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Site Characterization and Multipath Maps Using Zernike Polynomials

Andrea Gatti, Giulio Tagliaferro, Eugenio Realini

SOURCES OF “ERROR” IN GNSS PPP

When using a GNSS receiver we try to measure the distance between its antenna and a GNSS satellite.
Unfortunately this measurement is afflicted by some “disturbances”. These can be considered as a signal (if you want to study them) or as an error if the model you use for removing them from your observations is not accurate.
Other than the geometrical distance between the antenna and the satellite, different are the effects that influence the measurements:

- *Ionosphere - it can be reduced by having at least a dual frequency receiver, or by external models*
- *Relativity - this can be described accurately by a model*
- *Tides - modeled*
- ***Troposphere** - hydrostatic component can be estimate from a model, in this work we consider the **wet** component a **signal** that we want to estimate*
- ***Multipath** - we are interested in removing it and thus we would like to **build a model** to “mitigate” it’s effect*

← All the unmodeled effects outliers and other errors

Sources of “error” in GNSS PPP

Ionosphere
Accounted for
tens of meters

Relativity
Perfectly modelled
tens of meters

$E = mc^2$

Troposphere
Hydrostatic + Wet
the first is modelled
~2 meters

Tides
Modeled
Solid + Ocean + Athmosperic
tens of centimetres

Other sources
Antennas,
other model errors
centimeters

Phase multipath
Far field, near field reflections
up to tens of centimetres

Multipath mitigation techniques

01 **HARDWARE**

Choke ring antennas, receiver internal tracking mitigation

02 **MASKING**

Can be based on different techniques, e.g. SNR threshold, on-site measurements, …

A mask is generated to ignore observations affected by multi-path

03 **BASED ON SIGNAL QUALITY MEASUREMENTS**

Use the signal to noise ratio information to compute carrier phase multipath corrections

04 **SIDEREAL FILTERING**

This methods based on the orbit repeatability of GPS (mainly) analyse data satellite by satellite to detect temporally repeated patterns in the **residuals** and correct their observations

05 **MULTIPATH STACKING MAPS**

Using different gridding algorithms on the **residuals** of the PPP processing create maps of corrections to compensate for the spatial repeatable errors

MULTIPATH MITIGATION TECHNIQUES

In literature there are many methods to perform multipath management, from hardware techniques to models based on the knowledge and model of the installation environment.
Two of the less invasive procedure to deal with multipath rely on residuals to produce a reduction model. ***Sidereal filtering****, and* ***residuals gridding*** *are two techniques that do not require any access or modification to antennas and receivers and can be applied to any GNSS station.*

