Can stochastic resonance explain the amplification of planetary tidal forcing?

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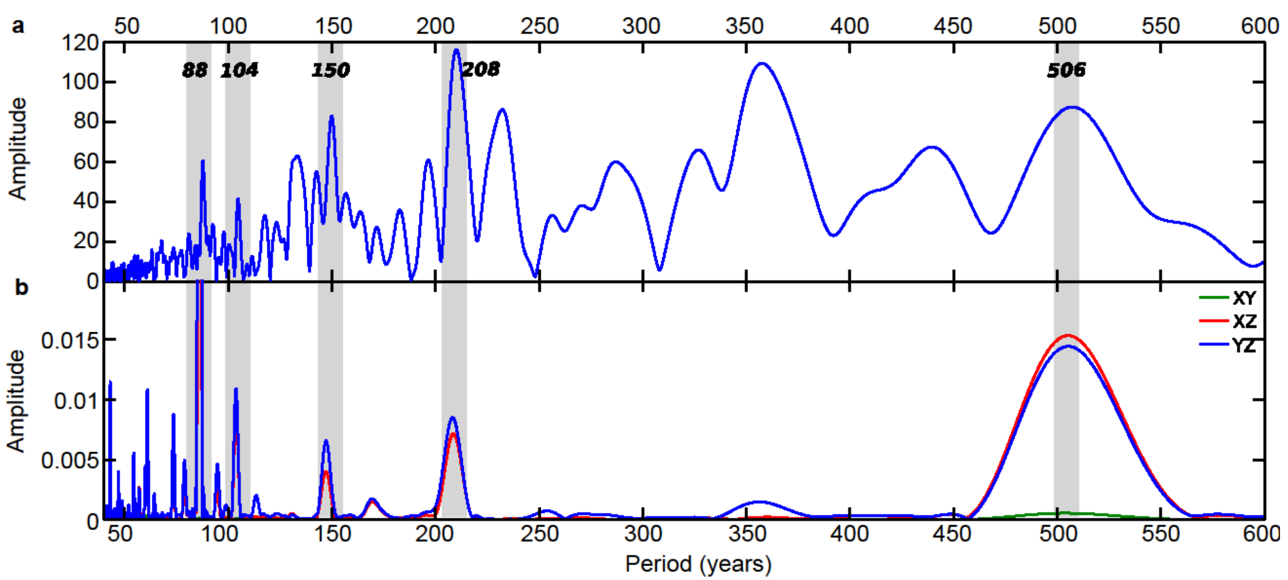
Long-period cycles in solar activity

Gleissberg de Vries



Solar activity from C^{14} and Be^{10} proxies.

Planetary torque

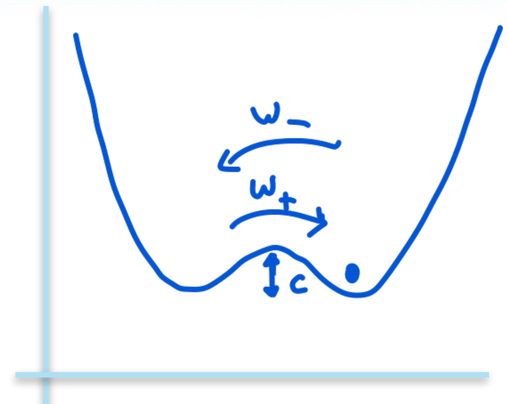
Hypothesis of planetary influence

Abreu et al. 2012

Stochastic Resonance

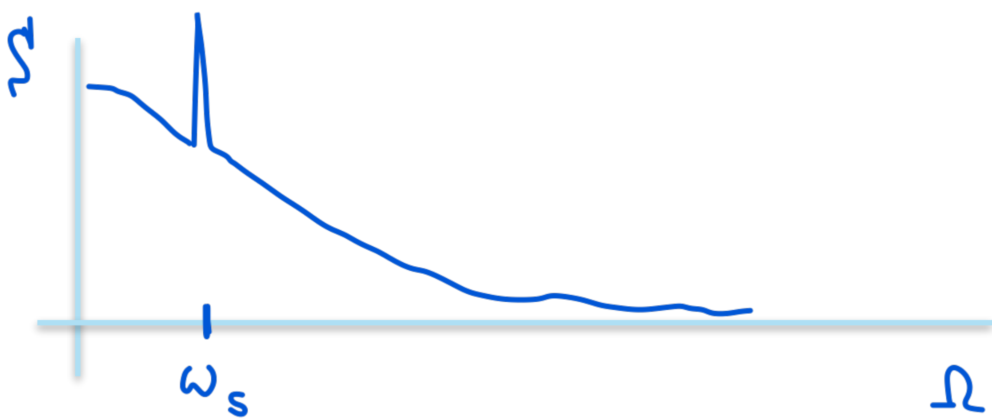
Transition probabilities:

