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Automated characterization of magnetic reconnection using particle distributions

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EGU 2020 – ST 1.9 "Theory and Simulation of Solar System plasmas"

Magnetic reconnection

Magnetic Reconnection

- **Complex physical phenomenon**
  - Break of the frozen-in magnetic field
  - Magnetic energy release
  - Transport mechanism, particle acceleration
  - Magnetic Electron Diffusion Region (EDR)

- **Occurring in many plasma environments**
  - Sun: flares, CME
  - Earth's magnetosphere: magnetopause, magnetotail
  - MMS mission

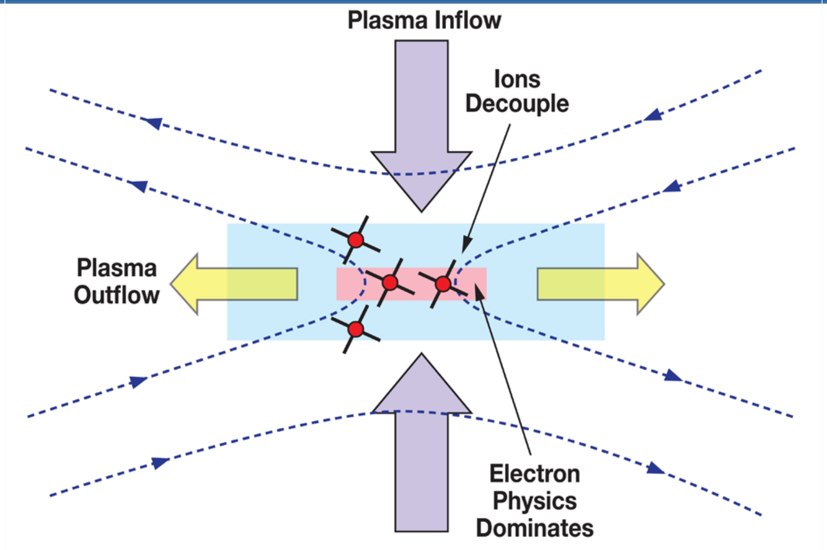
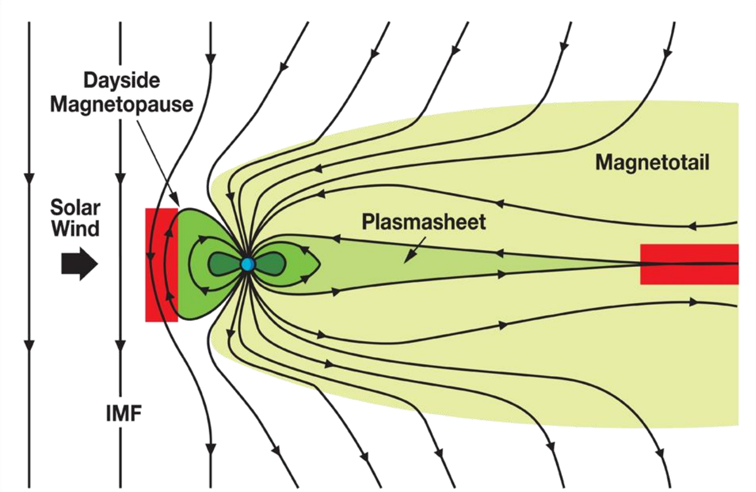


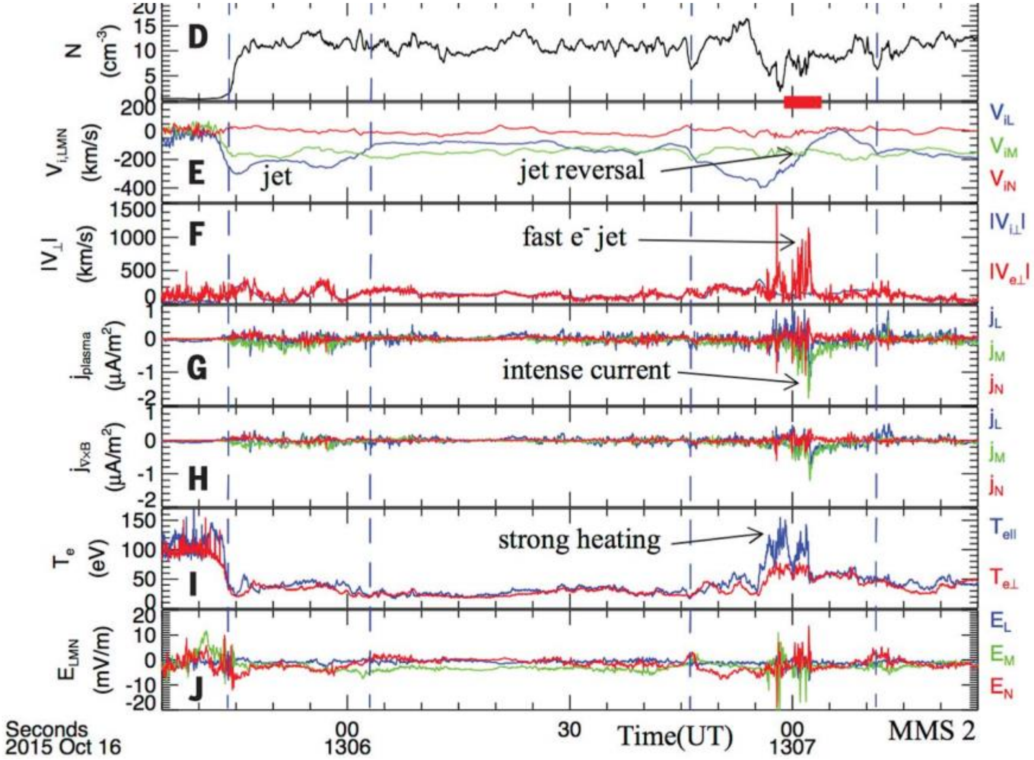
Image Credit: NASA MMS



Detection of reconnection

- **EDR is very small**
  - The precise detection is hard
  - Use of indirect signatures

- **Usually two groups**
  - Field quantities
  - Statistical moments



Burch et al. 2016

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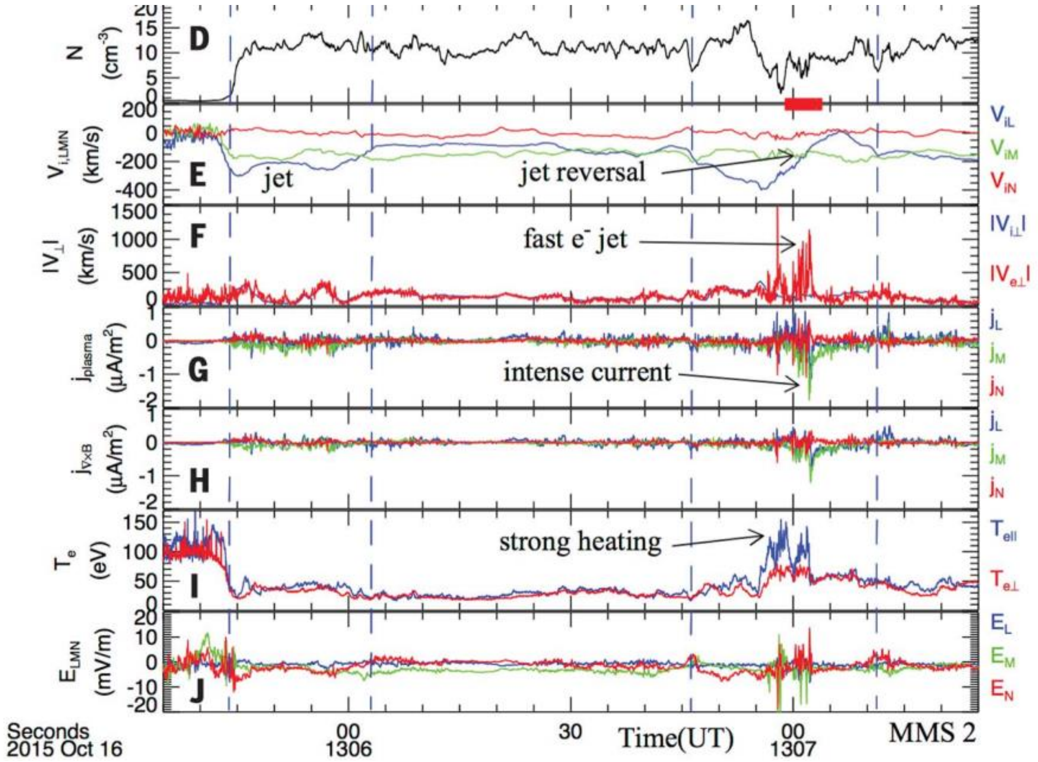


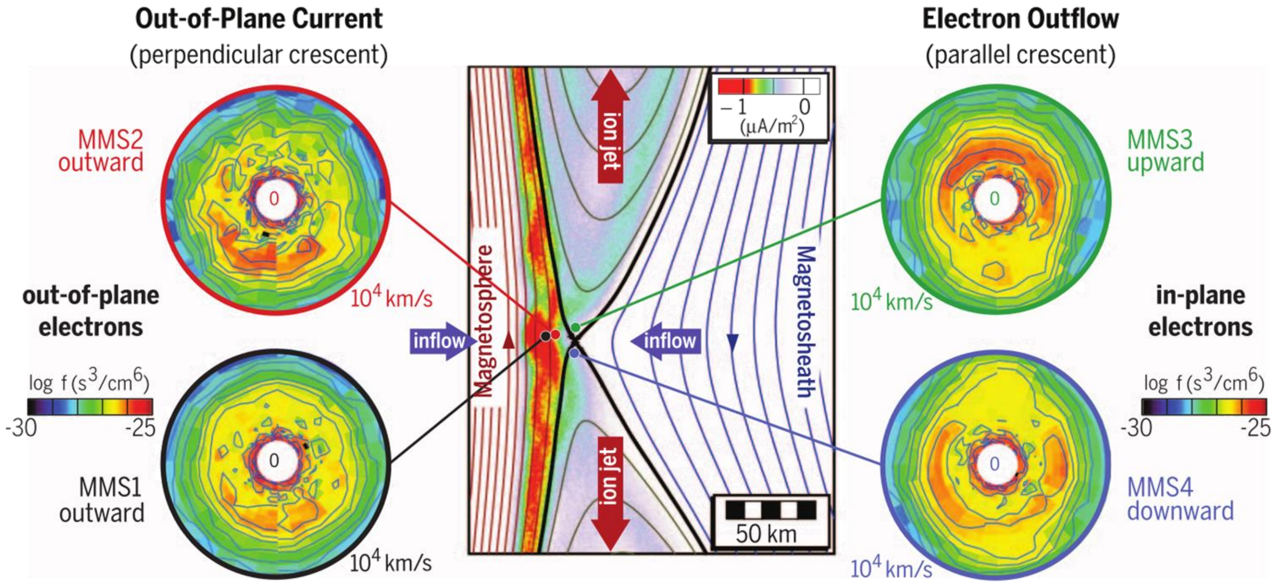
Burch et al. 2016

Third approach: using directly the distribution

Particle distributions

- **Rich part of the information**
  - MMS mission: crescent shape
  - Electron dynamics dominates
  - Beams, power law, top-hat

- **But very large data**
  - 3D velocity space
  - Spatial space
  - Temporal aspect



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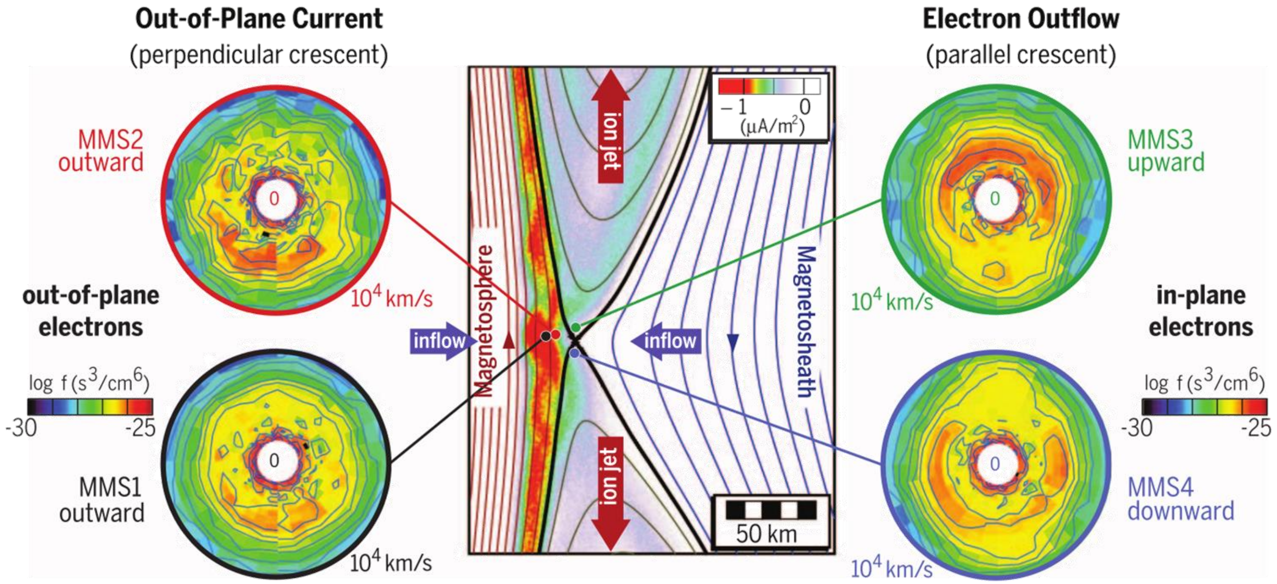


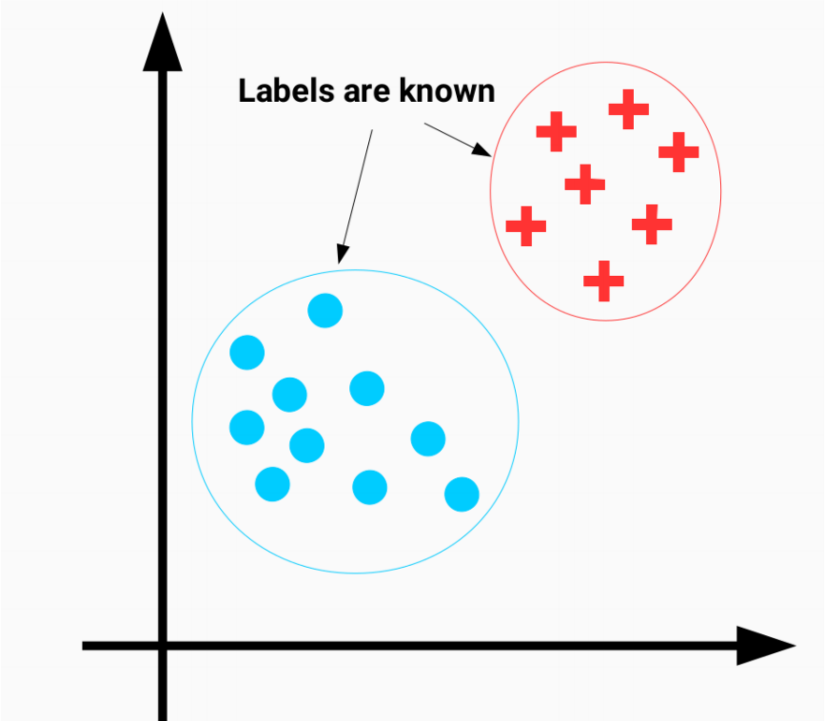
Burch et al. 2016

Extract automatically information from the particle distributions

Machine learning

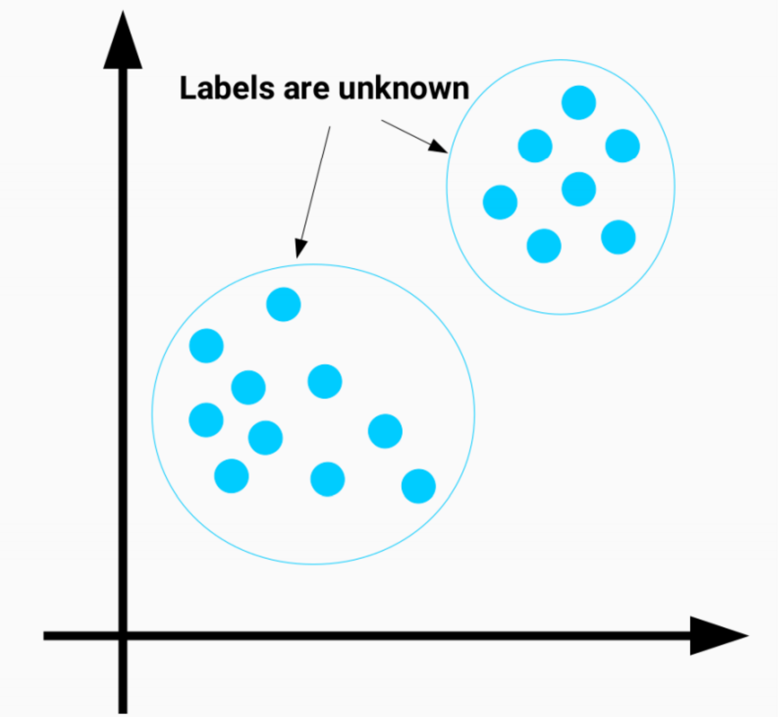
Supervised vs. unsupervised

- **Supervised learning**
  - Regression
  - Classification



Credit S. Carrazza

- **Unsupervised learning**
  - Clustering
  - Dimension reduction
  - Density estimation



Credit S. Carrazza

Related work

- *Automated classification of plasma regions using 3D particle energy distribution. MMS mission: crescent shape,* Olshevsky V. et al, 2019

- *Automatic Detection of Magnetospheric Regions around Saturn using Cassini Data, Yeakel, K et al., 2017*

- *Automatic detection of magnetopause reconnection diffusion regions, Garnier P. et al.*