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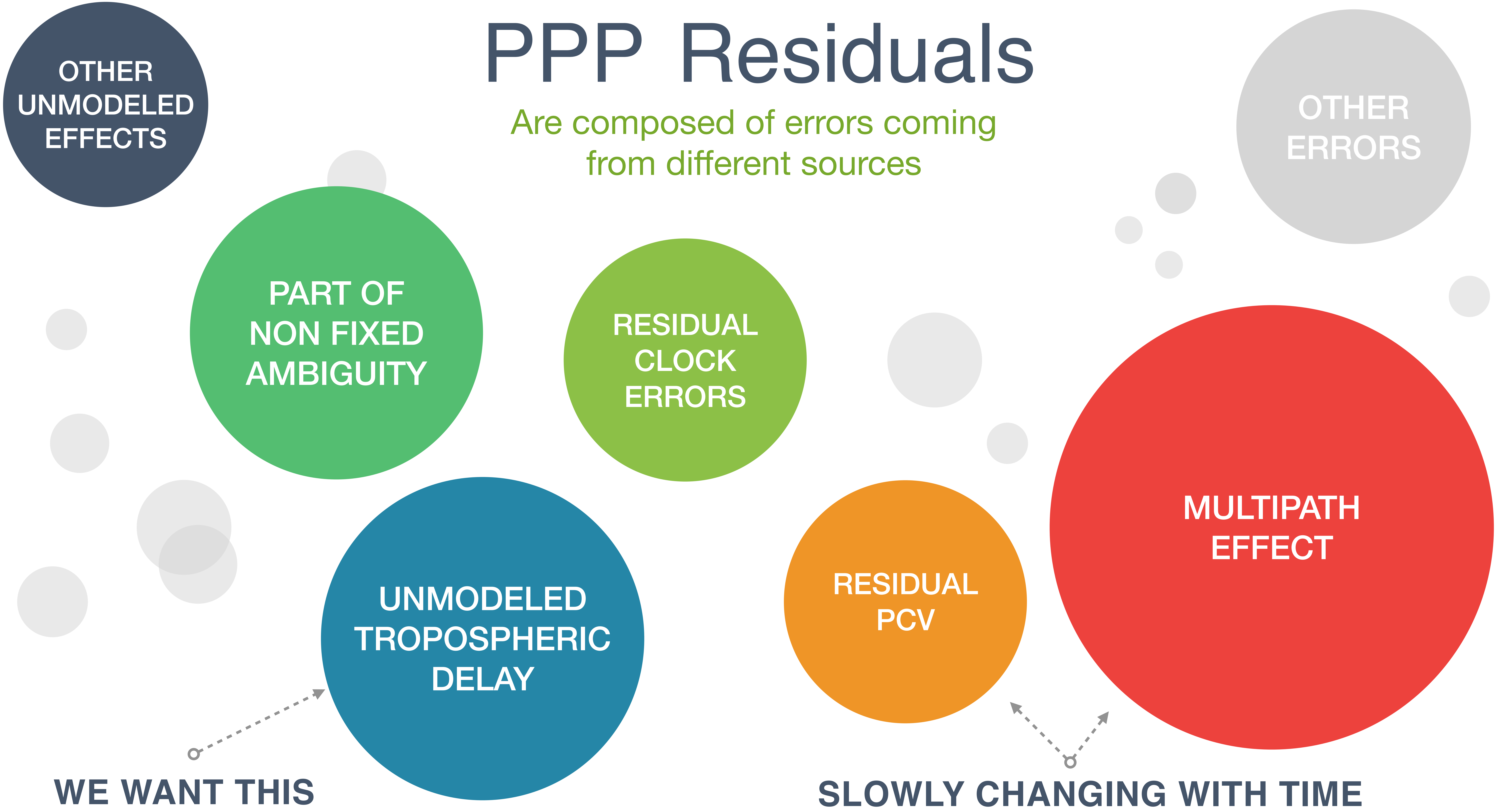
In a simplistic way (a more detailed paper on this work with all the necessary references is almost ready to submission) we can evaluate the two approaches by citing some of their characteristics:

- *From different tests in literature* ***sidereal filtering*** *demonstrated an higher accuracy in modelling high frequency multipath, but due to slow orbit repeatability or satellite manoeuvres it is not always applicable in any scenario (and any constellations).*
- ***Multipath maps*** *have been generally estimated with low resolutions (up to 1x1 degrees) that are not enough for high frequency multipath, and are limited by the number of samples per cell that can be averaged, but they avoid the problems on the determination of the exact orbit repetition time, and are generally more robust.*

PPP Residuals

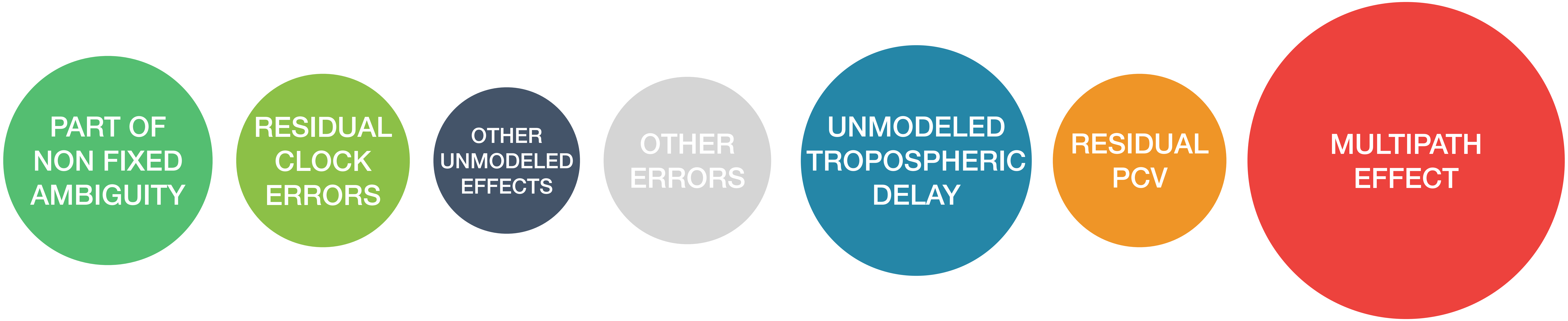
Are composed of errors coming from different sources



PPP RESIDUALS

In our undifferenced Precise Point Positioning (PPP) processing all the models of the known effects are applied to reduce the data from their influence (e.g. ionospheric delay, relativistic effects, ….) and tropospheric wet delay are estimated as Zenith Total Delay (ZTD) plus its gradients. The residuals of the PPP least squares (LS) batch adjustment contains errors and residual signals.

