heterogeneity structures. Yet, it remains largely unknown how geomorphology and geology shape the residence times in the aquifers and the transit times in the receiving stream water bodies.

We investigate this issue with 3D synthetic models of free aquifers. Aquifer models represent hillslopes from the river to the catchment divide with constant slopes, evolving widths and depths. They are submitted to uniform and constant recharge. All flows end up in the river either through the aquifer or through the surface as return flows and saturation excess overland flows. Steady-state flows and transit times to the river are simulated with Modflow and Modpath (Niswonger et al., 2011; Pollock, 2016). The mean and standard deviation of the transit time distribution (TTD) are systematically determined as functions of the hillslope shapes (convergent or divergent to the river, thinning or thickening to the river) and the ratio of recharge to hydraulic conductivity.

We show that the the TTD is fondamentally related to the repartition of the aquifer volume and the extend seepage area. The mean transit time distribution is a function of the geology through the volume of the aquifer divided by the recharge rate even in the presence of seepage areas. The standard deviation of the transit time distribution is a function of the geomorphology through the bulk organization of the groundwater body from the river to the catchment divide. Without seepage, the organization of the groundwater body is efficiently characterized by its barycenter. When seepage occurs, the standard deviation becomes also sensitive to the extent of the seepage zone.

We conclude that mean of the transit time distribution is primarily determined by geology through the accessible aquifer volume while the ratio of its standard deviation to the mean (coefficient of variation) is rather determined by geomorphology through the profile of the aquifer from the river to the catchment divide. We discuss how geophysical data might help to determine the groundwater body and assess the transit time distribution. We illustrate these findings on natural aquifers in the crystalline basements of Brittany-Normandy