Table 2. Training Dataset

Label	Region	# examples	percentage
−1	Unknown	0	0
0	Solar Wind	12,135	42.4
1	Foreshock	7,351	25.7
2	Magnetosheath	7,009	24.5
3	Magnetosphere	2,146	7.5
	Total	28,641	100

We want to privilege unsupervised approaches

Density estimation

- **Building an estimate of the probability density function**

Density estimation

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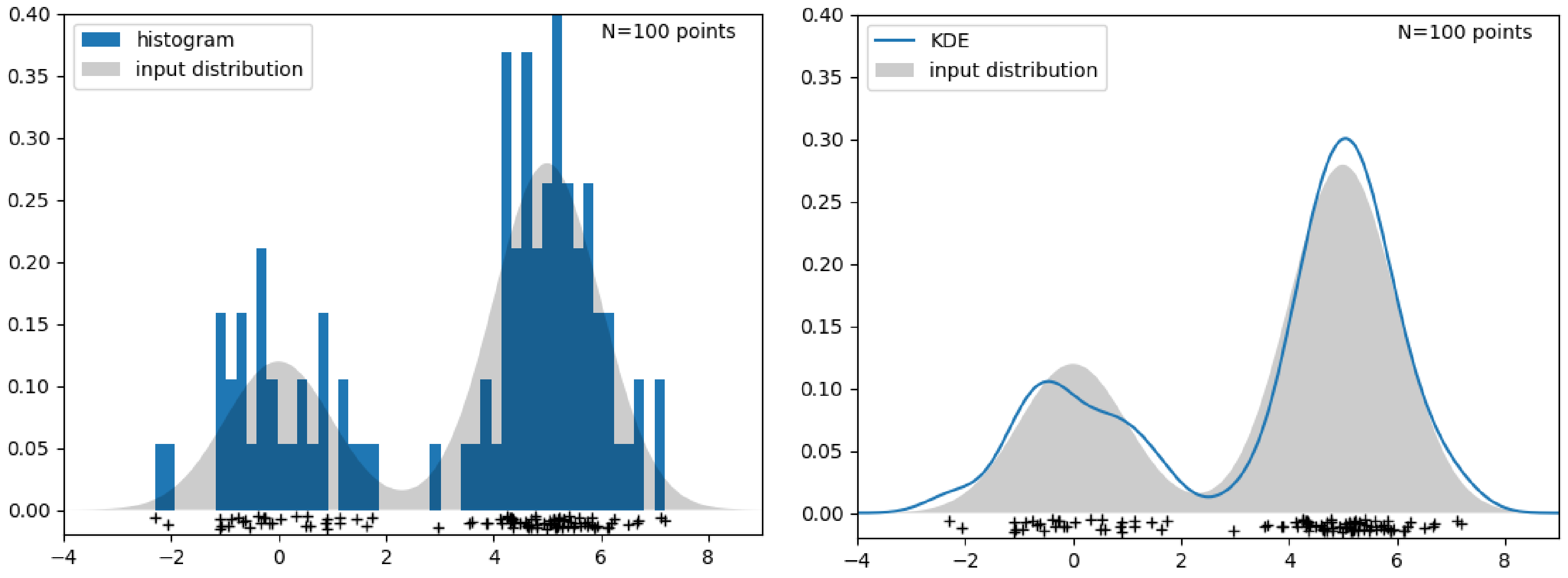
- **Non-parametric methods**
  - Histogram
  - Kernel Density Estimation



Density estimation

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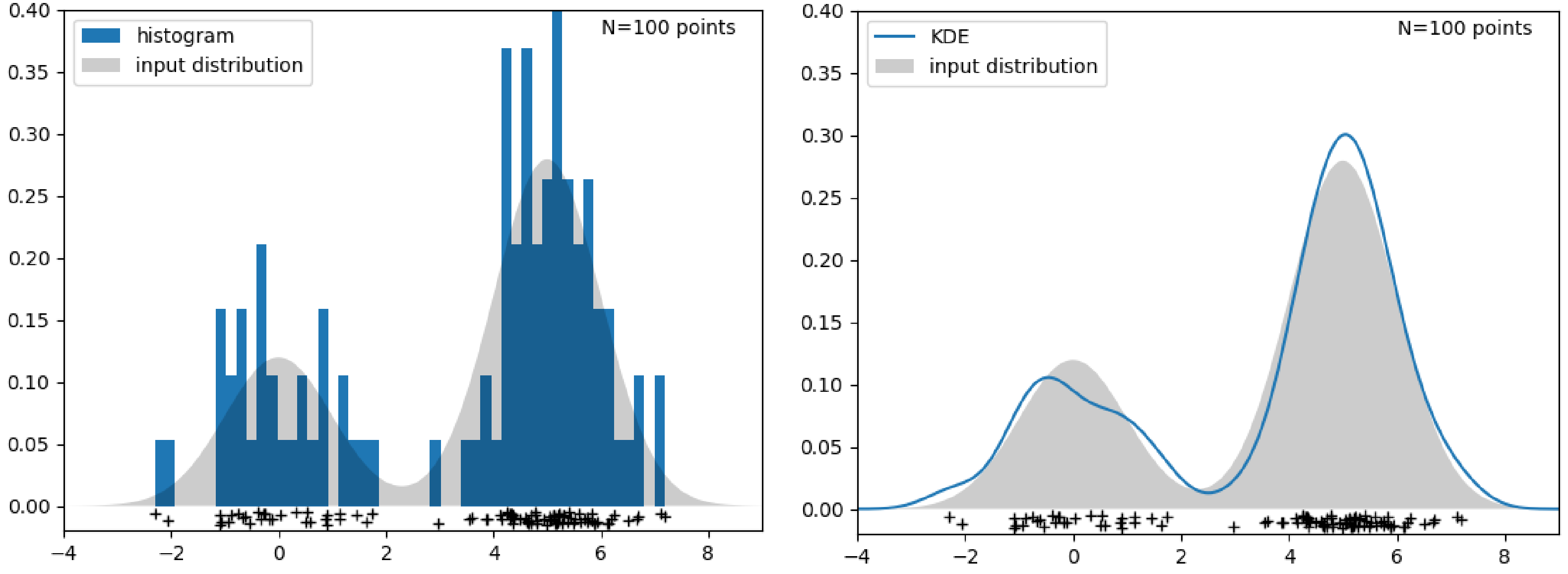

- **Parametric methods**
  - Fitting given distributions
  - Gaussian Mixture Models (GMM)

Density estimation

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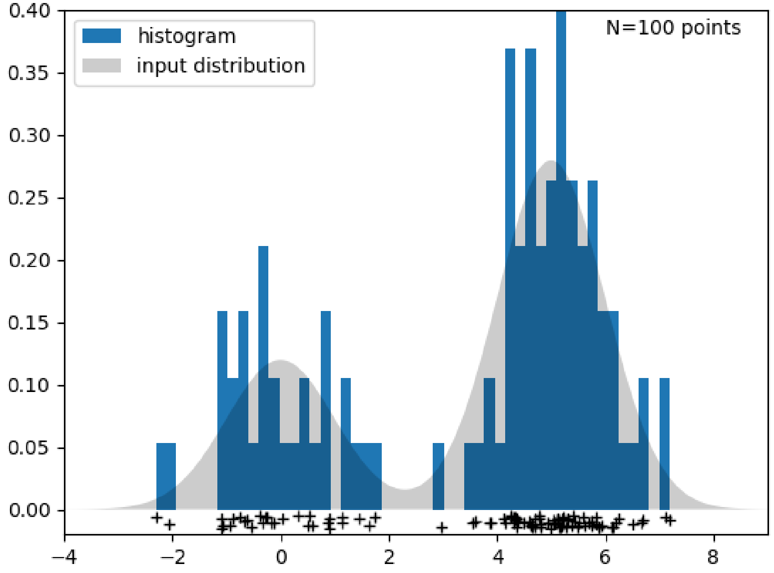

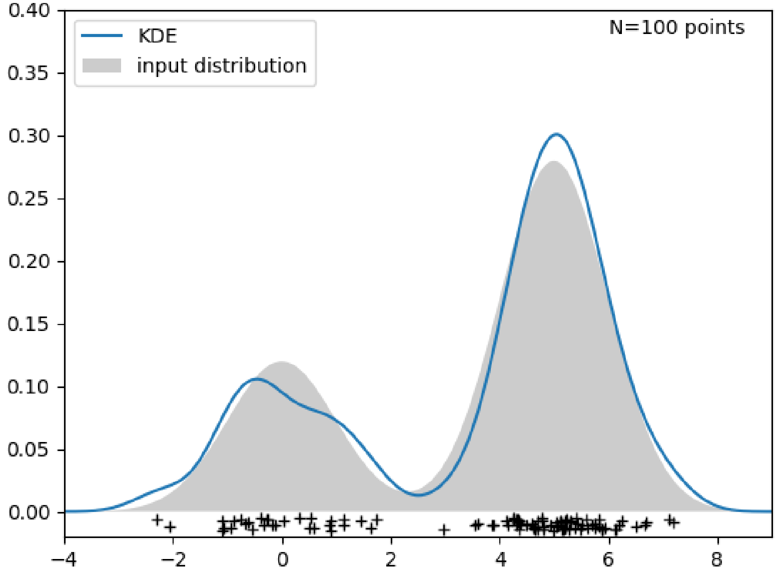

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Gaussian Mixture models

- **Gaussian probability distribution**

$$\mathcal{N}(\boldsymbol{x}|\boldsymbol{\mu},\, \boldsymbol{\Sigma}) = \frac{1}{\sqrt{(2\pi)^d|\Sigma|}} exp\left(-\frac{(\boldsymbol{x}-\boldsymbol{\mu})\boldsymbol{\Sigma}^{-1}(\boldsymbol{x}-\boldsymbol{\mu})}{2}\right)$$

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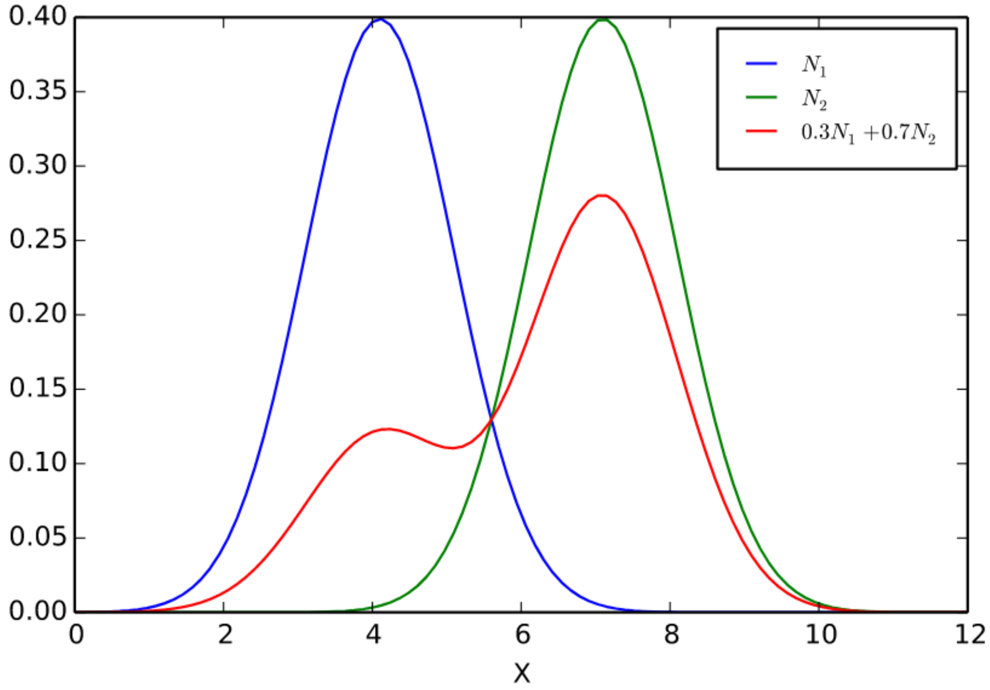
- **Sum of Gaussians (mixture)**

$$p(\boldsymbol{x}|\boldsymbol{\Phi}) = \sum_{k=1}^{K} w_k\,\mathcal{N}(\boldsymbol{x}|\boldsymbol{\theta_k})$$

- **Parameters to find**

$$\boldsymbol{\theta_k} = (w_k\,,\boldsymbol{\mu_k}\,,\boldsymbol{\Sigma_k})$$

$p(x) = 0.3N_1 + 0.7N_2$



Credit Rémi Emonet

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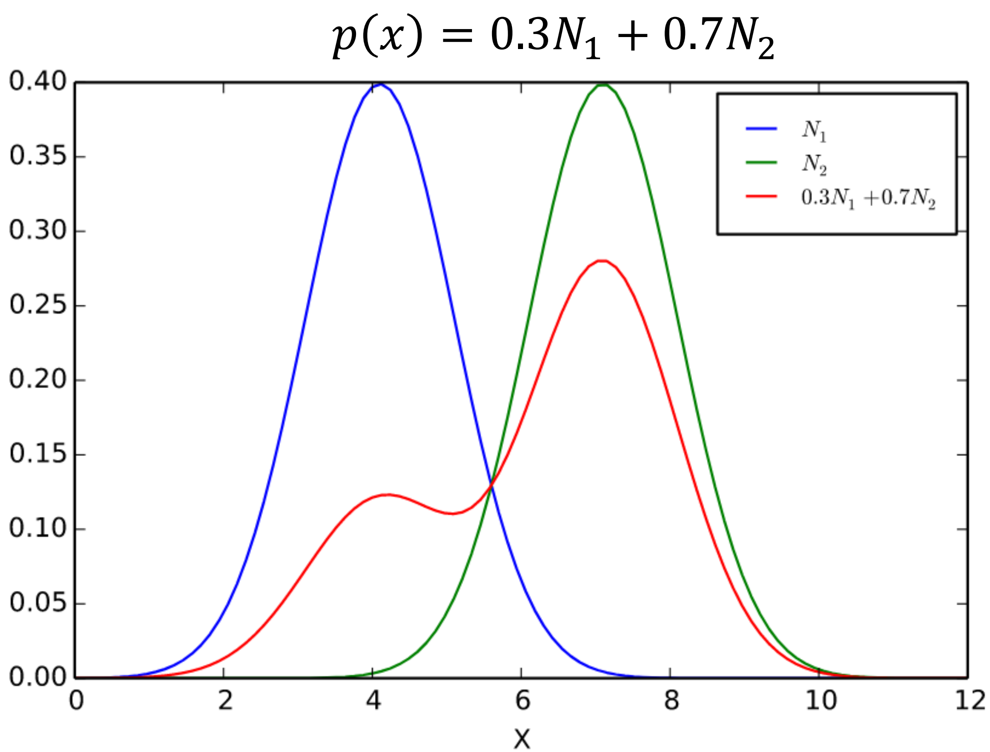


Credit Rémi Emonet

Gaussian Mixture Model

$$l(\phi;\boldsymbol{X}) = \sum_{i=1}^{n} \ln f(\boldsymbol{X_i}, \phi)$$

- **How to infer the best parameters ? Maximum likelihood estimation**

$$l(\phi;\boldsymbol{X}) = \sum_{i=1}^{n} \ln \left[\sum_{k=1}^{K} w_k \mathcal{N}(\boldsymbol{x_i}|\boldsymbol{\theta_k}) \right]$$

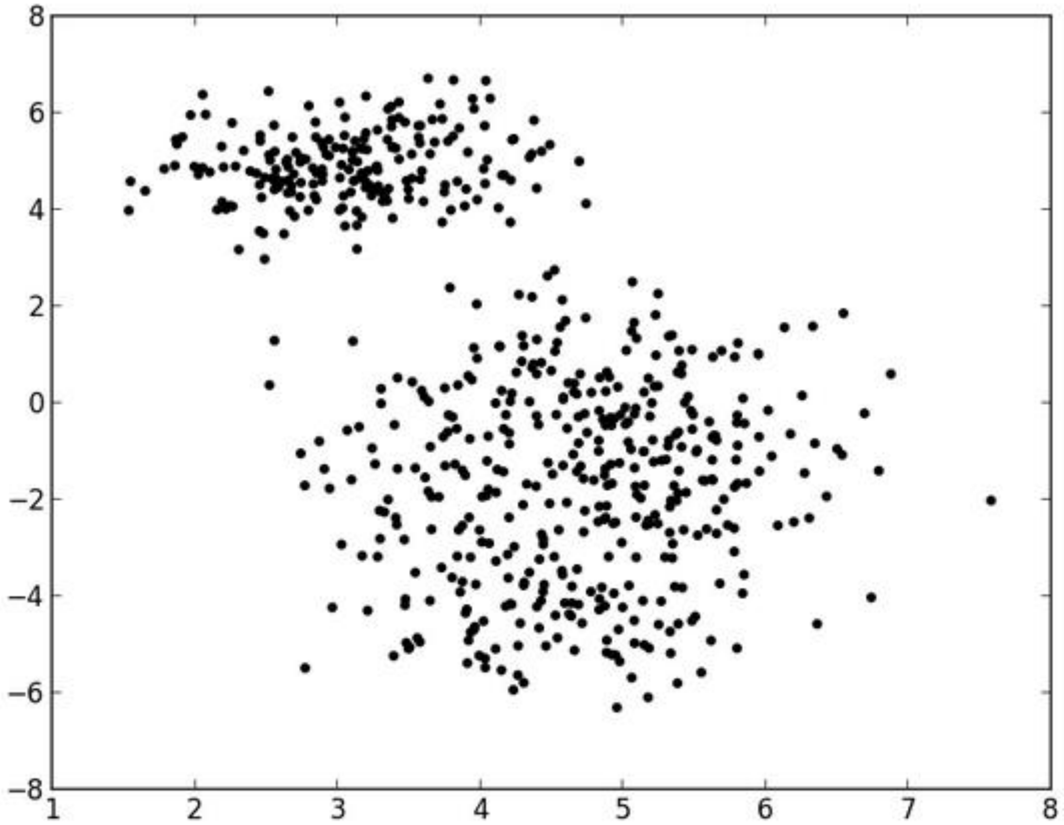
- **Maximizing the likelihood estimation**
  - Non linear maximization problem
  - No closed form

$$\frac{\partial l}{\partial \boldsymbol{\mu_k}} = 0 \qquad \frac{\partial l}{\partial \boldsymbol{\Sigma_k}} = 0 \qquad \frac{\partial l}{\partial w_k} = 0 \; s.t. \sum_{k=1}^{K} w_k = 1$$