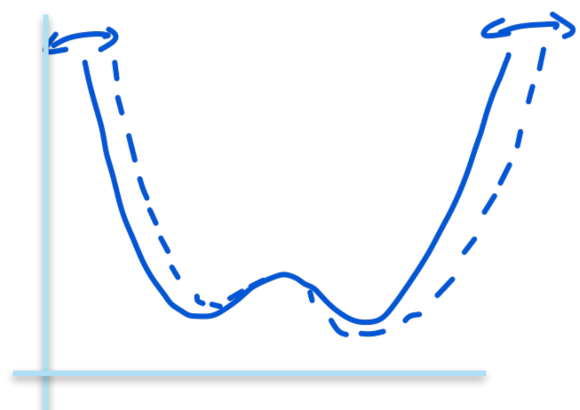
$$W_{\pm}(t) = \frac{1}{2}(\alpha_0 \mp \alpha_1 \cos \omega_S t)$$



Output power spectrum:

$$S(\Omega) = \left(1 - \frac{\alpha_1^2}{2(\alpha_0^2 + \omega_S^2)}\right)\left(\frac{4c^2\alpha_0}{\alpha_0^2 + \Omega^2}\right) + \frac{\pi c^2 \alpha_1^2}{\alpha_0^2 + \omega_S^2}\delta(\Omega - \omega_S)$$





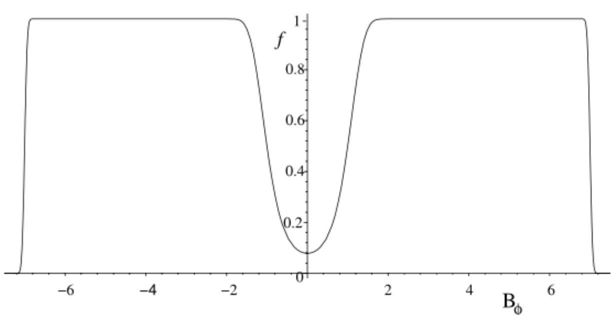
Archetypical example of a particle, subject to random fluctuations, in a double-well potential weakly modulated by an external force.

McNamara Wiesenfeld 1989

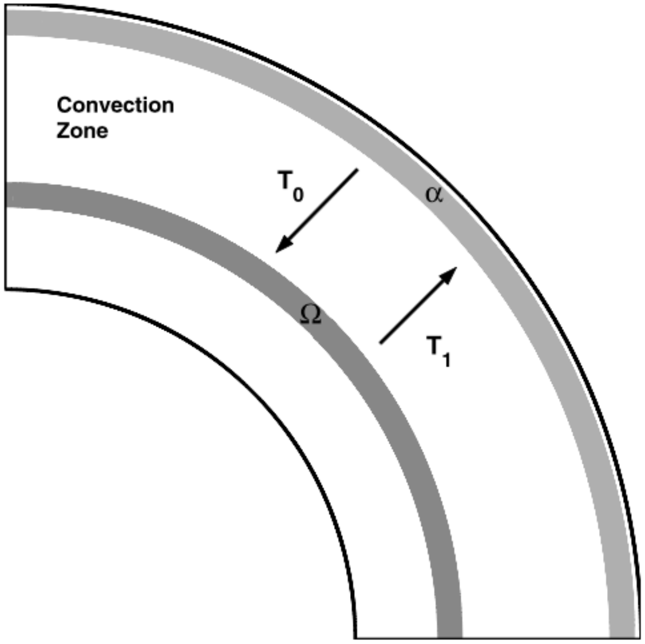
Babcock-Leighton-type dynamo models

$$\dot{B}(t) = -\frac{\omega}{L} A(t - T_0) - \tau^{-1} B(t),$$
$$\dot{A}(t) = \alpha_0 f(B(t - T_1)) B(t - T_1) - \tau^{-1} A(t),$$

$$\Rightarrow \boxed{\tau^2 \ddot{B}(t) + 2\tau \dot{B}(t) + B(t) = -N f(B(t - T)) B(t - T)},$$
$$N = \frac{\omega \alpha_0 \tau^2}{L}, \quad T = T_0 + T_1,$$
$$f(B) = \frac{1}{4}\left(1 + \mathrm{erf}\left(B^2 - B_{min}^2\right)\right)\left(1 - \mathrm{erf}\left(B^2 - B_{\max}^2\right)\right).$$