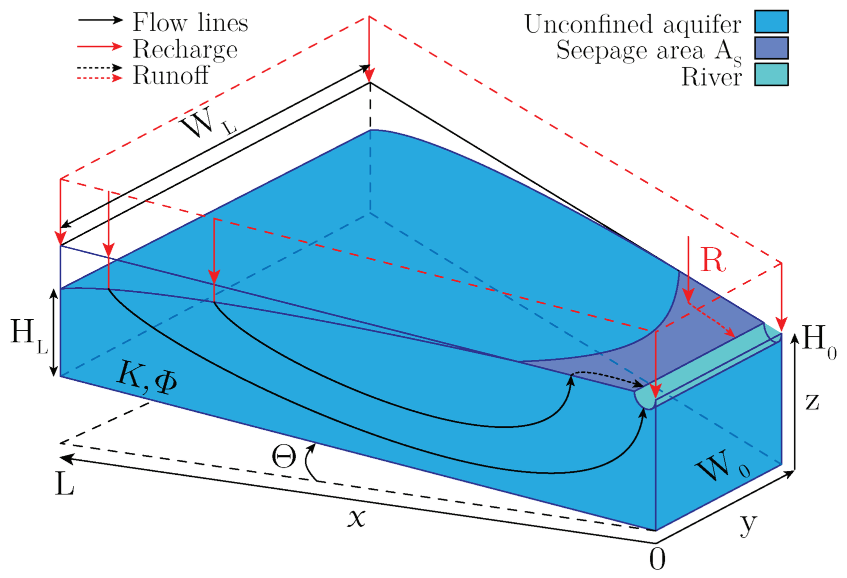
NUMERICAL HILLSLOPE MODELS

SUBSURFACE STRUCTURE

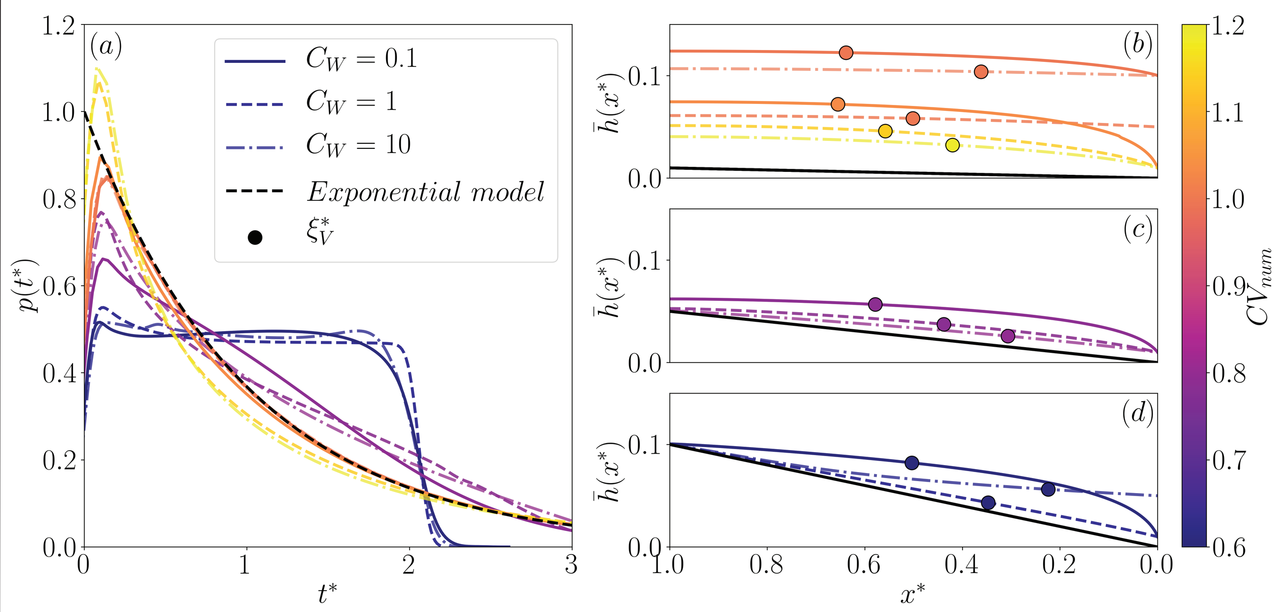
The aquifer is assumed uniform with its constant hydraulic conductivity K [L.T^{-1}] and porosity Φ [-]. The aquifer structure is defined by four parameters (Figure 1a). First, the model length L [L] extends from its downstream to upstream limits at respectively x=0 and x=L. Second, the surface and the base of aquifer are tilted equaly with the slope Θ [-]. Third, the subsurface thickness is uniform and equal too. Fourth, the hillslope may be convergent or divergent according to the shape coefficient C_W [-] equals to the ratio of downstream width W_0 [L] to upstream width W_L [L]. The width function W(x) [L] is a linear function expressed as:

$$W(x) = \frac{W_L - W_0}{L}x + W_0 \quad (1)$$

The recharge R [L.T^{-1}] to the aquifer is uniform. It drives subsurface flows, an non-linear aquifer thickness and a potential seepage downstream.



TRANSIT TIME DISTRIBUTION (TTD) & DISTRIBUTION OF AQUIFER VOLUME



Flows and transports are solved using Modflow [1] and Modpath [2]. TTD is characterized by its mean and its coefficient of variation CV. The coefficient of variation is the ratio of the standard deviation to the mean transit time of the transit time distribution:

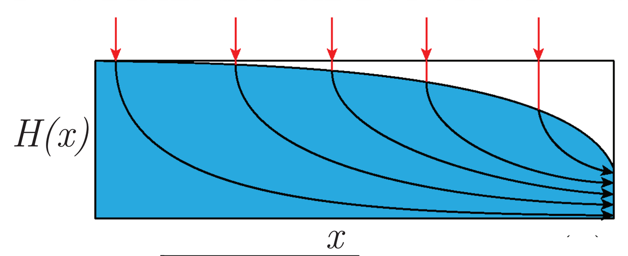
$$CV = \frac{\sigma}{\tau} = \sqrt{\frac{\sigma^2}{\tau^2}} = \sqrt{\frac{< t^2(x^*) > - < t^1(x^*) >^2}{< t^1(x^*) >^2}} \quad (2)$$

The aquifer volume distribution is characterized by the position of the barycenter. The barycenter of the aquifer volume represents the location where the volumes on either side of this point are equal:

$$\xi_V^* = \frac{\xi_V}{L} = \frac{\int_0^L x\,V(x)\,dx}{L\int_0^L V(x)\,dx} = \frac{\int_0^L x\,H(x)W(x)\,dx}{L\int_0^L H(x)W(x)\,dx} \quad (3)$$