- **Need to find numerical local maximum: Expectation Maximization**

Expectation Maximization

- **Very effective for models with unobserved latent variables**



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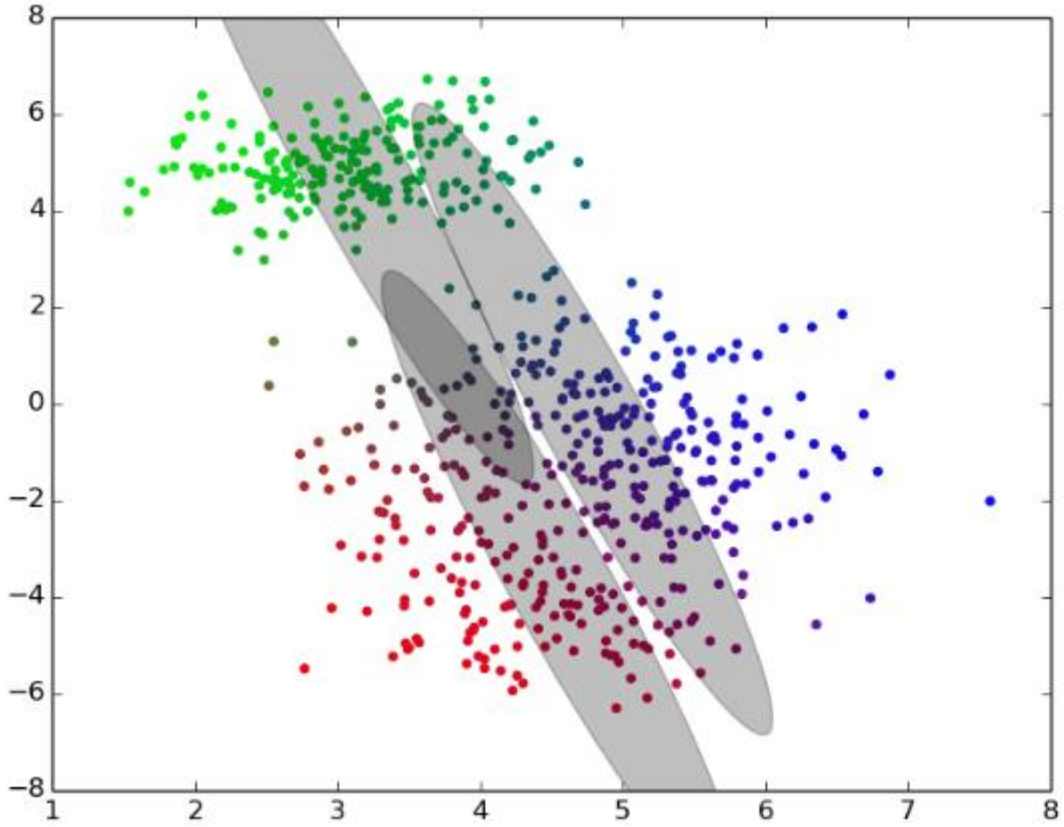

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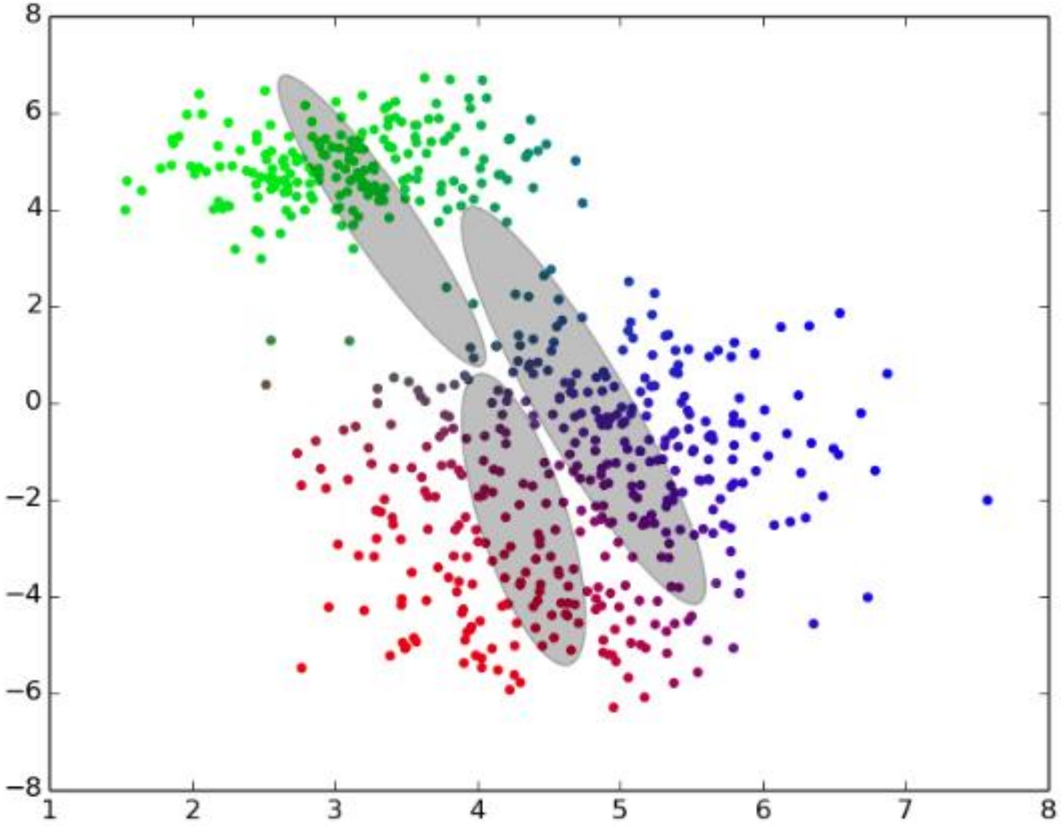

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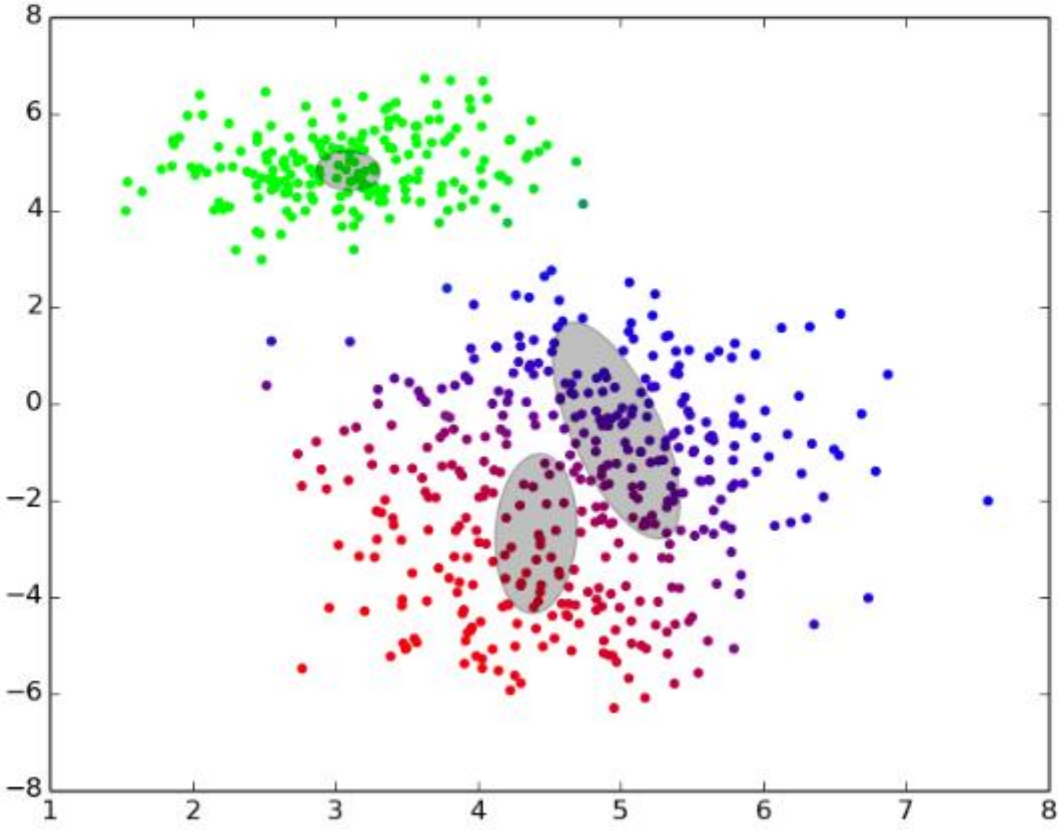

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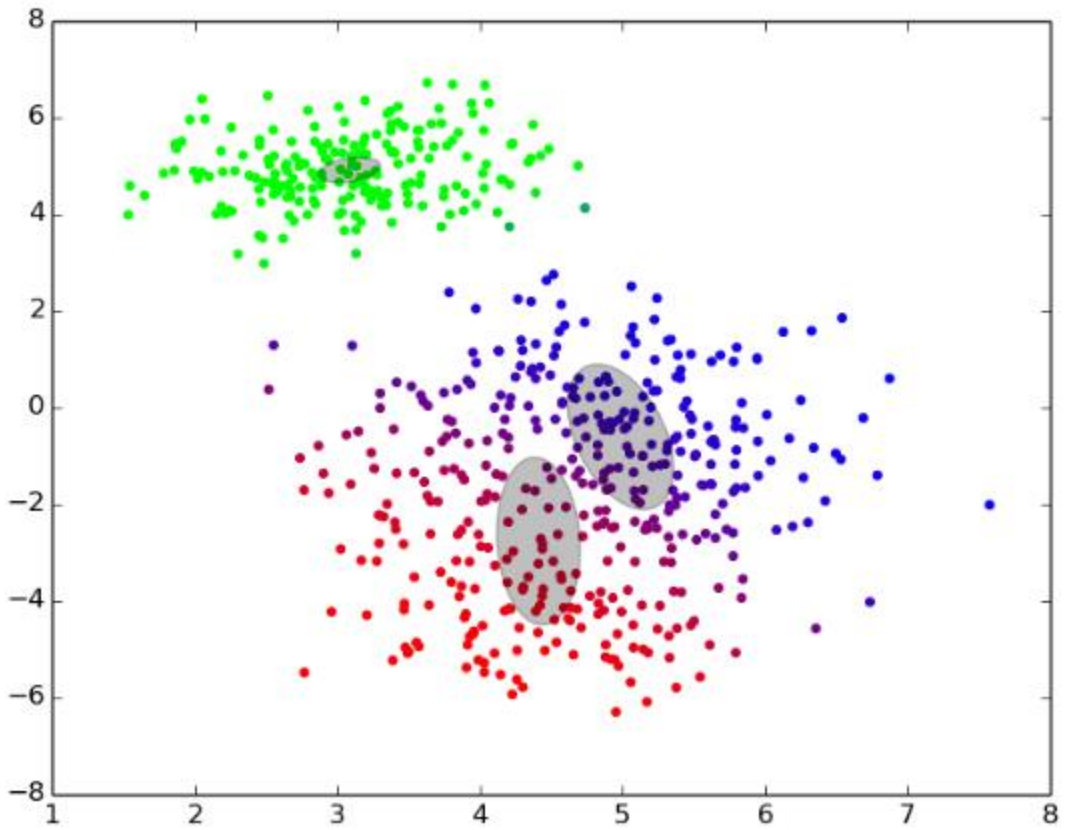

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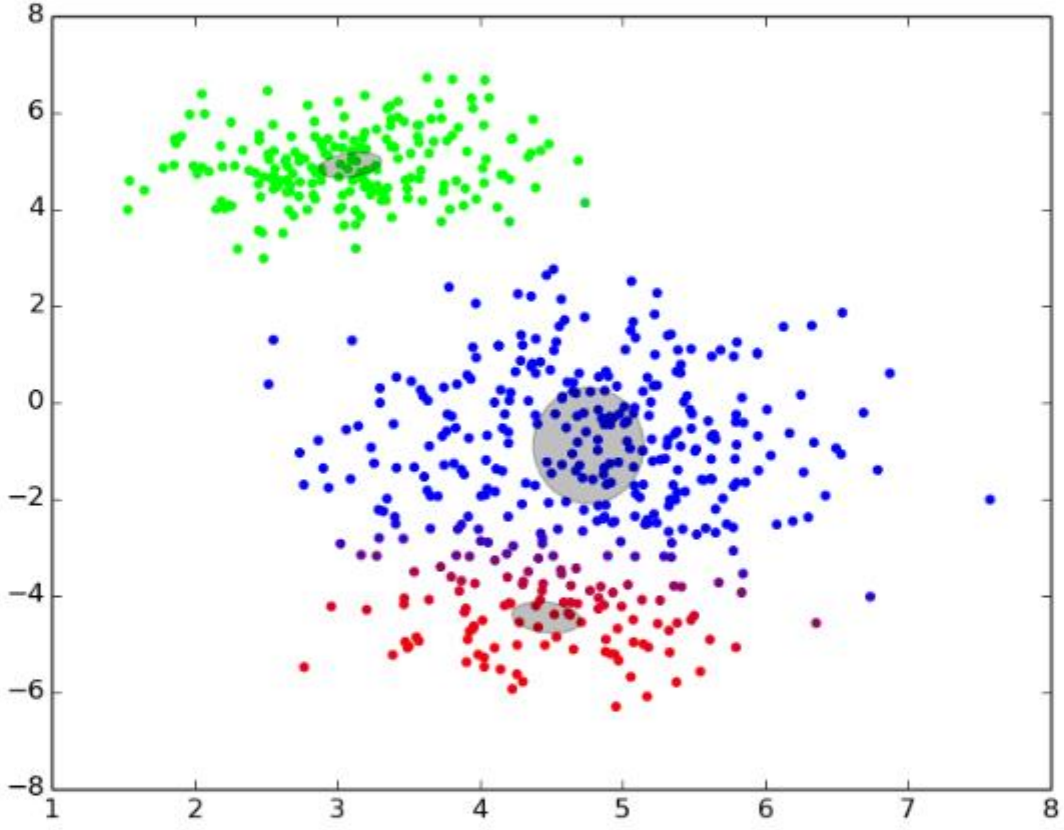

Model selection

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- **Information theory**
  - Aikaike Information Criterion
  - Bayesian Information Criterion

$$AIC = 2k - 2\ln(L)$$
$$BIC = \ln(n)k - 2\ln(L)$$

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Complexity

Goodness of fit

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Complexity

Goodness of fit

- **Various interpretation to the number of components K**
  - Beams/electron subpopulation
  - Complex distribution
  - Deviation from a Gaussian (tail, mode width, etc.)

Simulations

Simulations

- **Access to the complete description of the plasma over all the spatial grid**

- **2.5D collisionless Particle In Cell simulations**
  - iPic3D (Markidis et al.)
  - Double Harris sheet case, weak guide field
  - Grid: 769 x 1025 (30di x 40 di)
  - 196,000,000 particles (~250 particles/cell)
  - Weighted particle injection