- *Phase ambiguities are not estimated as integer by PPP (goGPS) unless precise code observations, and orbits with correct estimated biases are used. This means that the “float” error of each single arc is present in the residuals.*
- *Minor errors in the estimation of satellite and receiver clocks, other measurement errors and processing errors (outliers), and finally some mismodeling are also contributing to the residuals.*
- ***Tropospheric delays** are already described, but with a simple model (ZTD + gradients), other inhomogeneities of the water vapour distribution can be found in the residuals.*
- ***Multipath** and **PCV (residuals)** are one of the main component of the residuals, especially in non geodetic antennas.*



Residuals

- OTHER UNMODELED EFFECTS
- UNMODELED TROPOSPHERIC DELAY
- RESIDUAL CLOCK ERRORS
- OTHER ERRORS
- PART OF NON FIXED AMBIGUITY
- RESIDUAL PCV
- MULTIPATH EFFECT

Management

AVERAGE MANY OBSERVATION EPOCHS

With the residuals of a longer time span these effects can be drastically reduced but its a trade-off with multipath that can change in time

OUTLIER DETECTION

Performing an outlier detection is important to avoid introducing anomalous effects

IGNORE SMALL ARCS

Float ambiguity of longer arcs is better estimated and closer to a fixed solution

GEOMETRIC GRIDDING

Different methods are possible, interpolation can be used

Producing maps of residuals

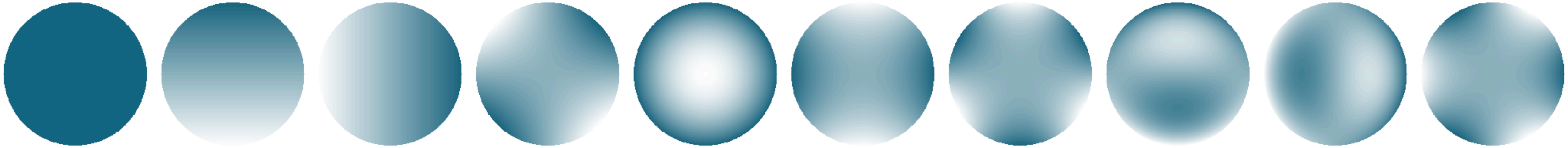
Method	PRO	CONS
Gridding with a **constant grid**	**Fast** Easy to implement	Close to zenith cells became small **Discrete** approach
Gridding with **congruent cells**	Cells with **homogenous** area	Does not interpolate in cells with no data **Discrete** approach
Interpolation with **spherical harmonics**	**Smooth** interpolation **Continuous** approach	Computationally **heavier**, designed on a sphere, **unstable** at high degrees of the expansion
Interpolation with **Zernike polynomials**	**Smooth**, **Defined on a disc** **Continuous** approach	Computationally **heavier**, designed on disc and not a hemisphere, requires regularisation

Zernike polynomials

The Zernike polynomials are a sequence of polynomials **orthogonal on** the unit **disc**. Invented by Fritz Zernike, a Dutch physicist and winner of the Nobel prize in physics. They are commonly used in different fields:

- in **optometry** and **ophthalmology**, to define aberrations of the cornea or lenses.
- in **adaptive optics**, to characterise atmospheric distortion
- in **computer vision** and **image processing** as basis functions of image moments

1888–1966



Z_0^0 Z_1^{-1} Z_1^1 Z_2^{-2} Z_2^0 Z_2^2 Z_3^{-3} Z_3^{-1} Z_3^1 Z_3^3

Zernike polynomials

There are even and odd Zernike Polynomials depending on their azimuthal degree:

even: $Z_n^m(\rho, \varphi) = R_n^m(\rho) * \cos(m\varphi)$

odd: $Z_n^{-m}(\rho, \varphi) = R_n^m(\rho) * \sin(m\varphi)$

where:

ρ is the radius [0, 1]