ANALYTICAL HILLSLOPE FLOW PROCESSES

TWO-DIMENSIONAL DUPUIT SYSTEM [3]



$$H(x) = \sqrt{\frac{R}{K}(x^2 - 2Lx) + H_0^2} \quad (4)$$

$$t(x) = \Phi\sqrt{\frac{\lambda}{KR}}\left[f\left(\frac{L-x}{\sqrt{\lambda}}\right) - f\left(\frac{L}{\sqrt{\lambda}}\right)\right] \quad (5)$$

$$f(u) = ln\left(\frac{1}{u} + \sqrt{\frac{1}{u^2} - 1}\right) - \sqrt{1-u^2} \quad (6)$$

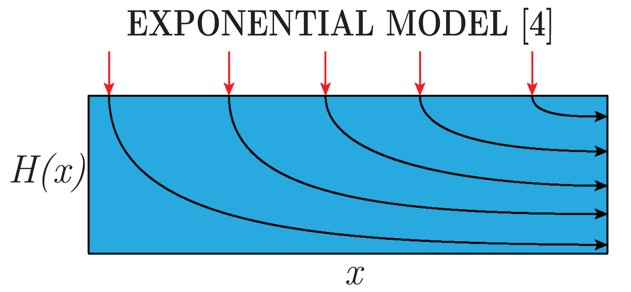
$$\lambda = L^2 + \frac{KH_0^2}{R} \quad (7)$$

$$< t^k(x) > = \int_0^L t^k(x)\,dx \quad (8)$$

$$CV = \sqrt{\frac{< t^2(x^*) > - < t^1(x^*) >^2}{< t^1(x^*) >^2}} \quad (9)$$

$$\xi_V^* = \frac{\int_0^L x\,H(x)\,dx}{L\int_0^L H(x)\,dx} \quad (10)$$

EXPONENTIAL MODEL [4]



$$H(x) = constant \quad (11)$$

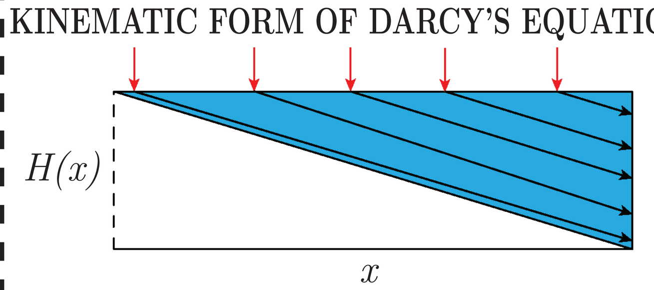
The TTD of an aquifer with a constant thickness is an exponential. The characteristic of an exponential distribution is that the standard deviation is equal to the mean:

$$CV = 1 \quad (12)$$

H(x) being constant, the barycenter equation can be simplified and expressed as:

$$\xi_V^* = \frac{\int_0^L x\,W(x)}{L\int_0^L W(x)} = \frac{C_W + 2}{3C_W + 3} \quad (13)$$

KINEMATIC FORM OF DARCY'S EQUATION [5]



$$H(x) = -\frac{H_0}{L}x + H_0 \quad (14)$$

$$\frac{\partial S}{\partial t} + \frac{\partial Q}{\partial x} = RW(x) \quad (15)$$

$$S(x) = H(x)W(x)\Phi \quad (16)$$

$$Q(x) = -K\frac{S(x)}{\Phi}\frac{\partial z}{\partial x} = -K\frac{S(x)}{\Phi}\frac{z_L}{L} \quad (17)$$