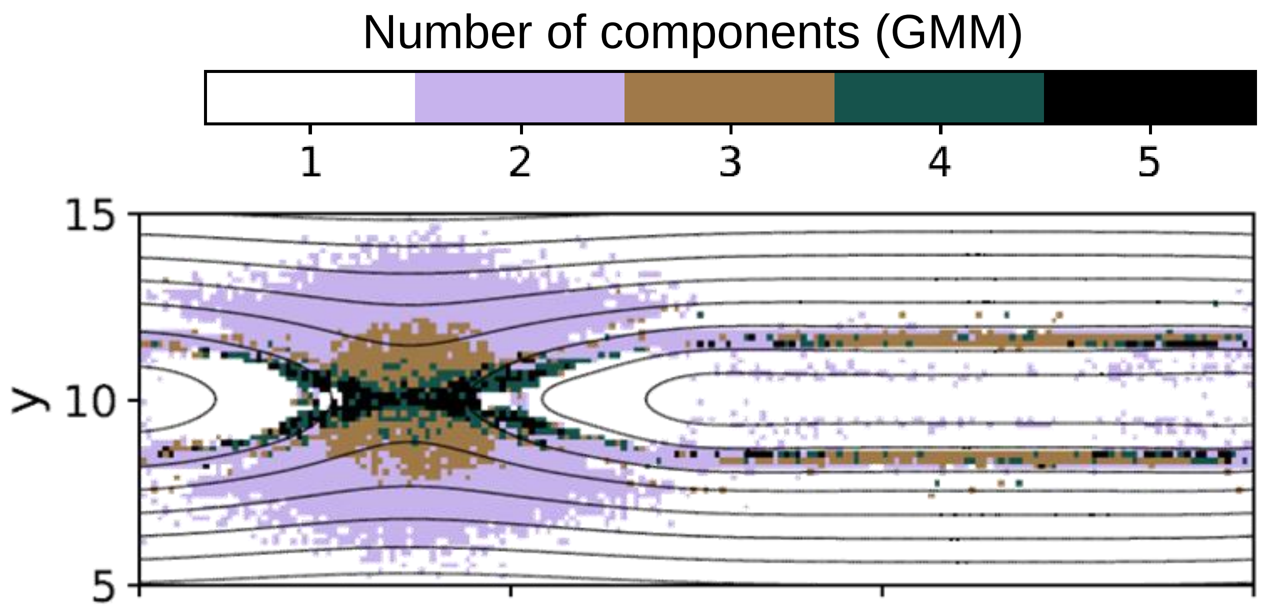
- **B-field-aligned basis**

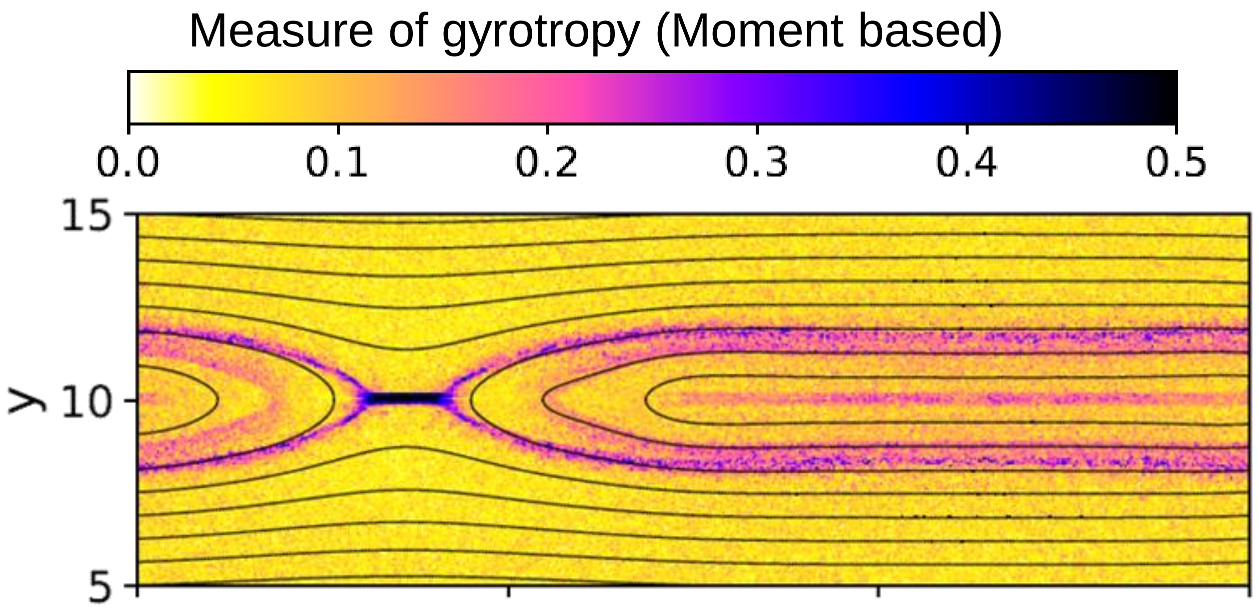
$$e_{\|} := \widehat{\boldsymbol{B}}, \text{ where } \widehat{\boldsymbol{B}} = \frac{\boldsymbol{B}}{\|\boldsymbol{B}\|}$$

$$\boldsymbol{e}_{\perp 1} := \widehat{\boldsymbol{B}} \times \boldsymbol{e_z}$$

$$\boldsymbol{e}_{\perp 2} := \widehat{\boldsymbol{B}} \times \boldsymbol{e}_{\perp 1} = -\widehat{B}^2 \boldsymbol{e_z} + (\boldsymbol{e_z}.\widehat{\boldsymbol{B}})\widehat{\boldsymbol{B}}$$

Number of components





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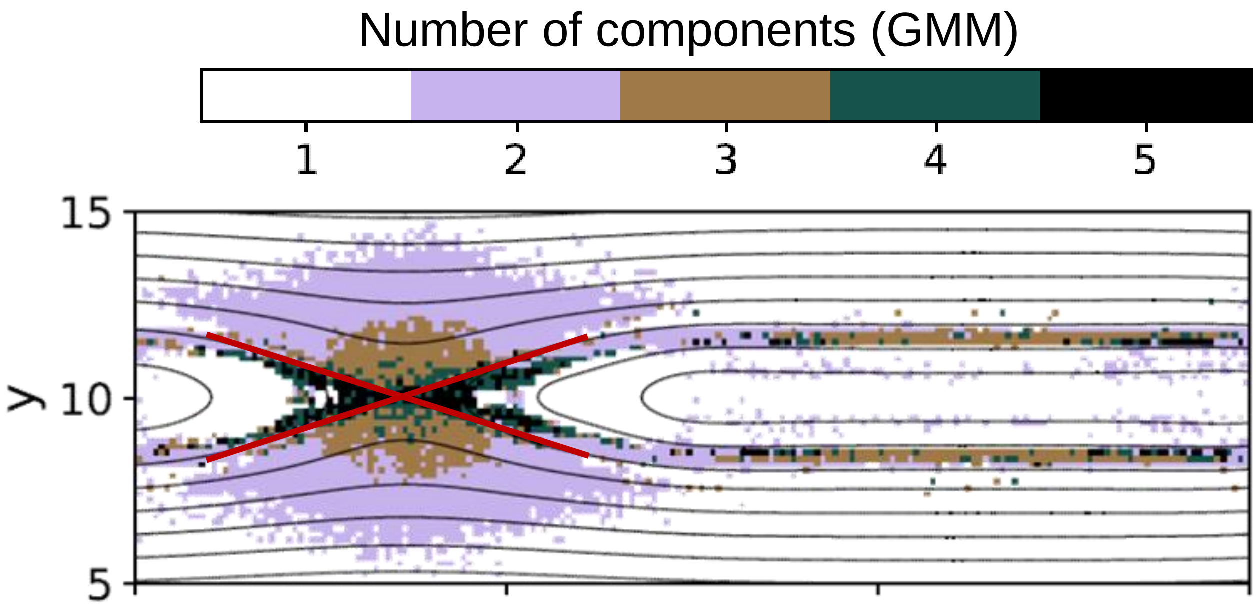

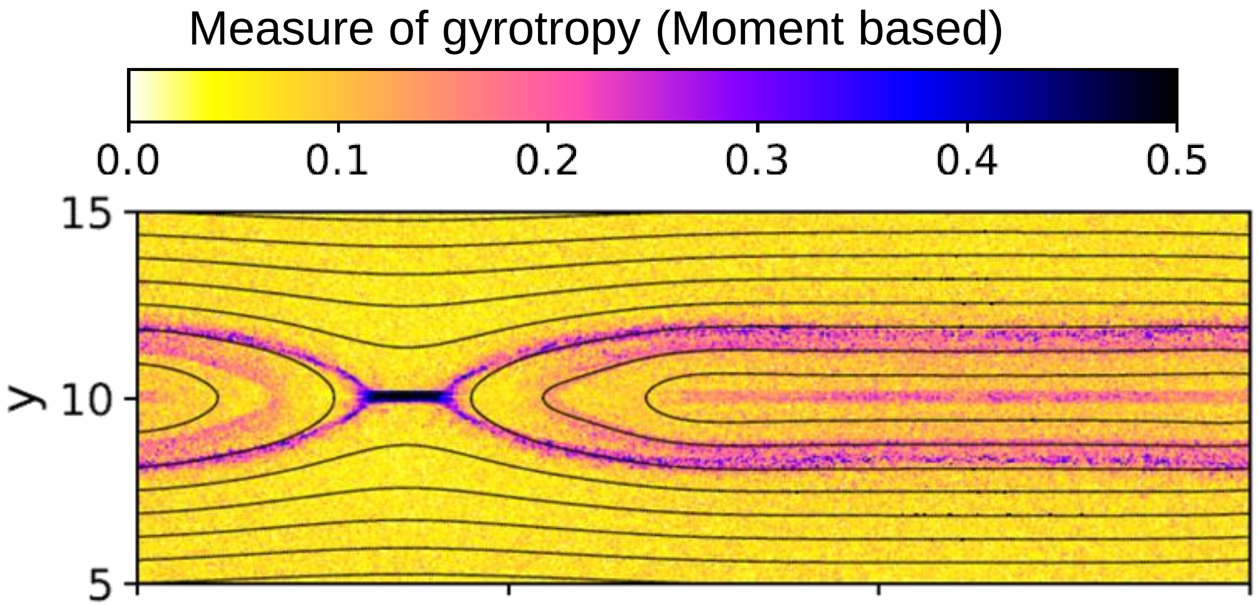

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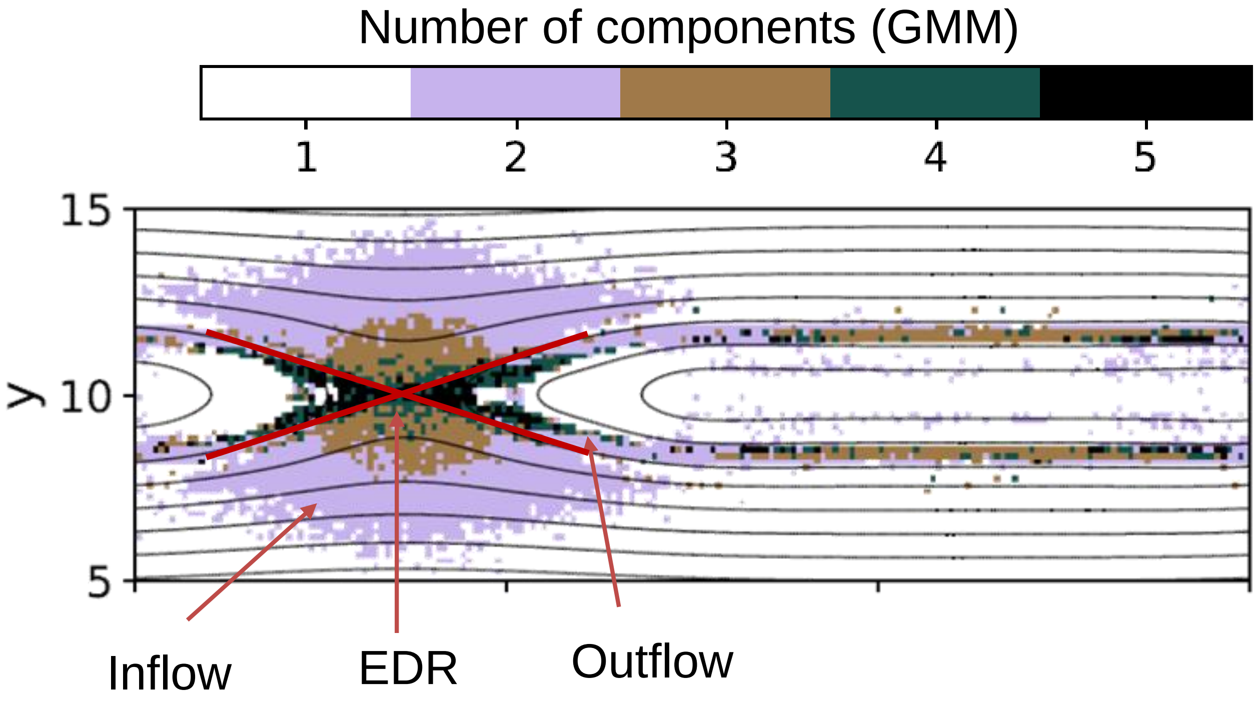

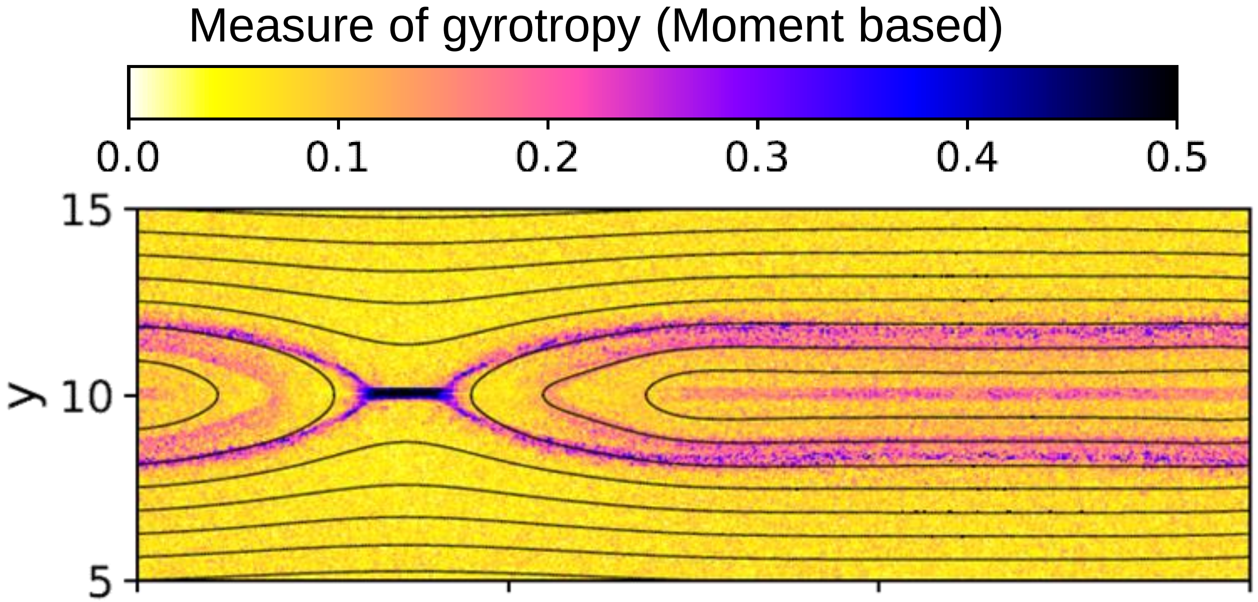

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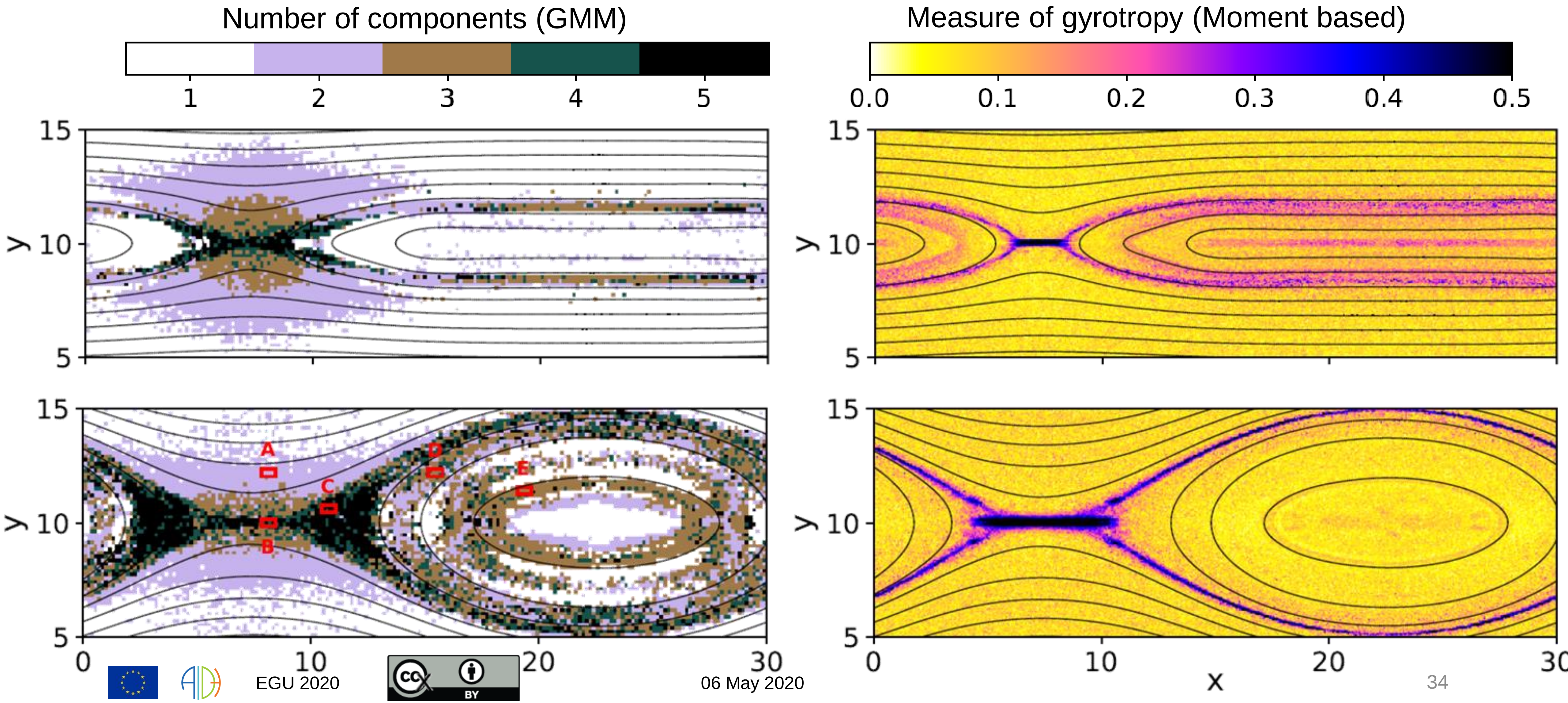

Analyzing the mixtures

- **Thermal energy of the distribution**

$$E_{thermal} = \frac{1}{N_p} \sum_{i=1}^{3} \left[\sum_p \left(\boldsymbol{V_p} - \langle \boldsymbol{V_p} \rangle \right)^2 \right]_i , \text{ with } \langle \boldsymbol{V_p} \rangle = \sum_p \frac{\boldsymbol{V_p}}{N_p}$$

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- **Variance of the mixture:**

$$(\sigma^2)^{(K)} = \sum_{i=1}^{3} \left[\sum_{k=1}^{K} w_k^2 \left(\boldsymbol{\sigma_k} \right)^2 + \sum_{k=1}^{K} w_k \left(\boldsymbol{\mu_k} \right)^2 - \left(\sum_{k=1}^{K} w_k (\boldsymbol{\mu_k}) \right)^2 \right]_i$$

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Analyzing the mixtures: energy drop and deviation