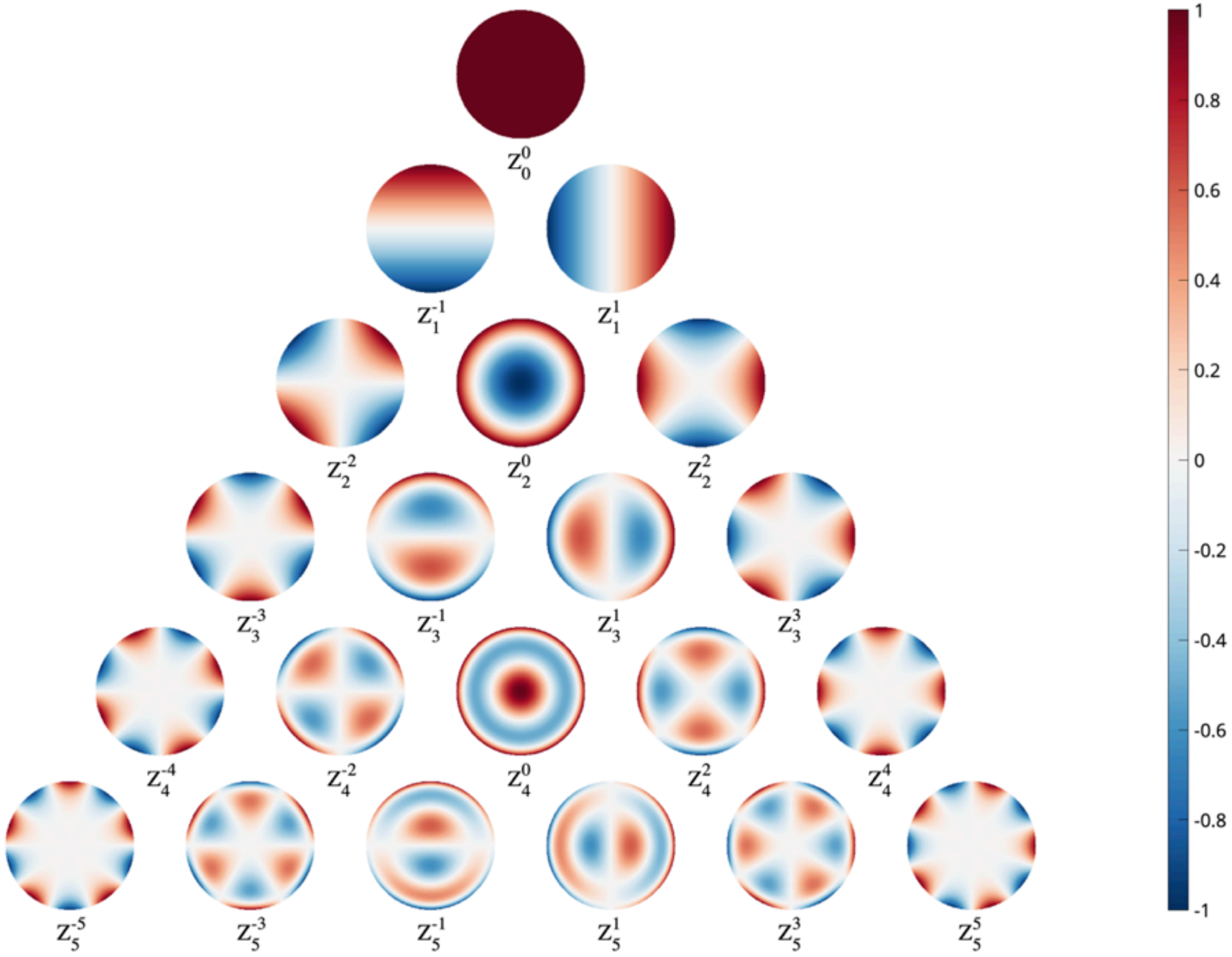
φ the azimuth

n the radial degree

m the azimuthal degree

and the radial polynomial is:

$$R_n^m = \sum_{k=0}^{\frac{n-m}{2}} \frac{(-1)^k (n-k)!}{k!(\frac{n+m}{2}-k)!(\frac{n-m}{2}-k)!} \rho^{n-2k}$$