$$\frac{dt}{dx} = \frac{S(x)}{Q(x)} = -\frac{\Phi}{K}\frac{L}{z_L} \quad (18)$$

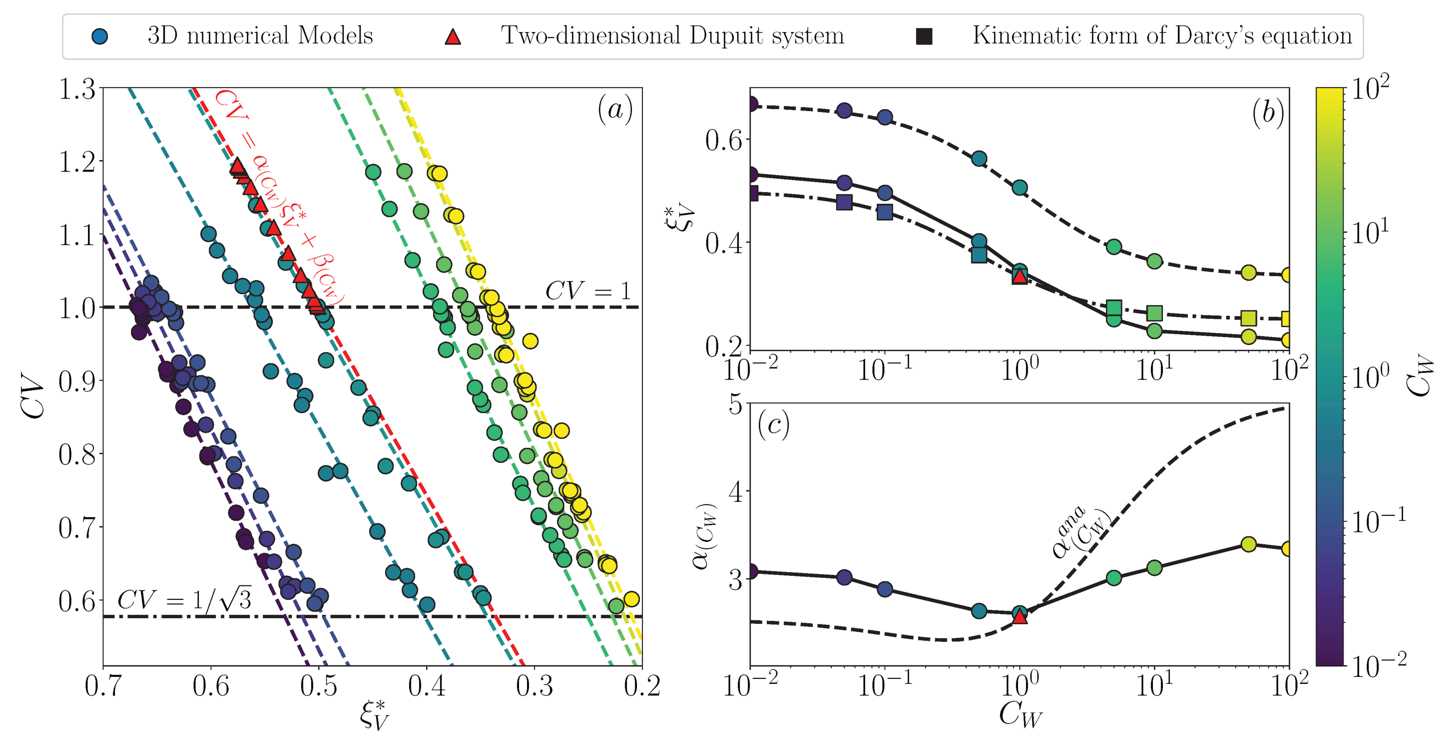
$$t(x) = \frac{\Phi}{K}\frac{L}{z_L}x \quad (19)$$

$$CV = \sqrt{\frac{< t^2(x^*) > - < t^1(x^*) >^2}{< t^1(x^*) >^2}} = \sqrt{\frac{L^2/12}{L^2/4}} = \frac{1}{\sqrt{3}} \quad (20)$$

$$\xi_V^* = \frac{\int_0^L x\,H(x)W(x)\,dx}{L\int_0^L H(x)W(x)\,dx} = \frac{C_W + 1}{4C_W + 2} \quad (21)$$