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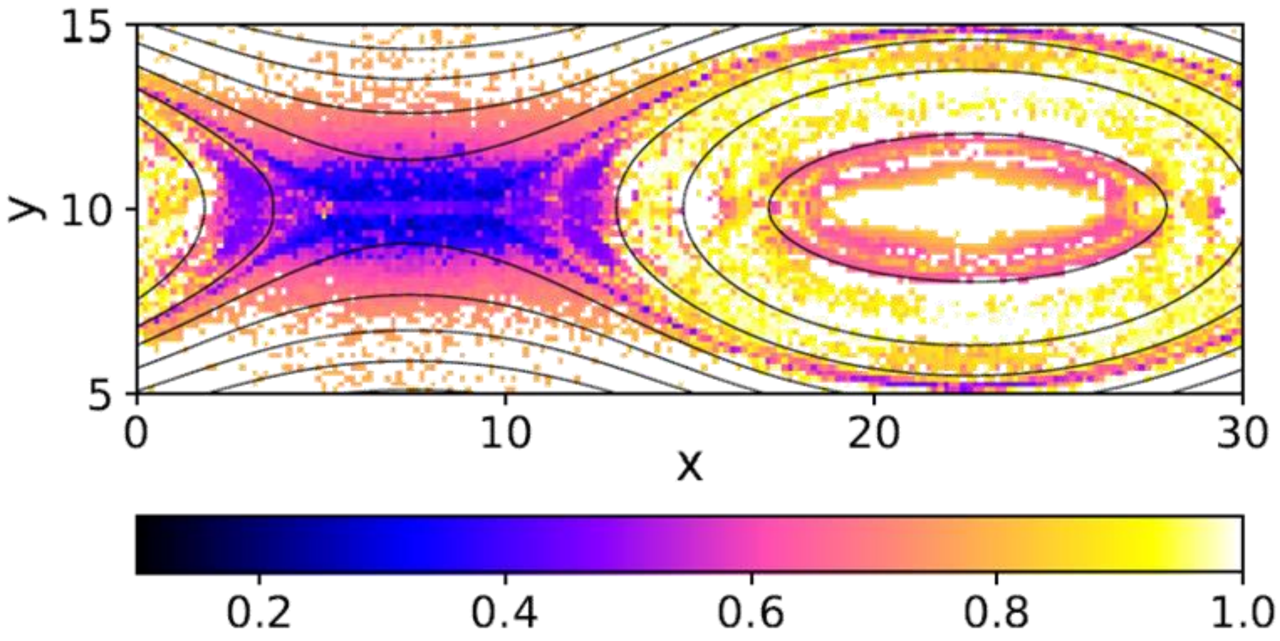
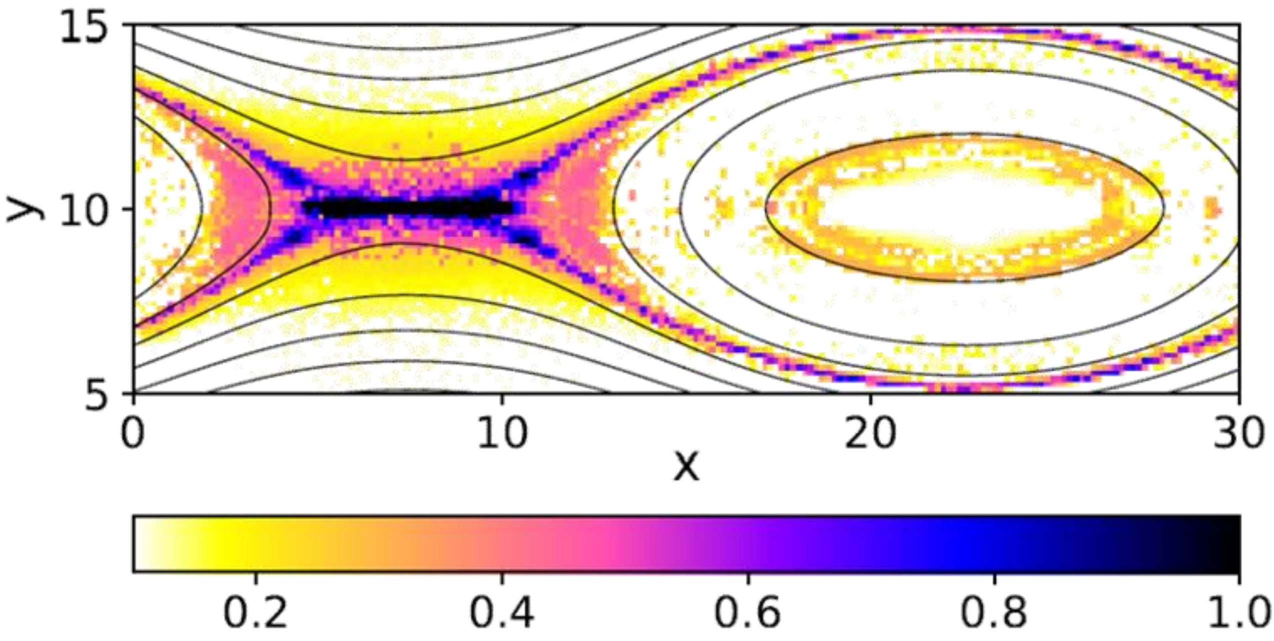
- **Diagnostic quantities**

$$E_{drop} = \frac{E^{(K)}_{thermal}}{E_{thermal}} \qquad E_{dev} = \frac{E^{(K)}_{dev}}{E_{thermal}}$$

Analyzing the mixtures: energy drop and deviation





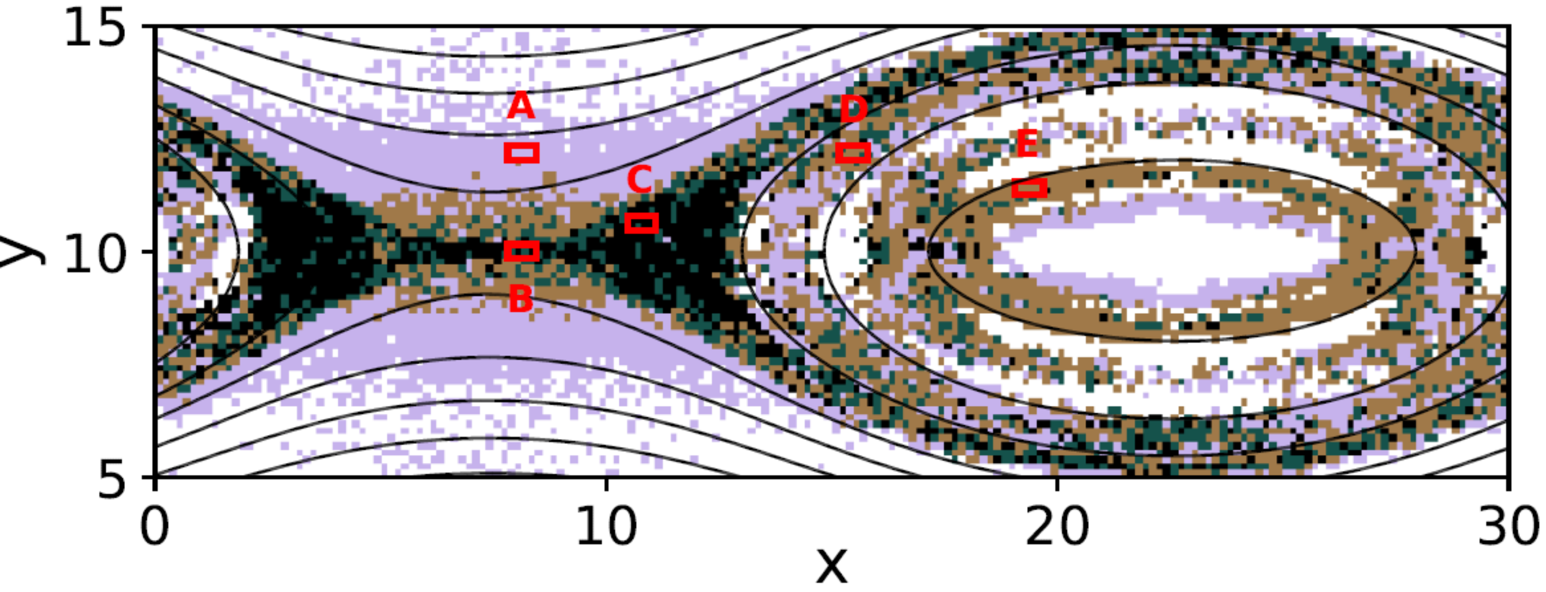
$$E_{drop} = \frac{E^{(K)}_{thermal}}{E_{thermal}}$$

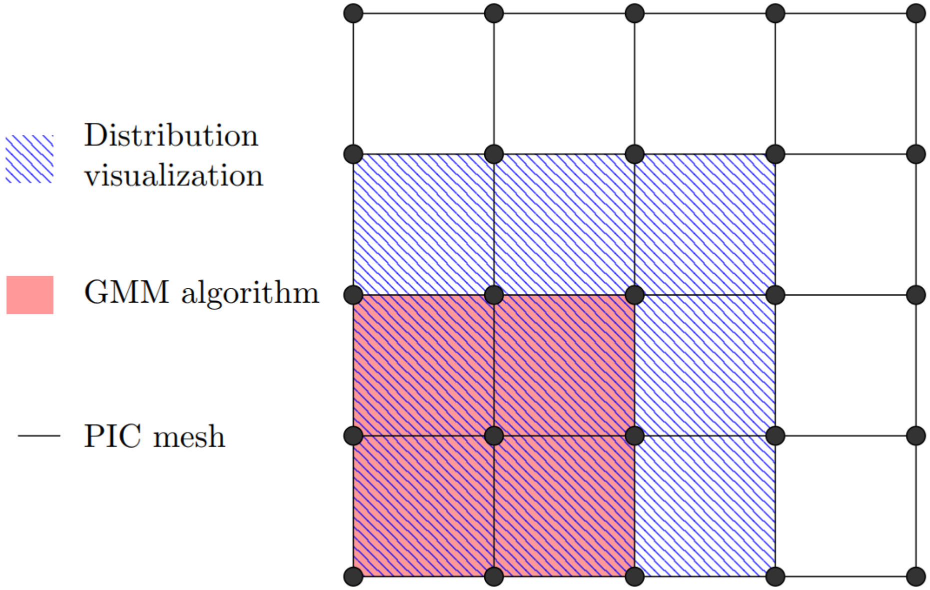
$$E^{(K)}_{thermal} = \frac{1}{2} \sum_{i=1}^{3} \sum_{k=1}^{K} w_k^2 \left[\boldsymbol{\sigma}_{\boldsymbol{k}}^2\right]_i$$

$$E_{dev} = \frac{E^{(K)}_{dev}}{E_{thermal}}$$

$$E^{(K)}_{dev} = \sum_{i=1}^{3} \left[\sum_{k=1}^{K} w_k \left(\boldsymbol{\mu}_{\boldsymbol{k}}\right)^2 - \left(\sum_{k=1}^{K} w_k (\boldsymbol{\mu}_{\boldsymbol{k}}) \right)^2 \right]_i$$

Closer look to the distributions





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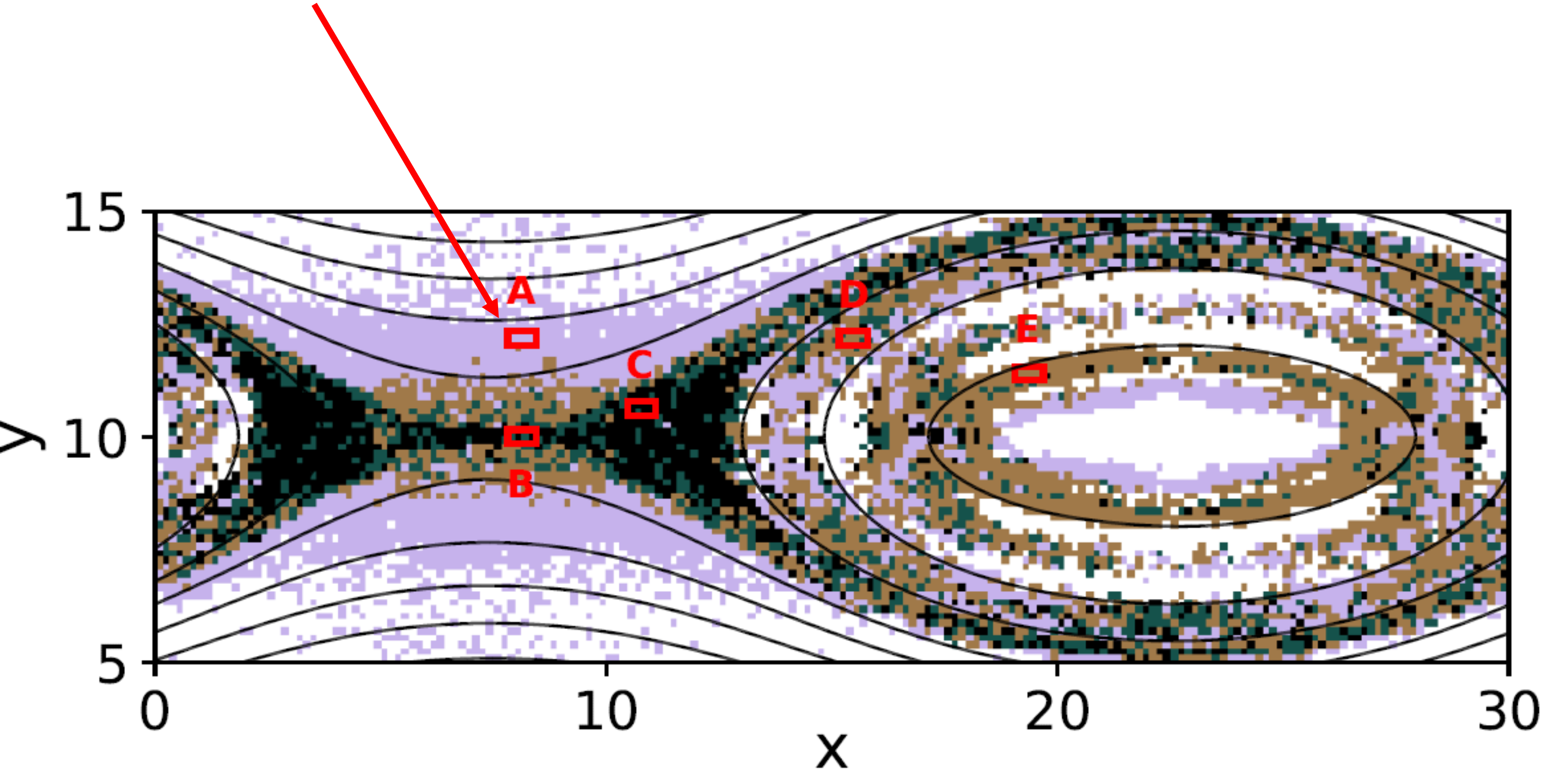

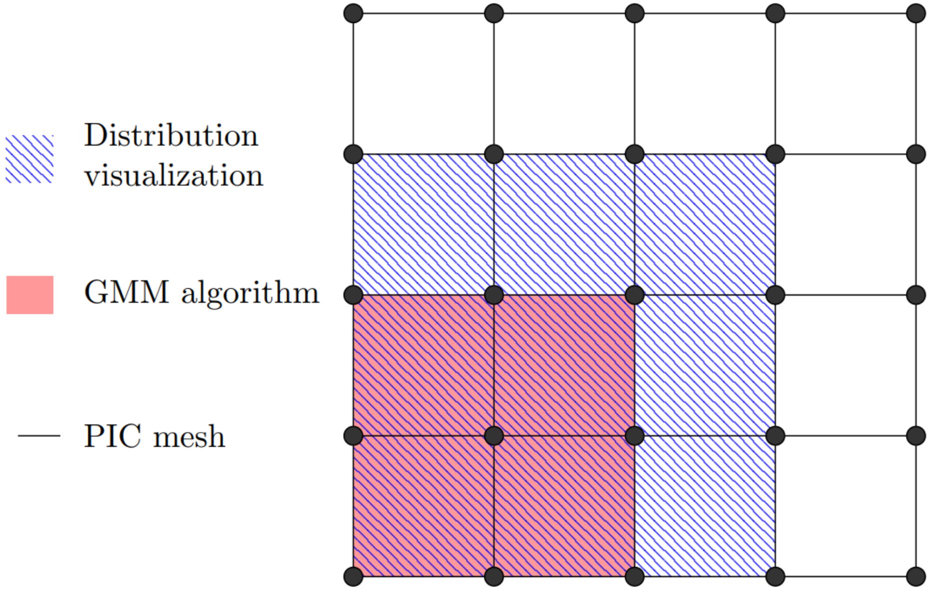

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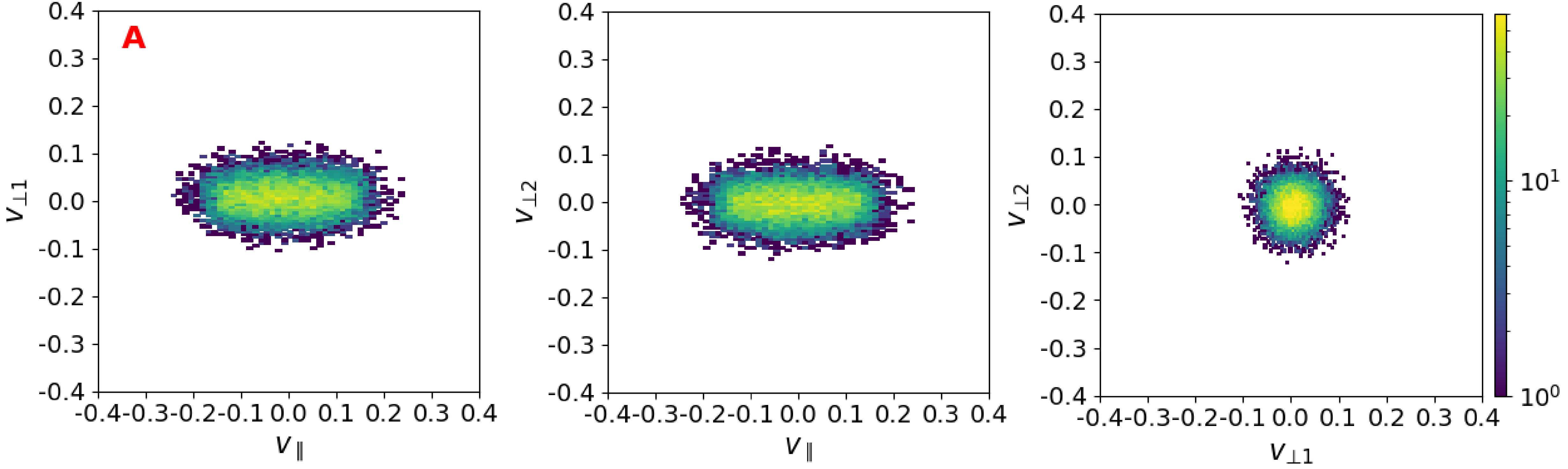