$R_0^0 = 1$

$R_1^1 = \rho$

$R_2^0 = 2\rho^2 - 1$

$R_2^2 = \rho^2$

$R_3^1 = 3\rho^2 - 2\rho$

$R_3^3 = \rho^3$

$R_4^0 = 6\rho^4 - 6\rho^2 + 1$

$R_4^2 = 4\rho^4 - 3\rho^2$

$R_4^4 = \rho^4$

$R_5^1 = 10\rho^5 - 12\rho^3 - 3\rho$

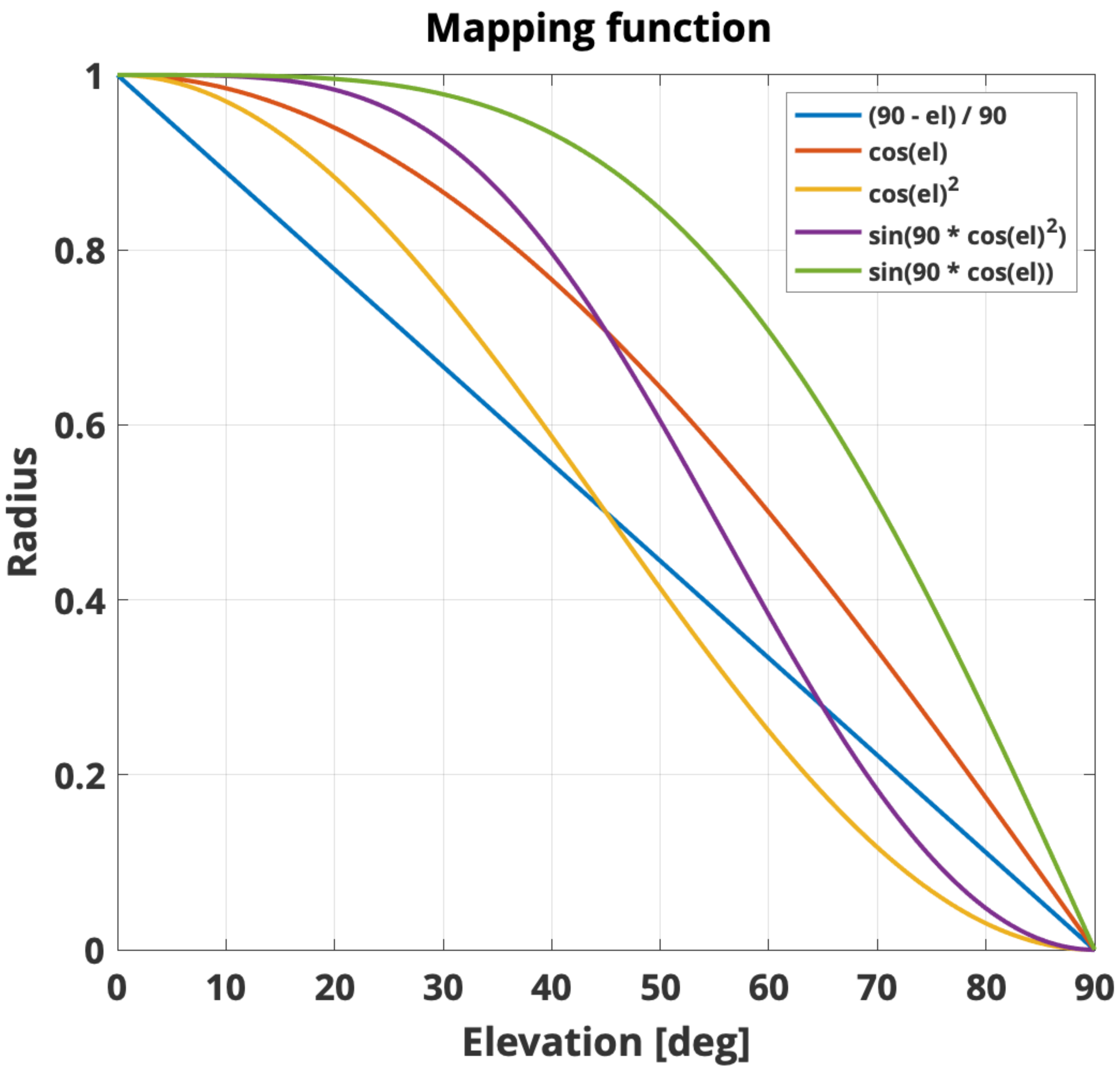
$R_5^3 = 5\rho^5 - 4\rho^3$

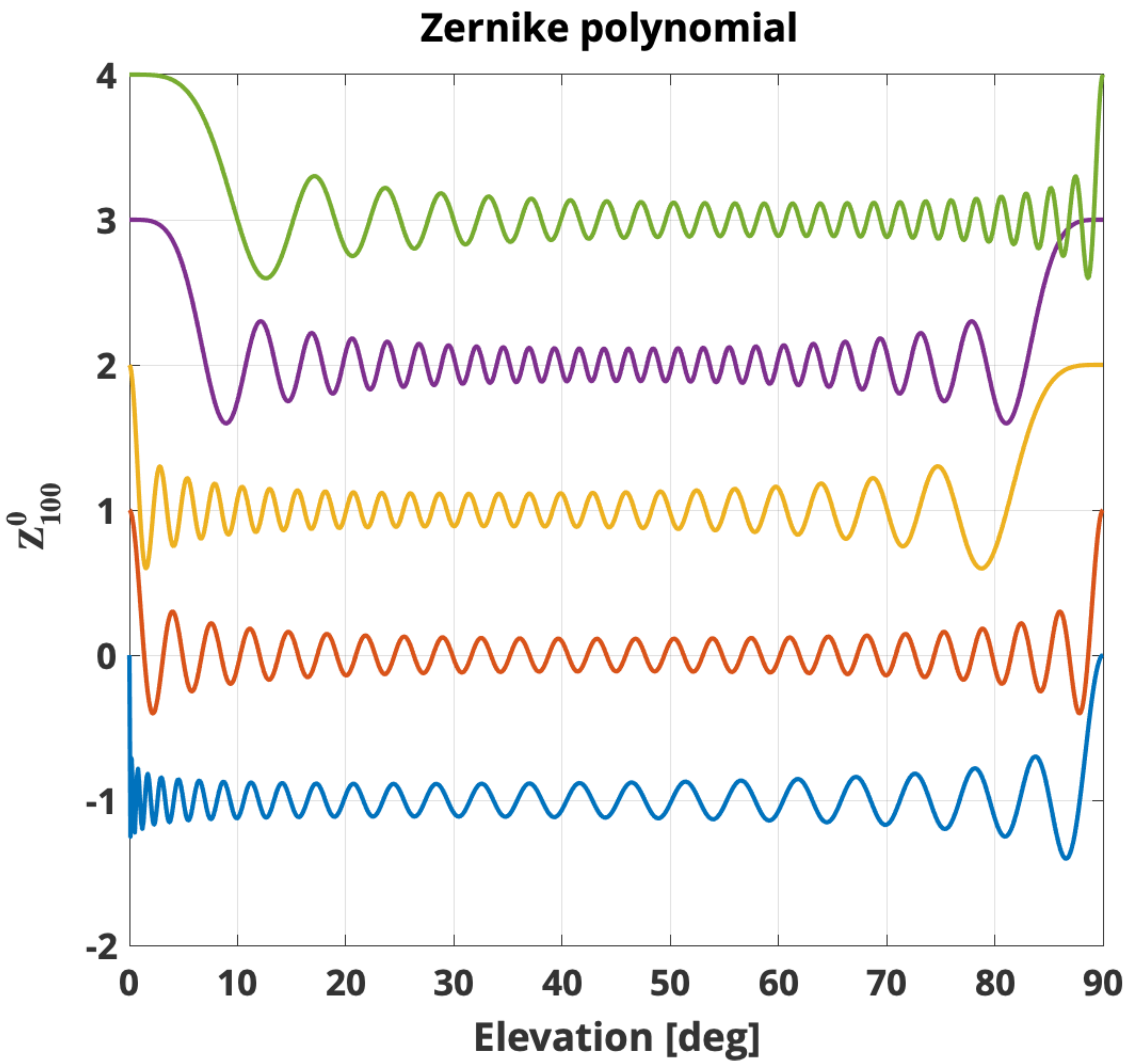
$R_5^5 = \rho^5$

...

ZERNIKE POLYNOMIALS

Zernike polynomials are defined on a disk with a unitary radius, by introducing a mapping function from elevation to ρ, the Zernike expansion can be used with azimuth and elevation. Different are the possible choices: one or a sequence of these can be used iteratively to extract higher frequency features in different ranges of elevation.





From the residuals to the grids

The gridding procedure have been implemented in **goGPS,** an open source software for the processing of GNSS CORS stations written in MATLAB and developed at GReD. Zernike polynomials have been chosen instead of spherical harmonics since they are easier to be implemented, faster to be computed.