The Alpha effect is assumed to be limited to $B_{min} \lesssim B \lesssim B_{max}$

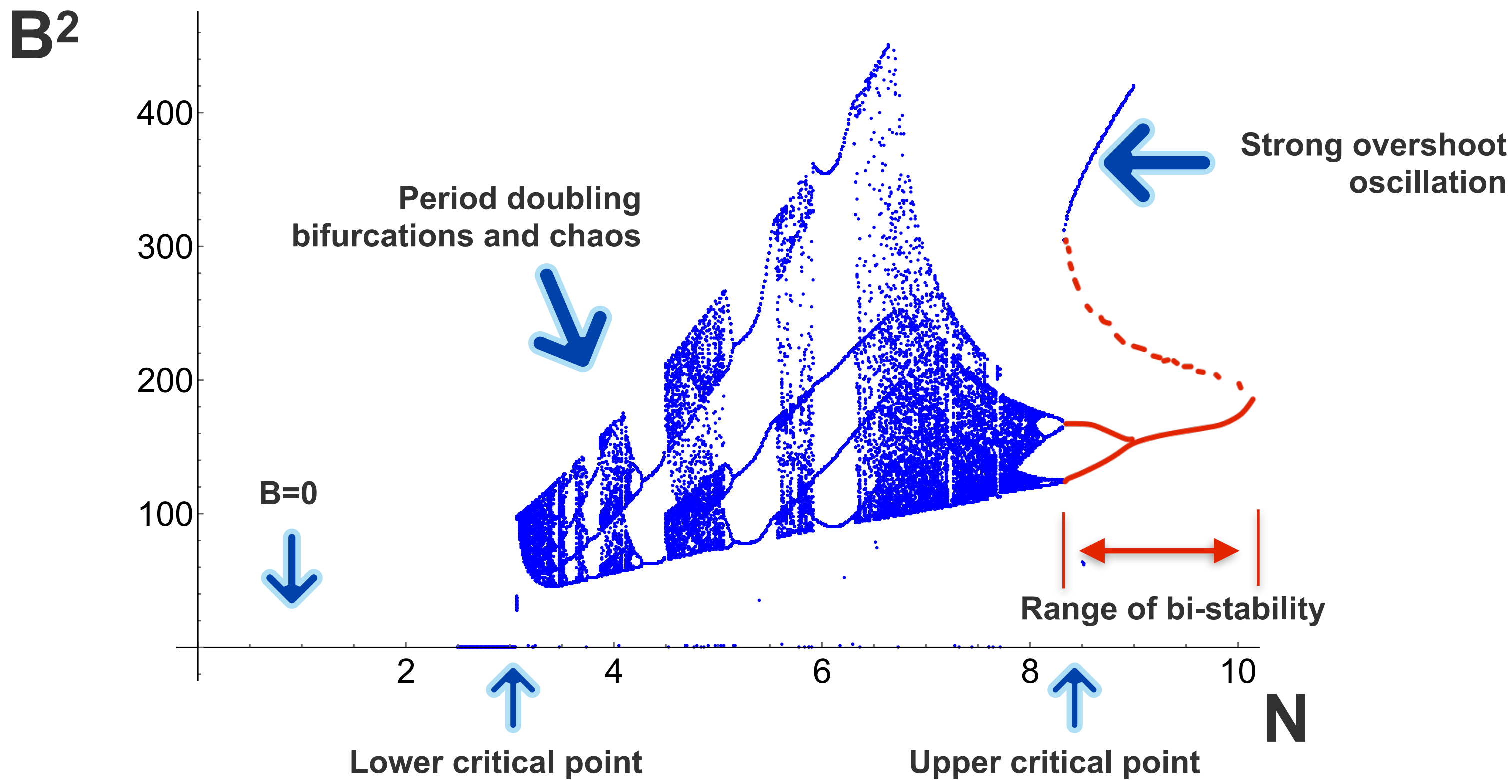


The sources of the Alpha and Omega effects are spatially segregated, which is modeled by effective delays in the ODE model

Wilmot-Smith et al. 2006

Babcock-Leighton-type dynamo models

Peaks as function of the dynamo number:



Babcock-Leighton-type dynamo models

- Three types of stable solutions:
  - Zero solution B = 0.
  - Oscillations with little overshoot w.r.t. B_{max}, a base frequency close to the linear solution, modulations due to period doubling bifurcations or chaos.
  - Periodic oscillations with strong overshoot.
- Near the upper critical point, all three solutions are stable.
- Adding noise allows the dynamo to switch between these modes.
- Near the critical points, the dynamo is very susceptible to external modulations (Stochastic Resonance).