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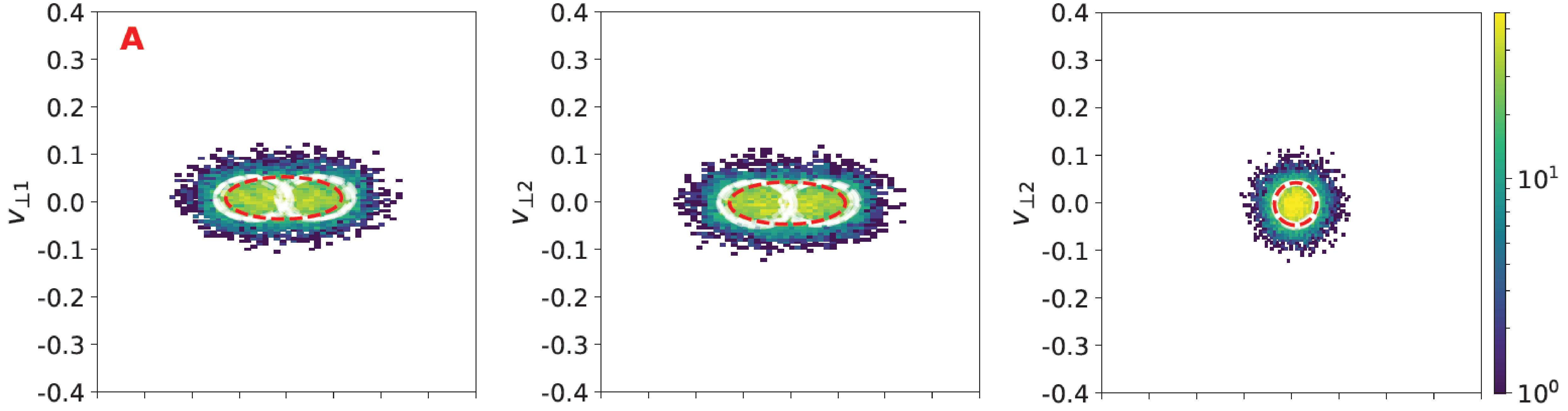

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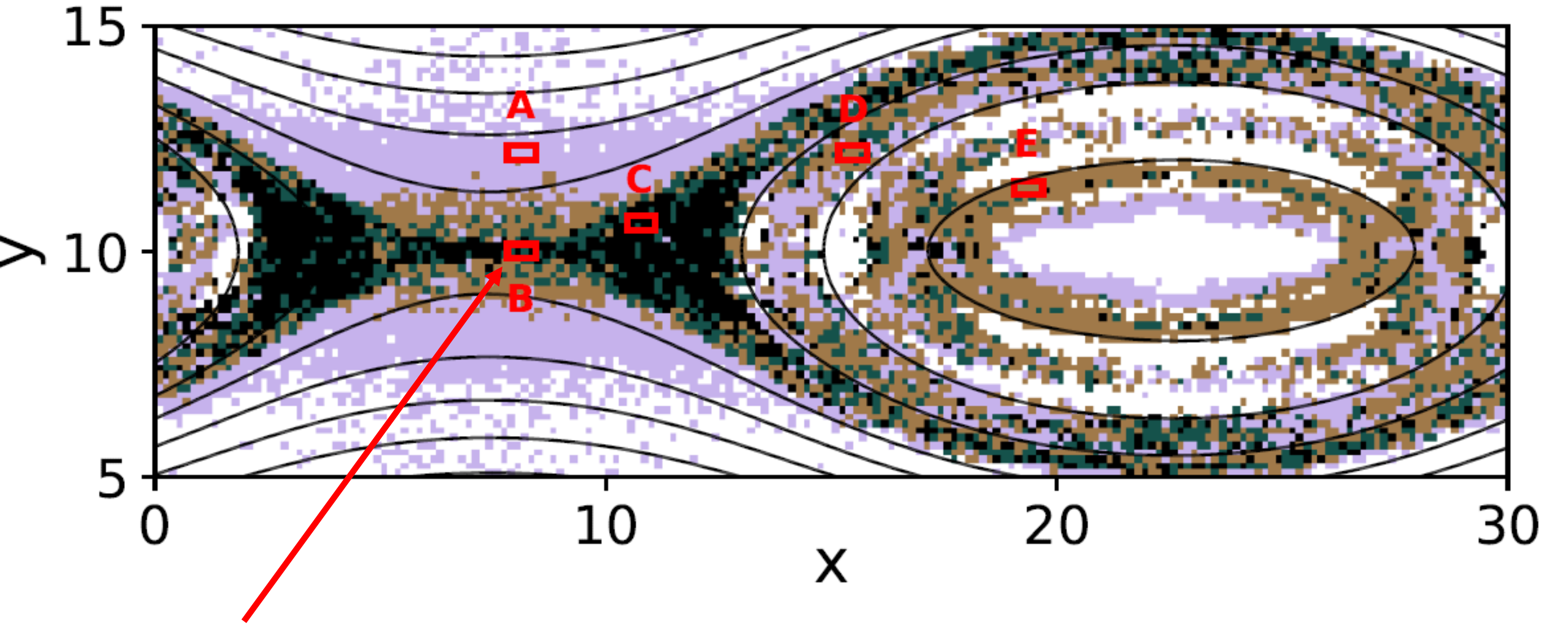

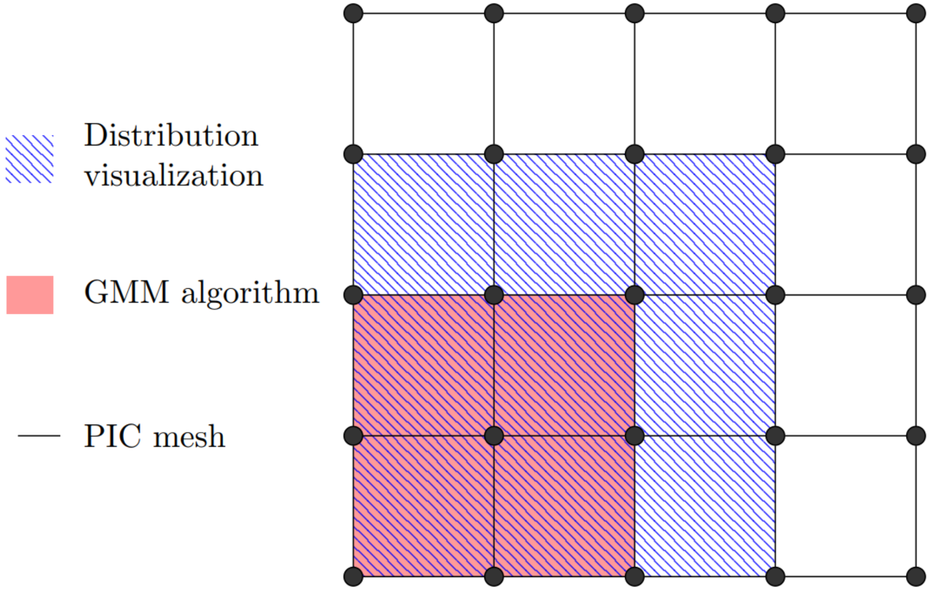

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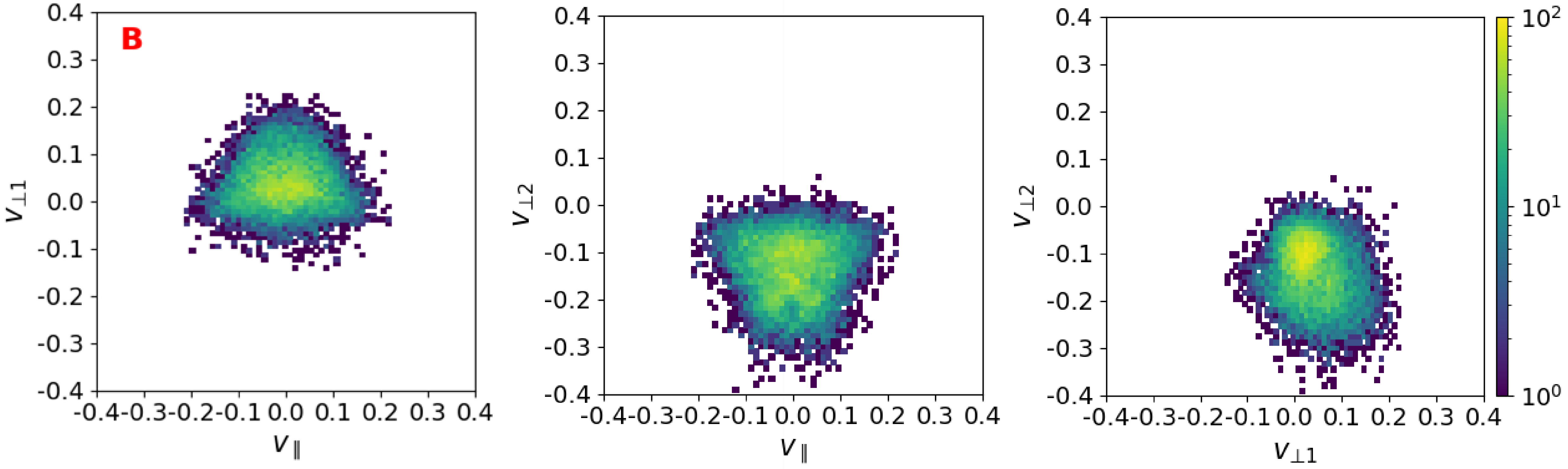

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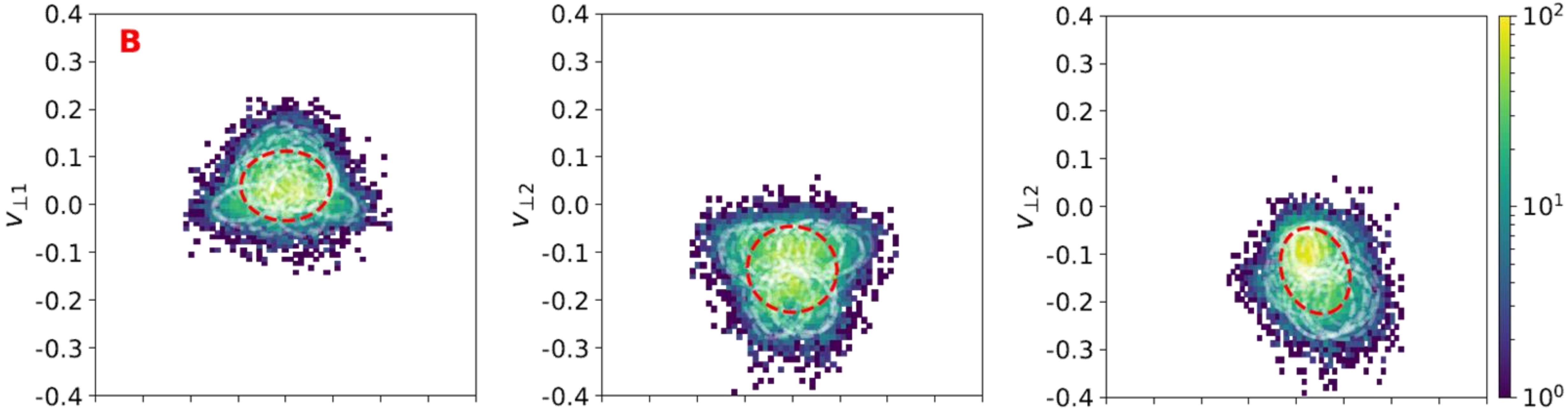

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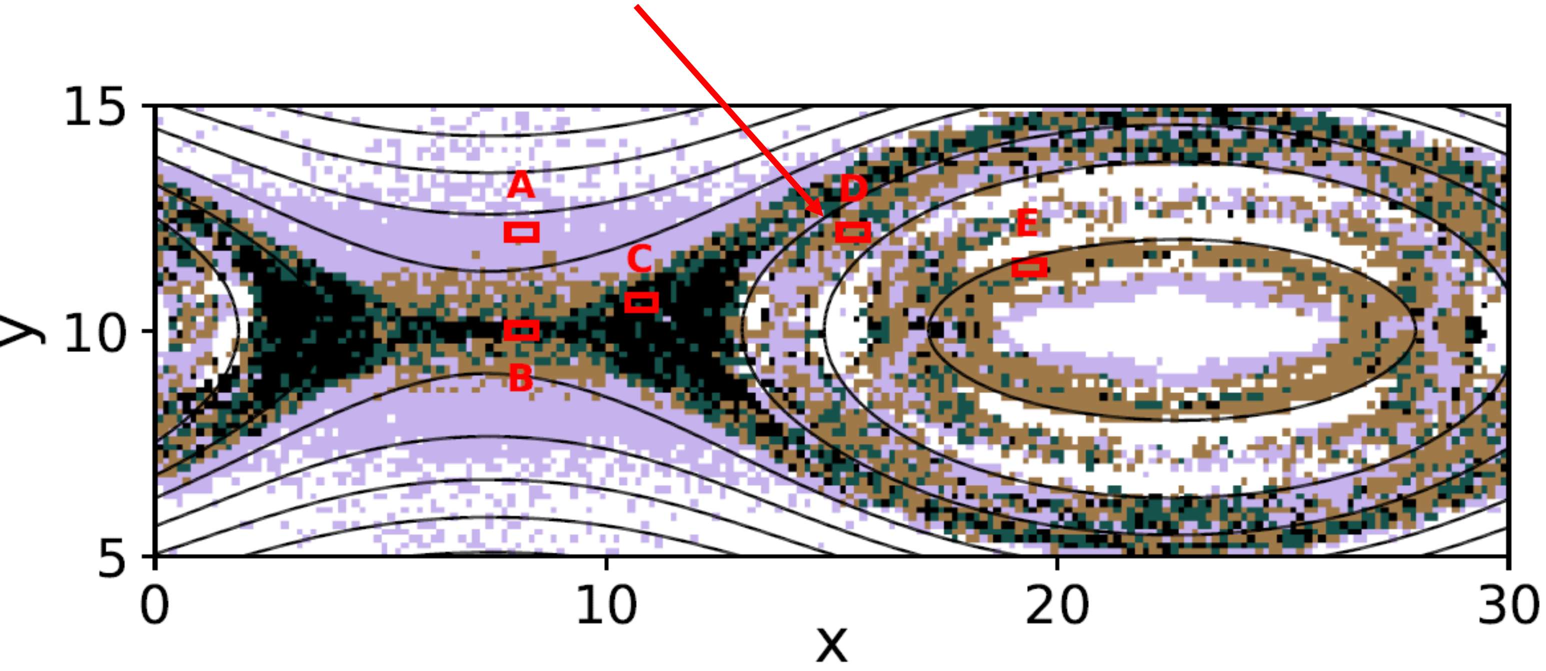

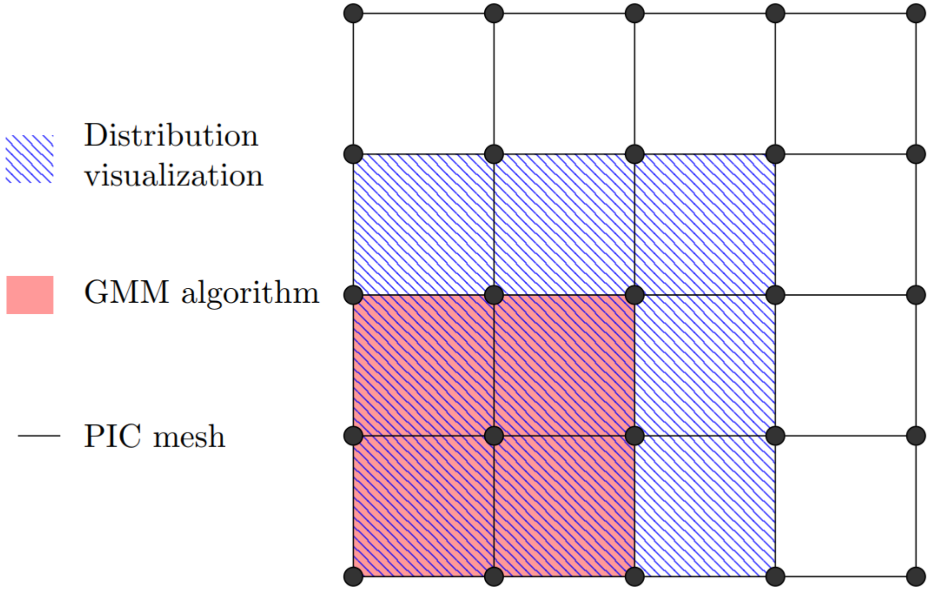

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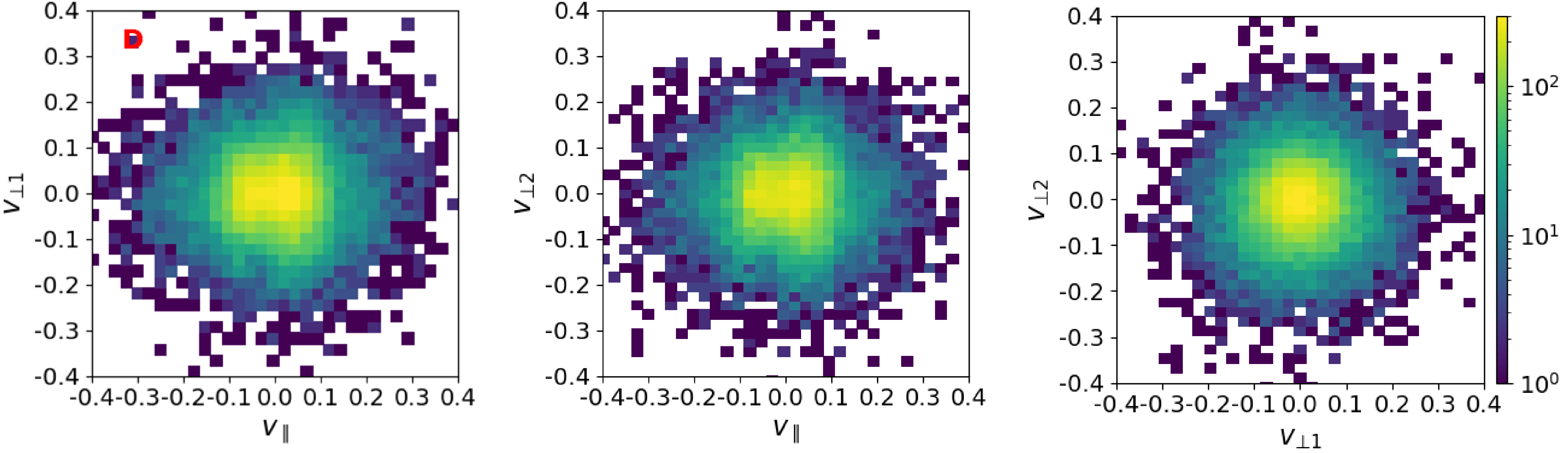