RESULTS

MODELS WITOUT SEEPAGE



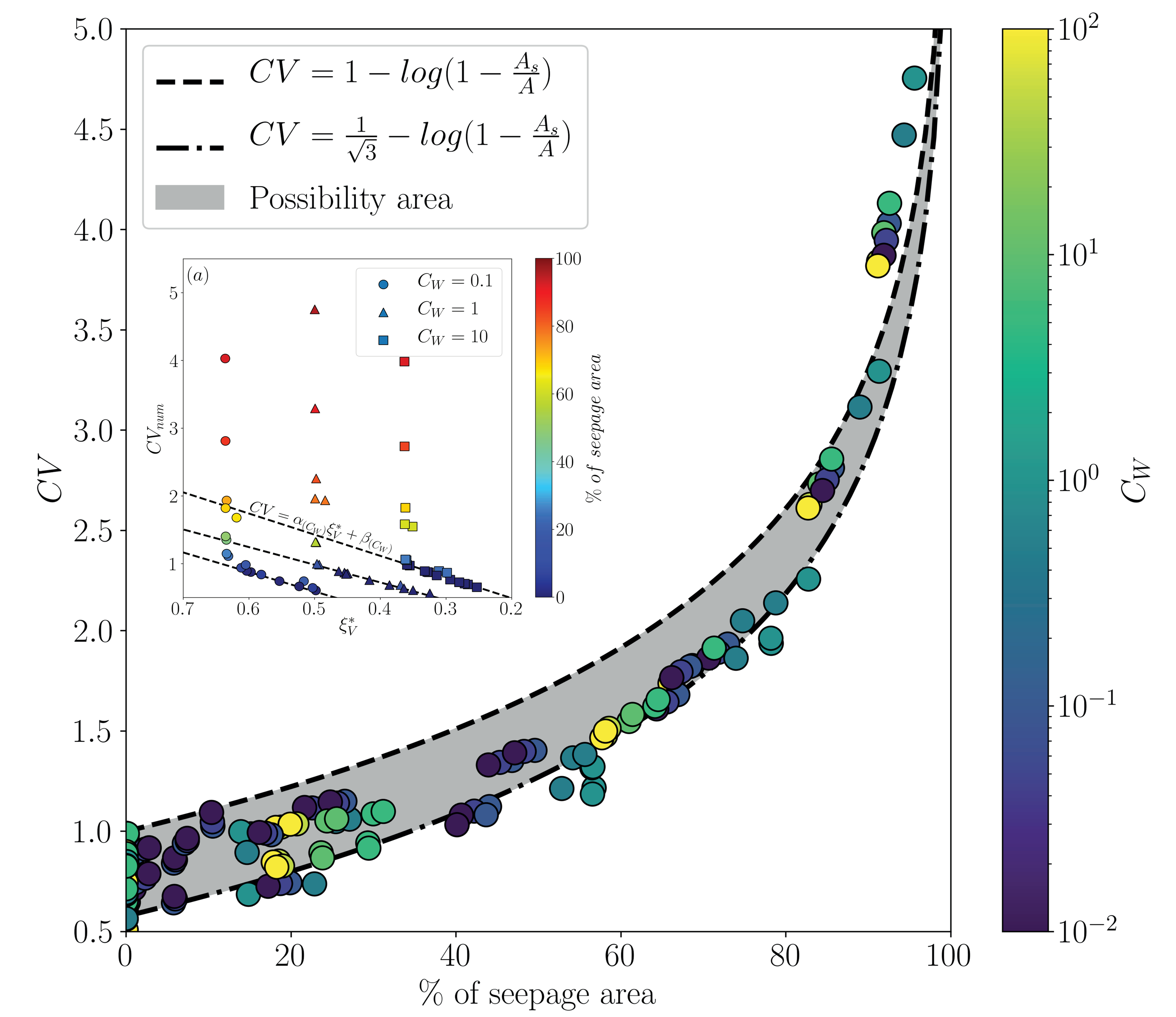
Linear relation between the coefficient of variation and the barycenter of the aquifer volume :

$$CV_{ana}(\xi_V^*) = \alpha_{C_W}\,\xi_V^* + \beta_{C_W} \quad (22)$$

The right fit between the barycenter and the hillslope shape coefficient C_W for the exponential model and the slight variation of the slope α_{C_W} allow to transform the equation (22) using a mean slope and the equation (13) as:

$$CV_{ana}(\xi_V^*, C_W) = 1 - \bar{\alpha}\left(\frac{C_W + 2}{3C_W + 3} - \xi_V^*\right) \quad (23)$$