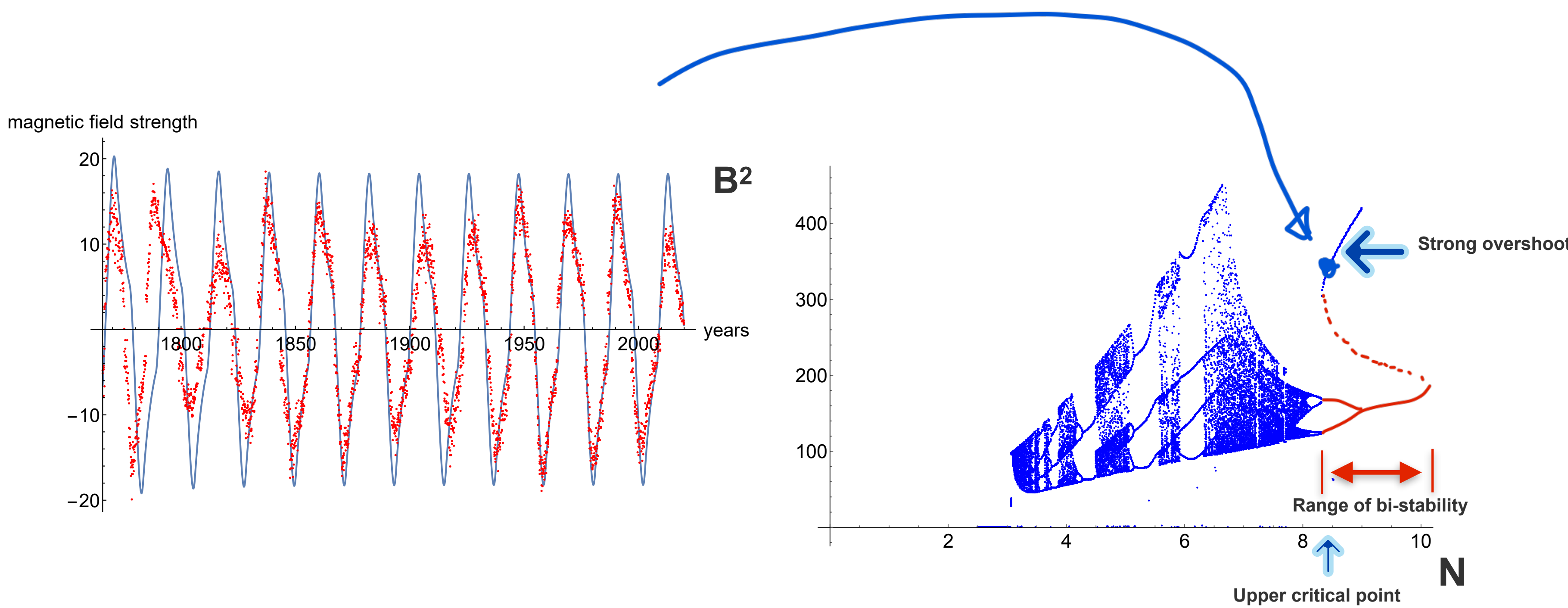
Evidence from records of solar activity

- Solar activity switches between different modes of oscillation: Both exceptionally large cycles and cycles with almost no sunspots are often clustered (Grand Maxima/Minima).

- Grand Minima themselves are clustered. During periods with Grand Minima, long-period cycles (e.g. 208y and 350y) are stronger than during periods without Grand Minima (McCracken et al 2013).

Evidence from records of solar activity

- Solar cycles observed through sunspot numbers (red dots) are characterized by a steep ascent, and a descent with a "kink". This shape fits well to the "strong overshoot" solution of the model, with a dynamo number near the upper critical point (blue line):



Conclusions

- BL-type dynamos display a critical point at high dynamo numbers, near which two stable oscillatory modes co-exist. The strong mode could be related to the strong cycles observed during Grand Maxima.
- Near this critical point, Stochastic Resonance could explain some of the long-period modulations of solar activity as an effect of a weak planetary forcing.
- The strong mode shows a characteristic kink in the falling limb (caused by the overshooting over B_{max}), which is also evident in the sunspot records of the modern Grand Maximum.
- Observations show a weak negative correlation between cycle amplitude and length. While our simple noisy dynamo model shows such a negative correlation, when in its low-overshoot mode, the strong-overshoot cycles are longer than the low-overshoot ones.

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