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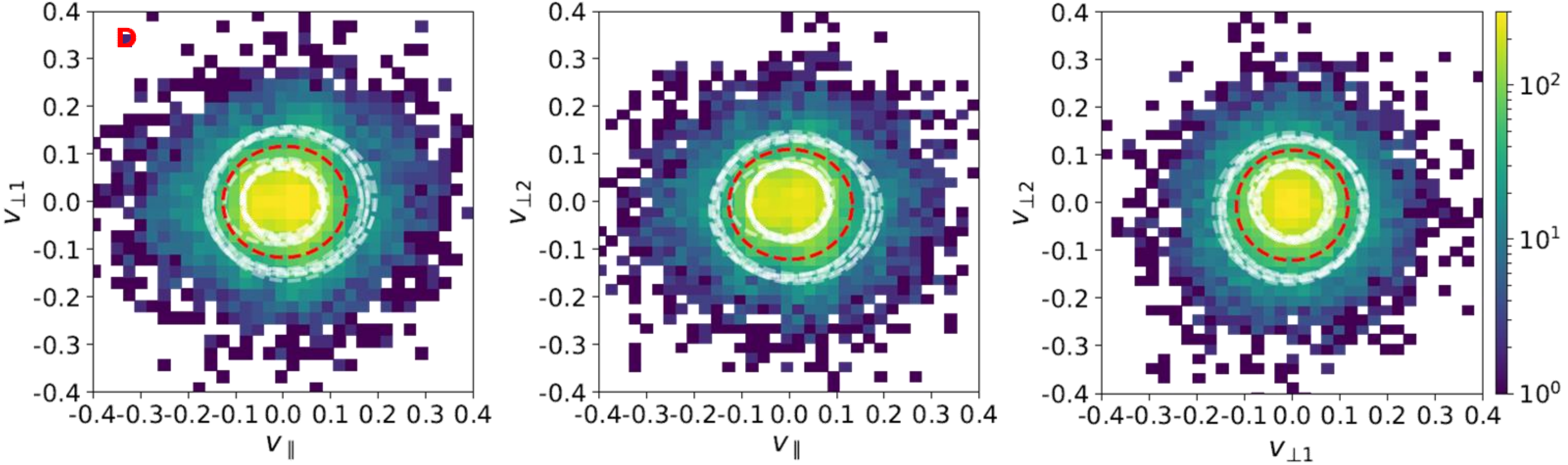

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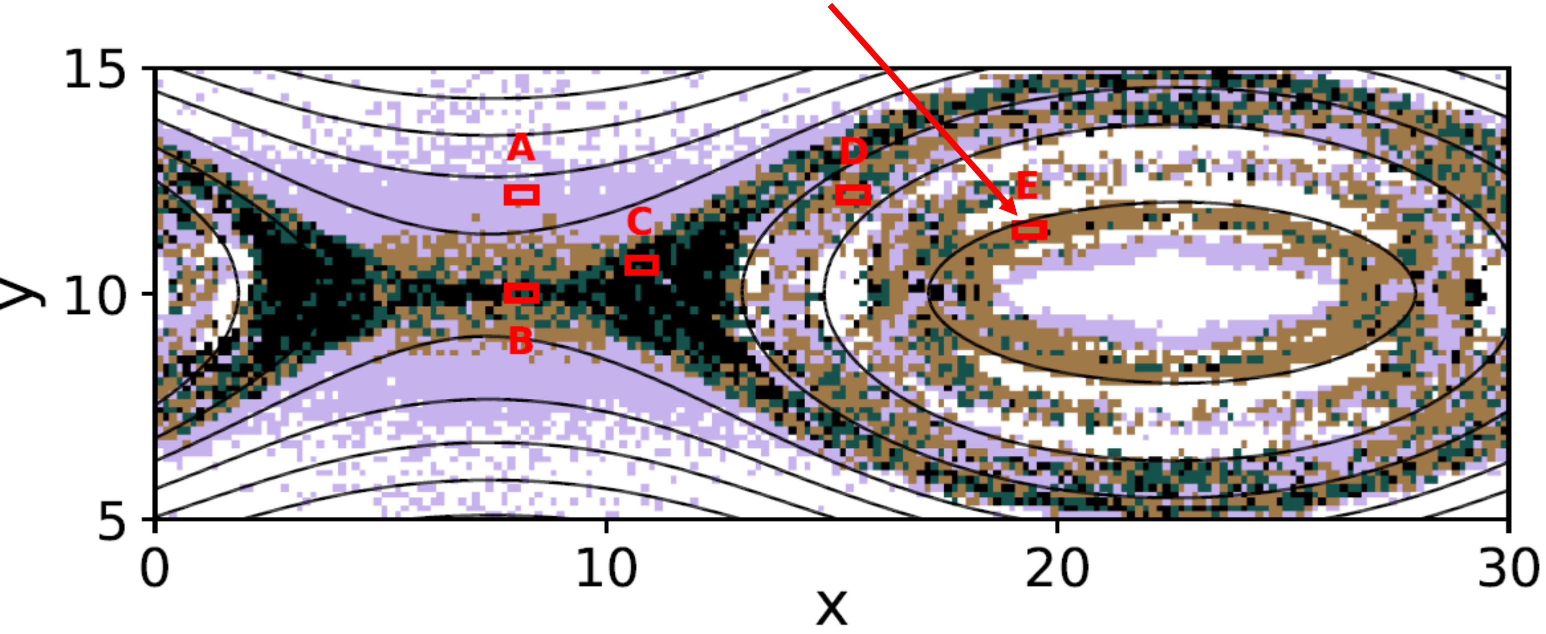
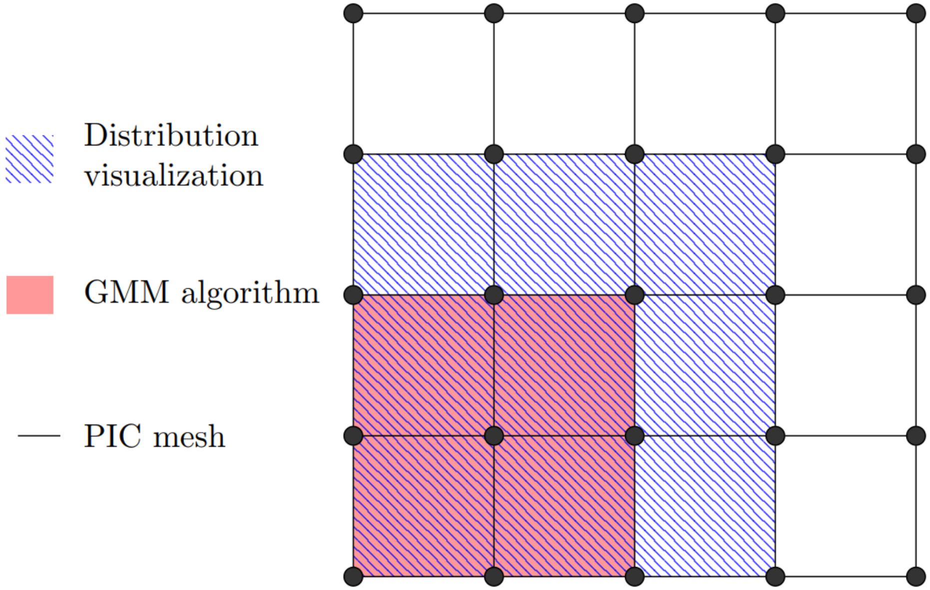
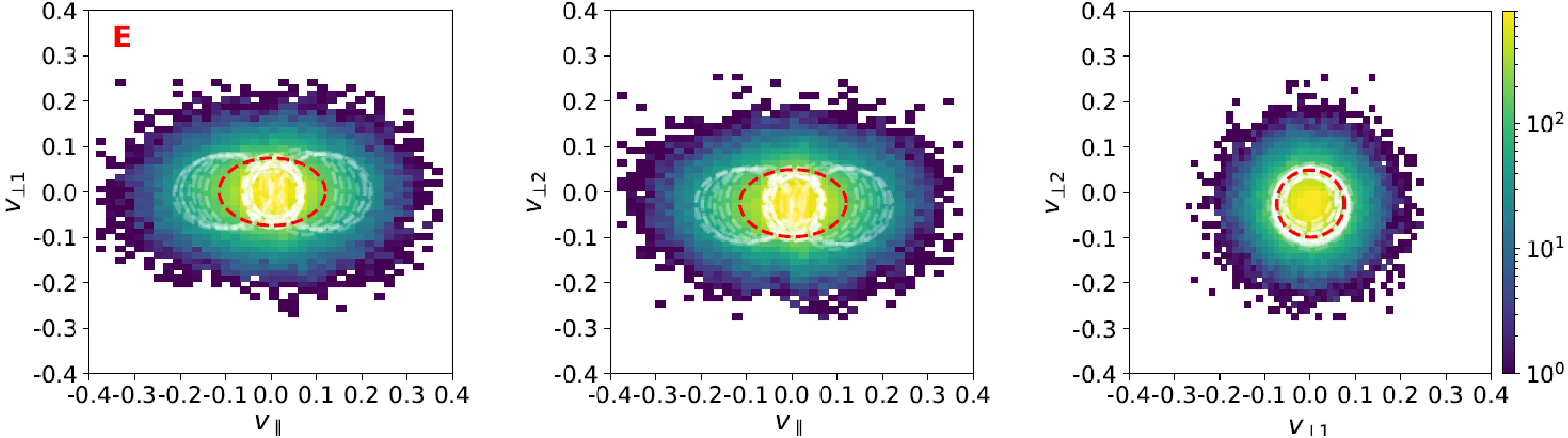




Closer look to the distributions



Next steps

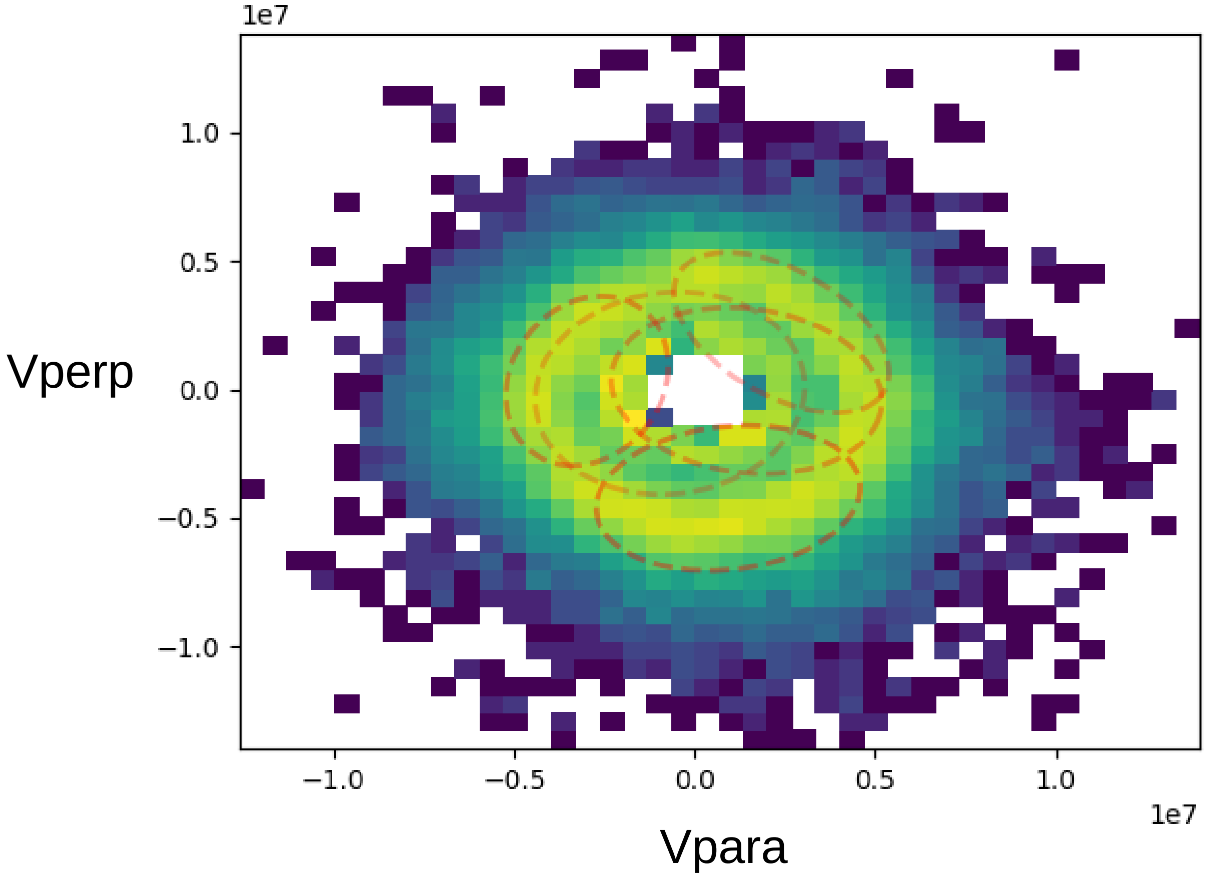
- **Simulation: focus on less documented cases**
  - 2D turbulent reconnection
  - 3D reconnection
  - Other kind of simulations

- **Observation: distributions from in situ space missions**
  - MMS data
  - Reconstruction of the particle sampling

- **Machine learning: potential improvement**
  - Convolutional methods and auto encoder
  - Dual problem with kernel method