MODELS WITH SEEPAGE

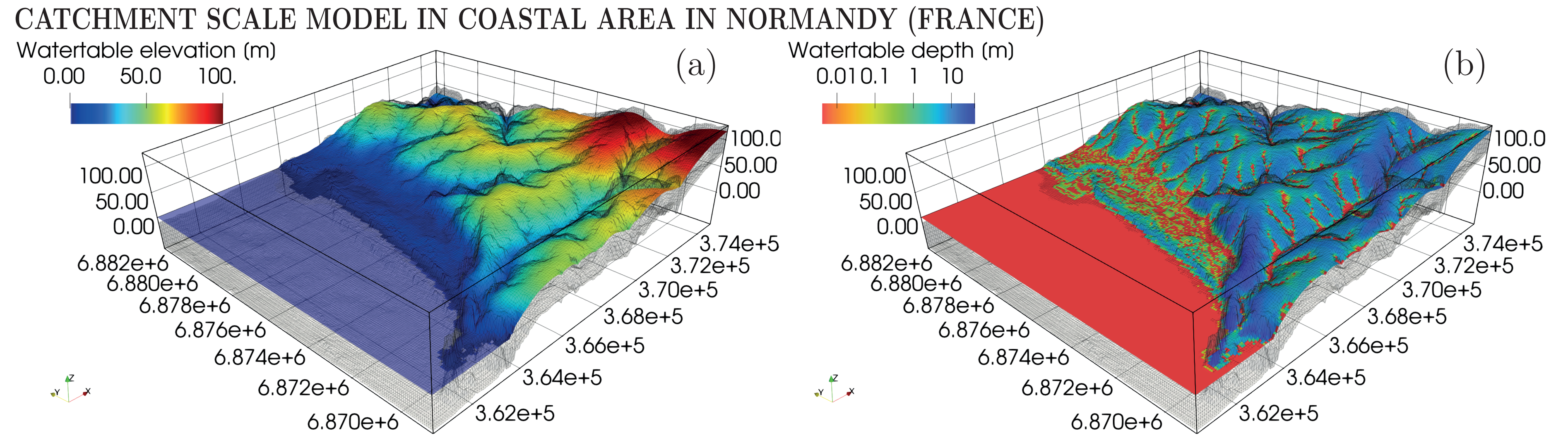


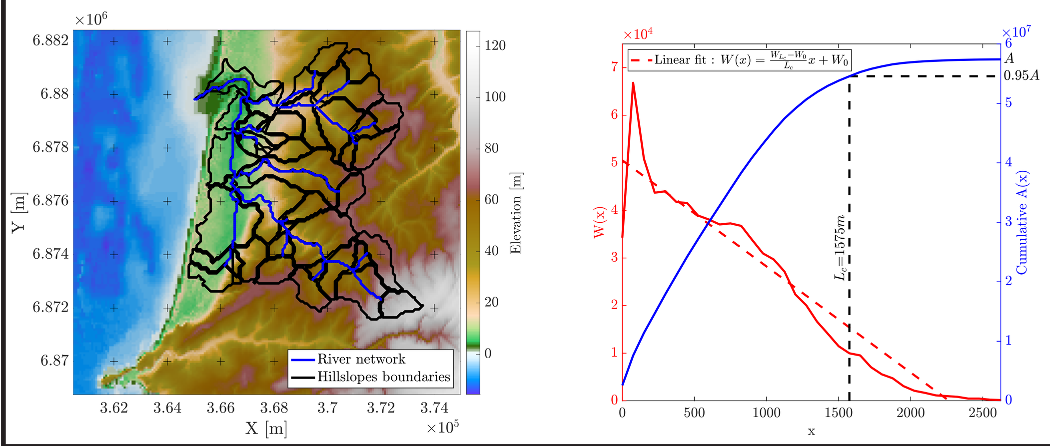
$$CV_{ana}(\xi_V^*, C_W, A_S/A) = 1 - \bar{\alpha}\left(\frac{C_W + 2}{3C_W + 3} - \xi_V^*\right) - \log\left(1 - \frac{A_S}{A}\right) \quad (24)$$

Impact of seepage (is the relative area A_S/A)

DISCUSSION

CATCHMENT SCALE MODEL IN COASTAL AREA IN NORMANDY (FRANCE)