COMPUTE RESIDUALS

Using the goGPS engine obtain the residuals

COMPUTE AZIMUTH AND ELEVATION

Remove all the residuals above 3 sigma (latitude by latitude)

OUTLIER DETECTION

Remove all the residuals above 3 sigma (latitude by latitude)

COMPUTE REGULAR GRIDS

With two grid size 1x1 and 2x1 degrees then upscaled at 0.5x0.5

COMPUTE CONGRUENT GRIDS

With two grid size 1x1 and 2x1 degrees then upscaled at 0.5x0.5

COMPUTE ZERNIKE POLYNOMIALS

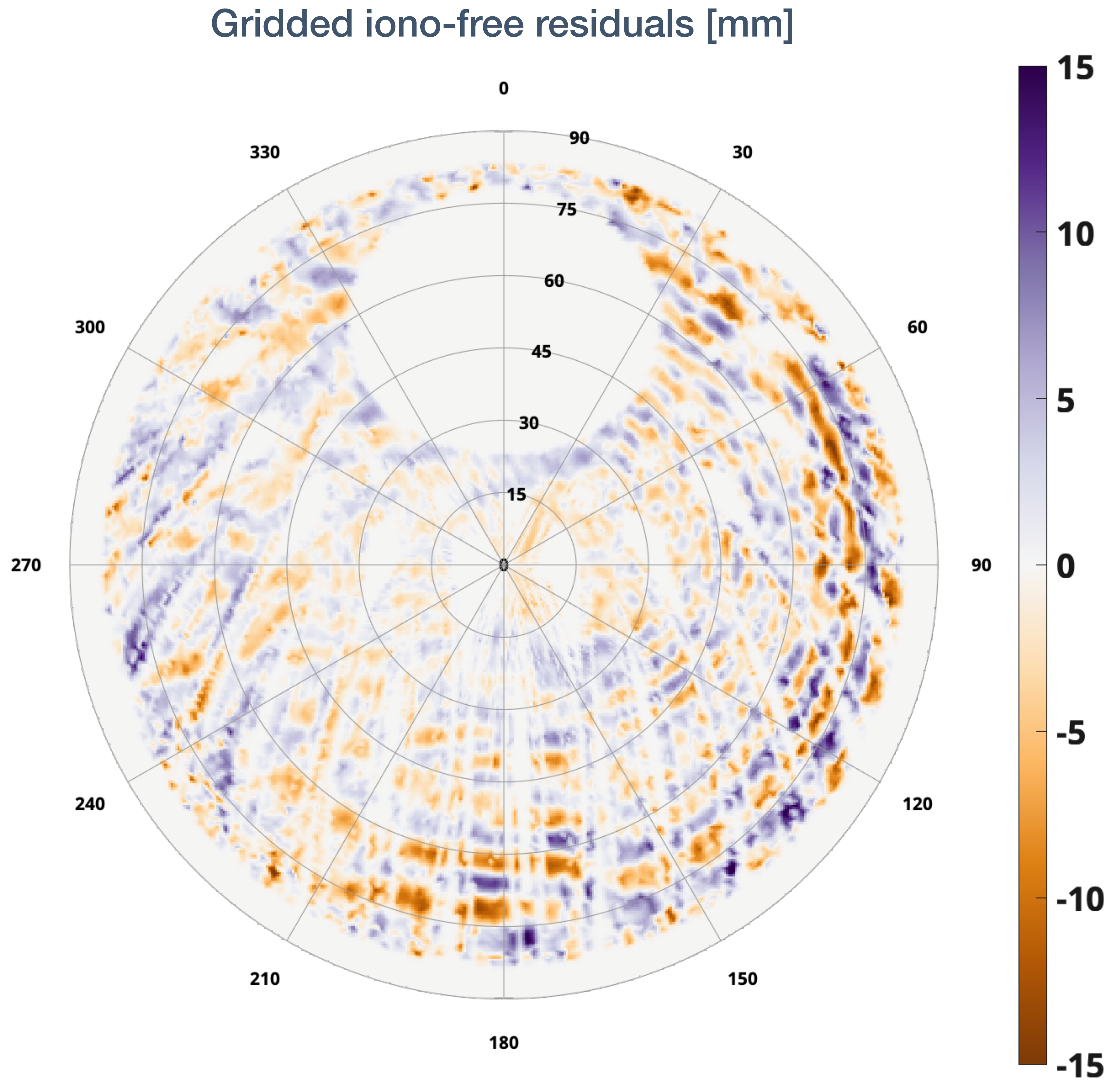
With up to 3 steps changing the mapping function for the radius.
Synthesis at 0.5x0.5 degrees (or higher resolutions if required)