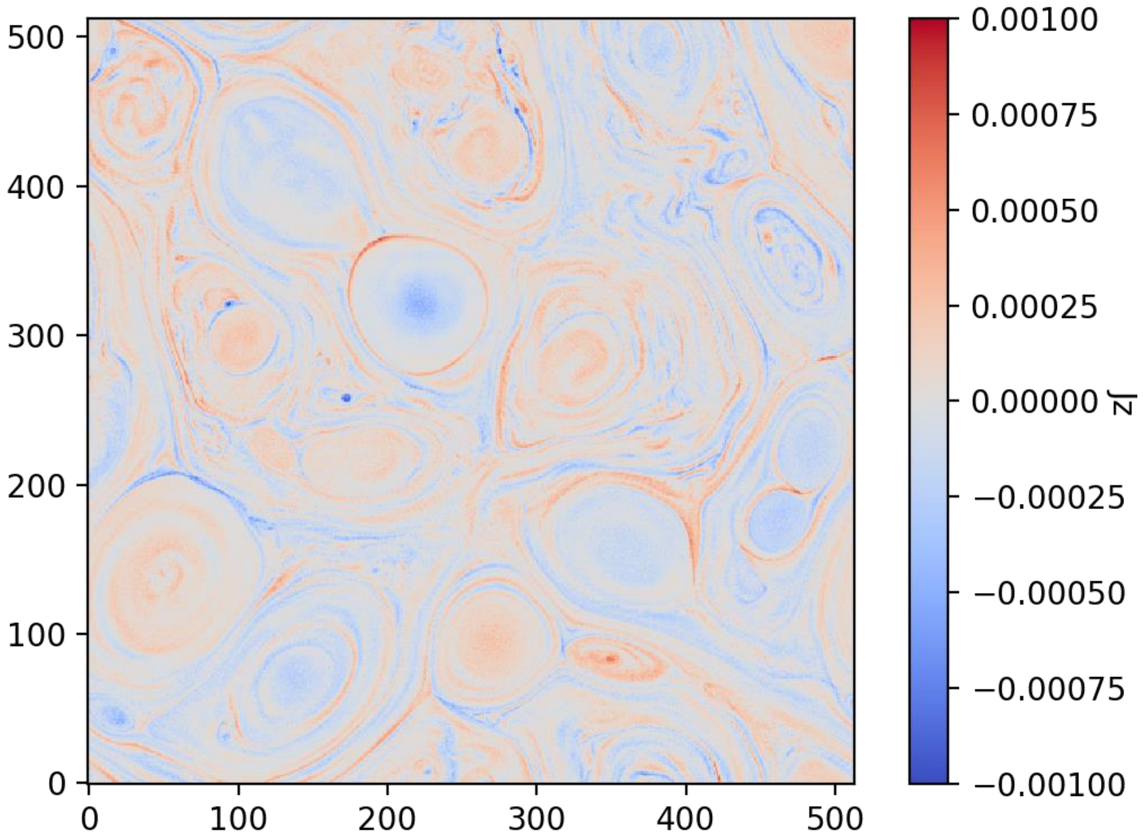
MMS observation: day side 2015

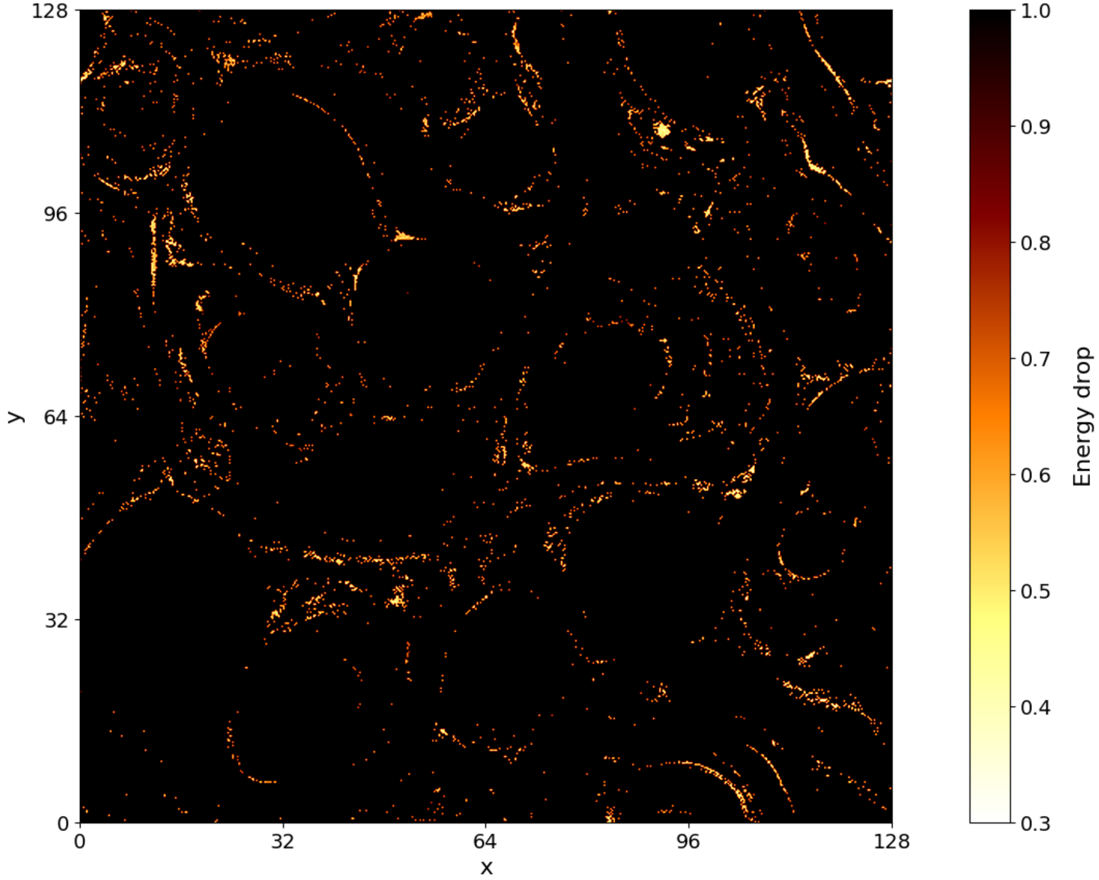
- **Reconnection event: 16 October 2015-13:07:02.235 (Burch et al.)**
  - Gap due to low energy channels
  - Complex preprocessing pipeline
  - Identify crescent shape
- **Auto encoder may help (**Olshevsky et al)
  - Extract 3D features
  - Use the PDF directly



Turbulent simulation

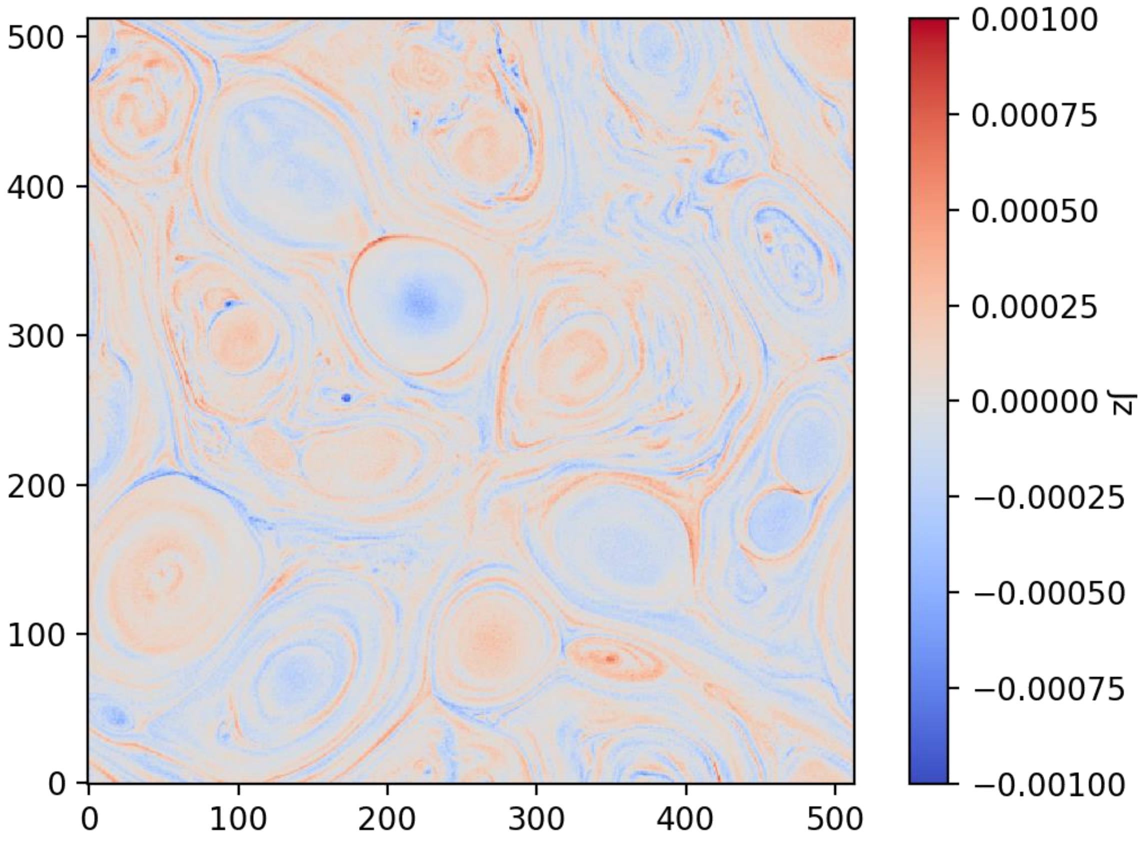
- **Identification of potential current layers**

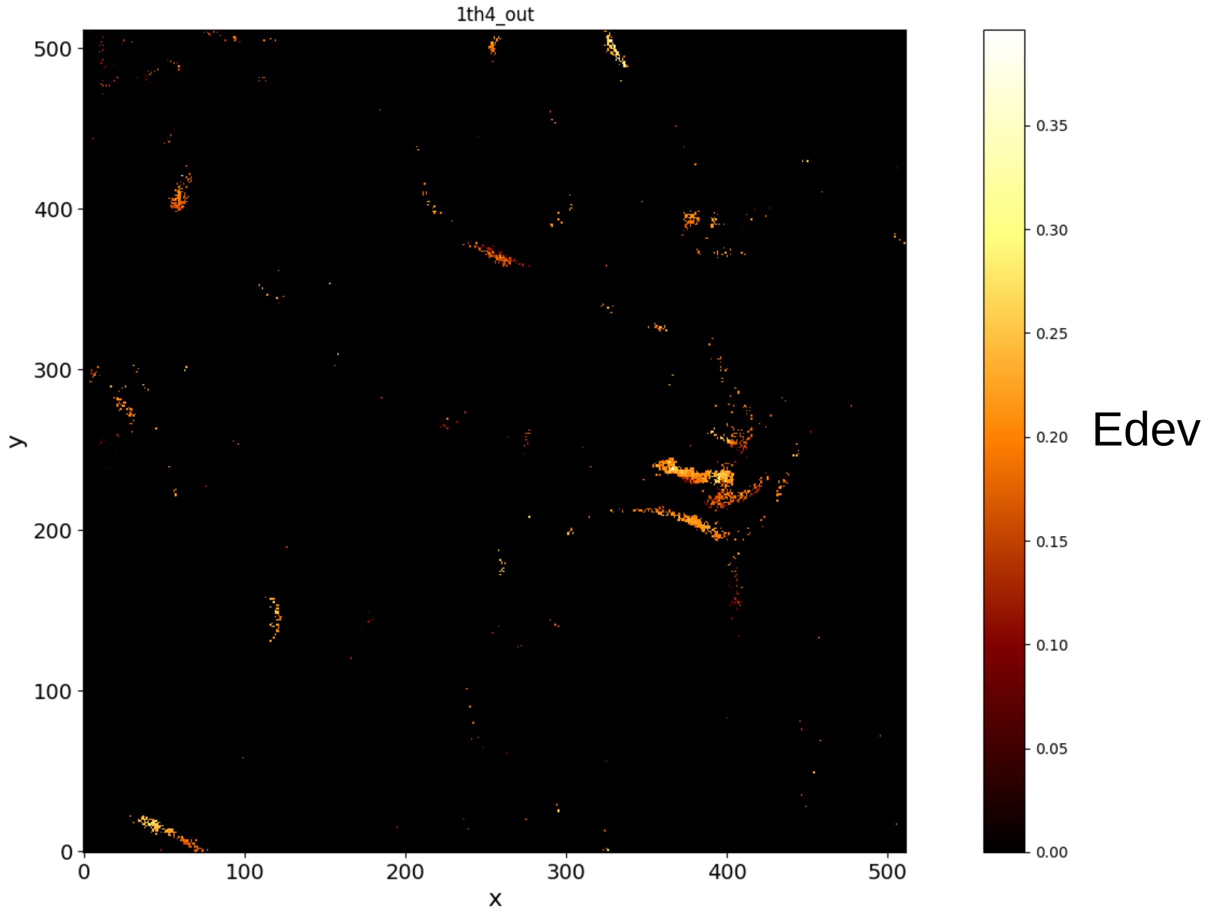




Turbulent simulation

- **Identification of potential current layers**





Conclusion

- **GMM is able to identify complex distributions in various cases**
  - Reconnection with weak and strong guide fields
  - Help to analyze and identify reconnection
  - No real physical interpretation and no unique solution

- **GMM as a start in applying ML on simulations and particles**
  - Other ML methods are considered
  - Auto encoder, SOM, etc.

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Thank you for your attention

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Gaussian Mixture Model

- **How to infer the best parameters ?**

$$l(\phi|\boldsymbol{X},\boldsymbol{Z}) = \sum_{i=1}^{n} \ln\left[\sum_{k=1}^{K} w_k \mathcal{N}(\boldsymbol{x_i}|\boldsymbol{\theta_k})\right]$$

- **Maximizing the likelihood estimation**
  - Non linear maximization problem
  - No closed form

$$\frac{\partial l}{\partial \boldsymbol{\mu_k}} = 0 \qquad \frac{\partial l}{\partial \boldsymbol{\Sigma_k}} = 0 \qquad \frac{\partial l}{\partial w_k} = 0 \;\; s.t. \sum_{k=1}^{K} w_k = 1$$

- **Need to find numerical local maximum: Expectation Maximization**

EM Algorithm

$$\gamma(z_k) = \frac{w_k \mathcal{N}(\boldsymbol{x}|\boldsymbol{\theta_k})}{\sum_{j=1}^{K} w_j \mathcal{N}(\boldsymbol{x}|\boldsymbol{\theta_j})}.$$

Expectation Step

$$\boldsymbol{\mu_k} = \frac{\sum\limits_{i=1}^{n} \gamma(z_{ik})\boldsymbol{x_i}}{\sum\limits_{i=1}^{n} \gamma(z_{ik})}, \ \forall k \in [1, \cdots, K],$$

$$\boldsymbol{\Sigma_k} = \frac{\sum\limits_{i=1}^{N} \gamma(z_{ik})(\boldsymbol{x_i} - \boldsymbol{\mu_k})(\boldsymbol{x_i} - \boldsymbol{\mu_k})^T}{\sum\limits_{i=1}^{N} \gamma(z_{ik})}, \ \forall k \in [1, \cdots, K]$$

$$w_k = \frac{1}{n}\sum_{i=1}^{n} \gamma(z_{ik}), \ \forall k \in [1, \cdots, K].$$

Maximization Step

Detection algorithm

Data: particles coordinates $\boldsymbol{X}$ and velocities $\boldsymbol{V}$, mesh Ω with n_x cells by n_y cells
Result: distribution function f_i

begin
  merge cells to increase the number of particles by subset, new mesh Ω' is of size $\frac{n_x}{r} \times \frac{n_y}{r}$
  for $i \in \Omega'$ **do**
    for $k = 1$ *to* $k = 6$ **do**
      $GMM_k = GMM(\mathbf{V_i}; k)$;
      $BIC_k = BIC(GMM_k)$;
    end
    $K = \arg\max_{k \in [1,6]} BIC_k$;
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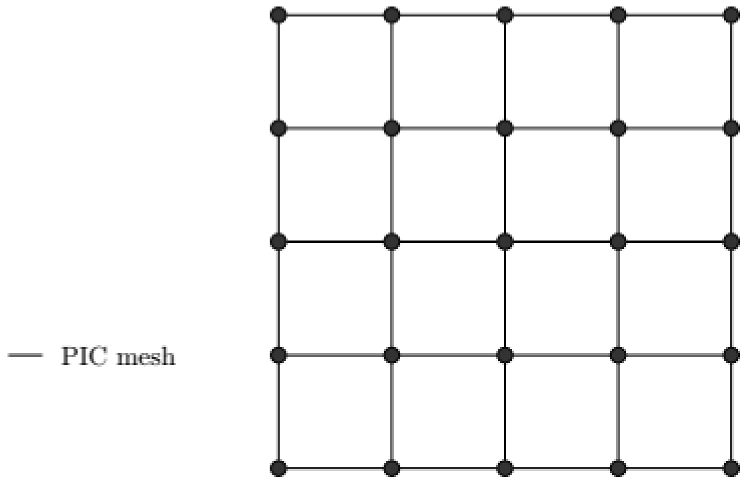

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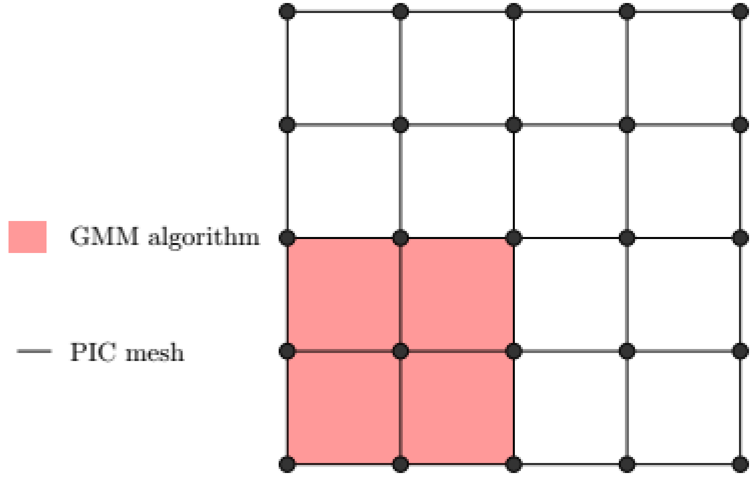

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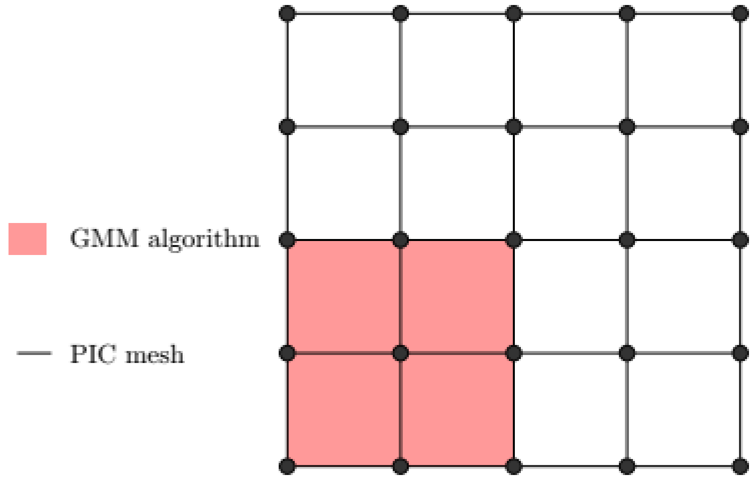

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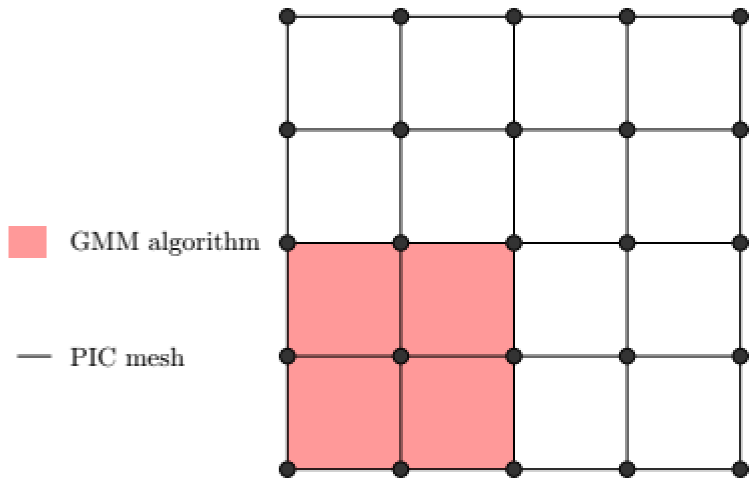

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