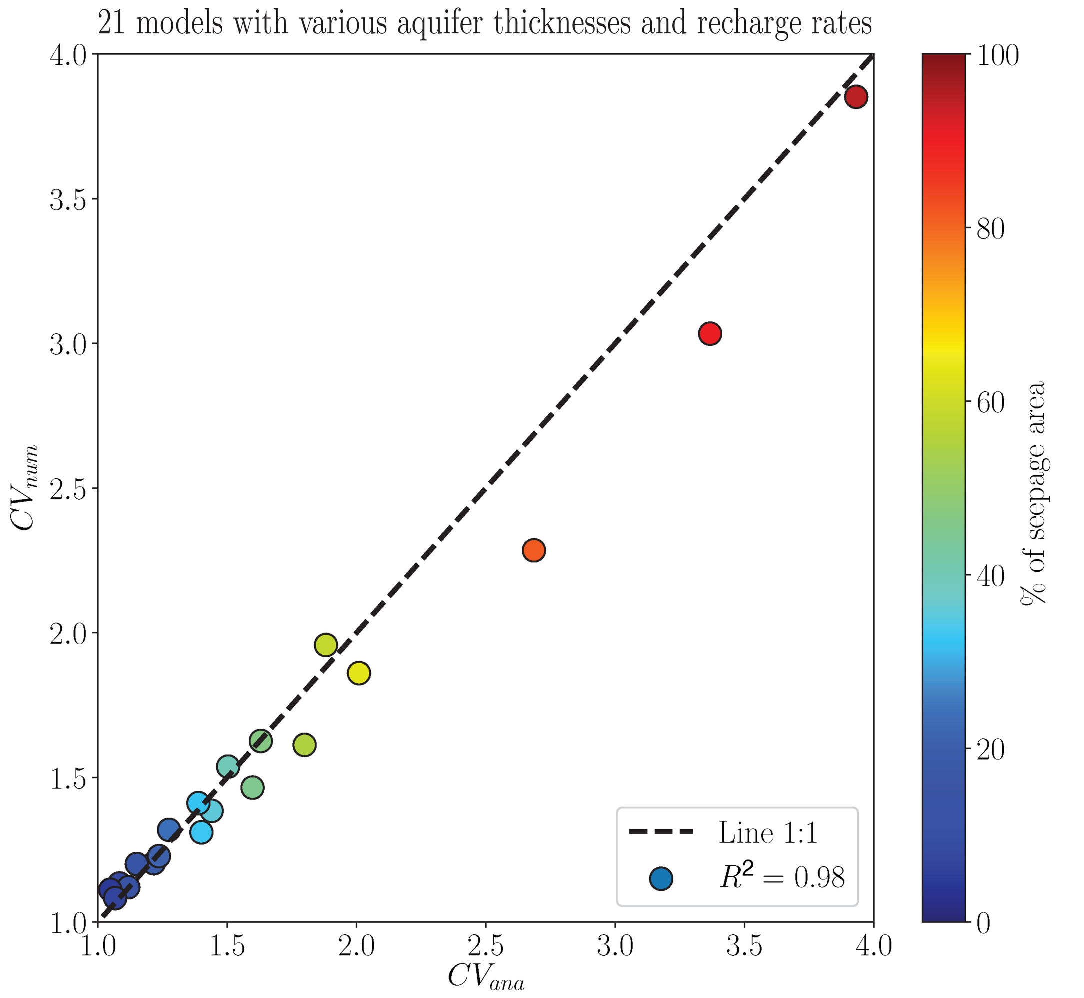
$$A = \int_0^{L_c} W(x)\,dx = \int_0^{L_c} \frac{W_{L_c} - W_0}{L_c}x + W_0\,dx \quad (25)$$

$$A = L_c\frac{W_{L_c} + W_0}{2} \quad (26)$$

$$W_{L_c} = \frac{2A}{L_c} - W_0 \quad (27)$$

$$C_{W_{ana}} = \frac{W_0}{W_{L_c}} = 2.26 \quad (28)$$

21 models with various aquifer thicknesses and recharge rates



CONCLUSION

(1) The volume repartition and seepage extend shape the transit time distribution.
(2) The coefficient of variation is a good proxy for the transit time distribution.
(3) The transit time distribution broadly evolves from the uniform to power law shapes.
(4) The coefficient of variation can be approximated from the geomorphological data.

REFERENCES

[1] Niswonger, R.G., Panday, S., Ibaraki, M., 2011. MODFLOW-NWT, A Newton formulation for MODFLOW-2005.
[2] Pollock, D.W., 2016. User guide for MODPATH Version 7—A particle-tracking model for MODFLOW (Report No. 2016–1086), Open-File Report. Reston, VA. https://doi.org/10.3133/ofr20161086
[3] Chesnaux, R., Molson, J.W., Chapuis, R.P., 2005. An Analytical Solution for Ground Water Transit Time through Unconfined Aquifers. Ground Water 43, 511–517. https://doi.org/10.1111/j.1745-6584.2005.0056.x
[4] Haitjema, H.M. 1995. Analytic Element Modeling of Ground-water Flow. San Diego, California: Academic Press.
[5] Sabzevari, T., Talebi, A., Ardakanian, R., Shamsai, A., 2010. A steady-state saturation model to determine the subsurface travel time (STT) in complex hillslopes. Hydrol. Earth Syst. Sci. 14, 891–900. https://doi.org/10.5194/hess-14-891-2010