COMPUTE A GRIDDING AFTER

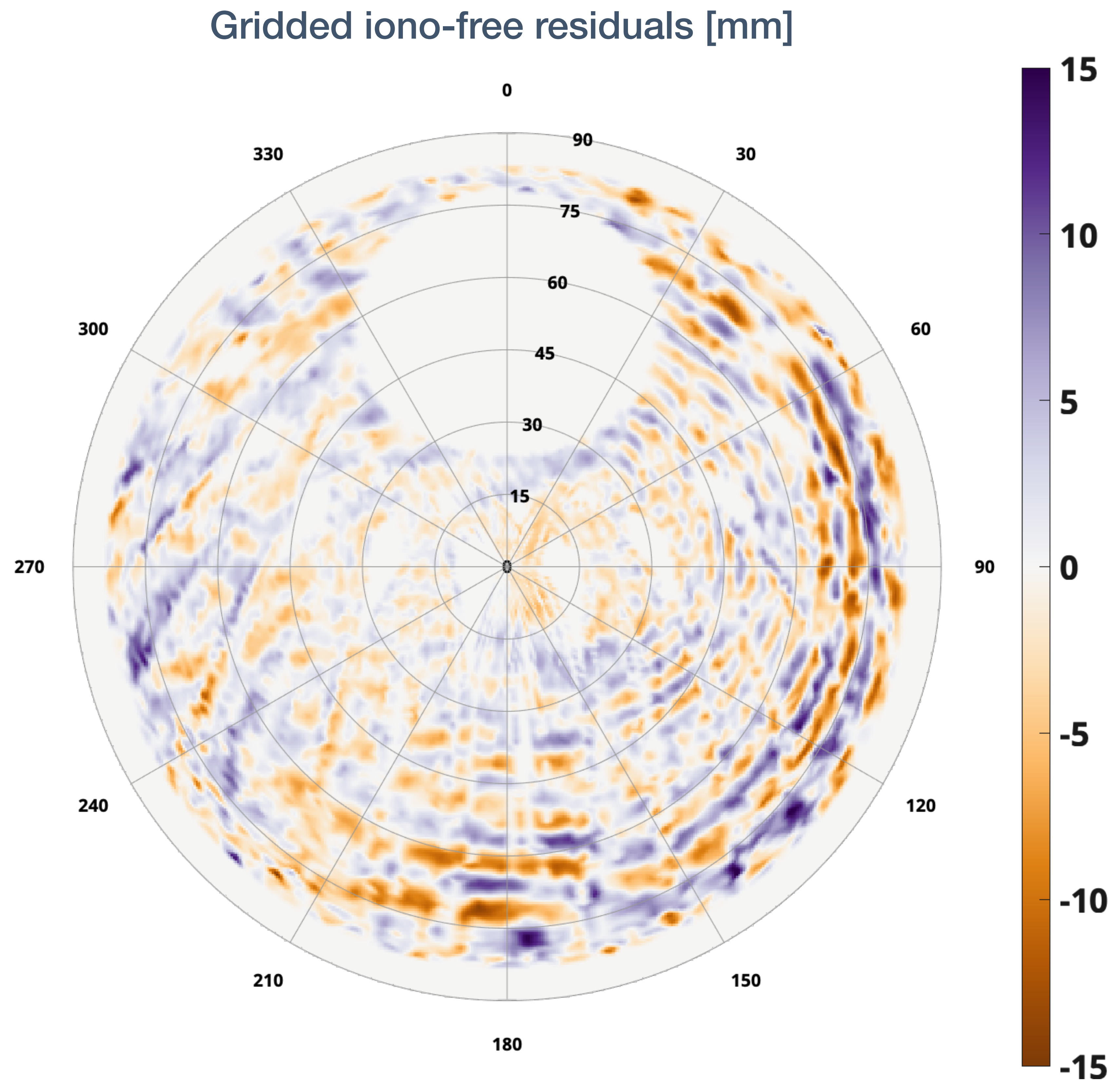
On the residuals after Zernike polynomials
With a multi-resolution approach up to a custom cell size (default 1 x 0.1) for cell with a minimum set number of observations

Gridding on a regular grid

This is the faster and simpler approach: **average** the residuals data in a cell with regular size in both elevation and azimuth, cells with less than `n_min` data will be set to zero.



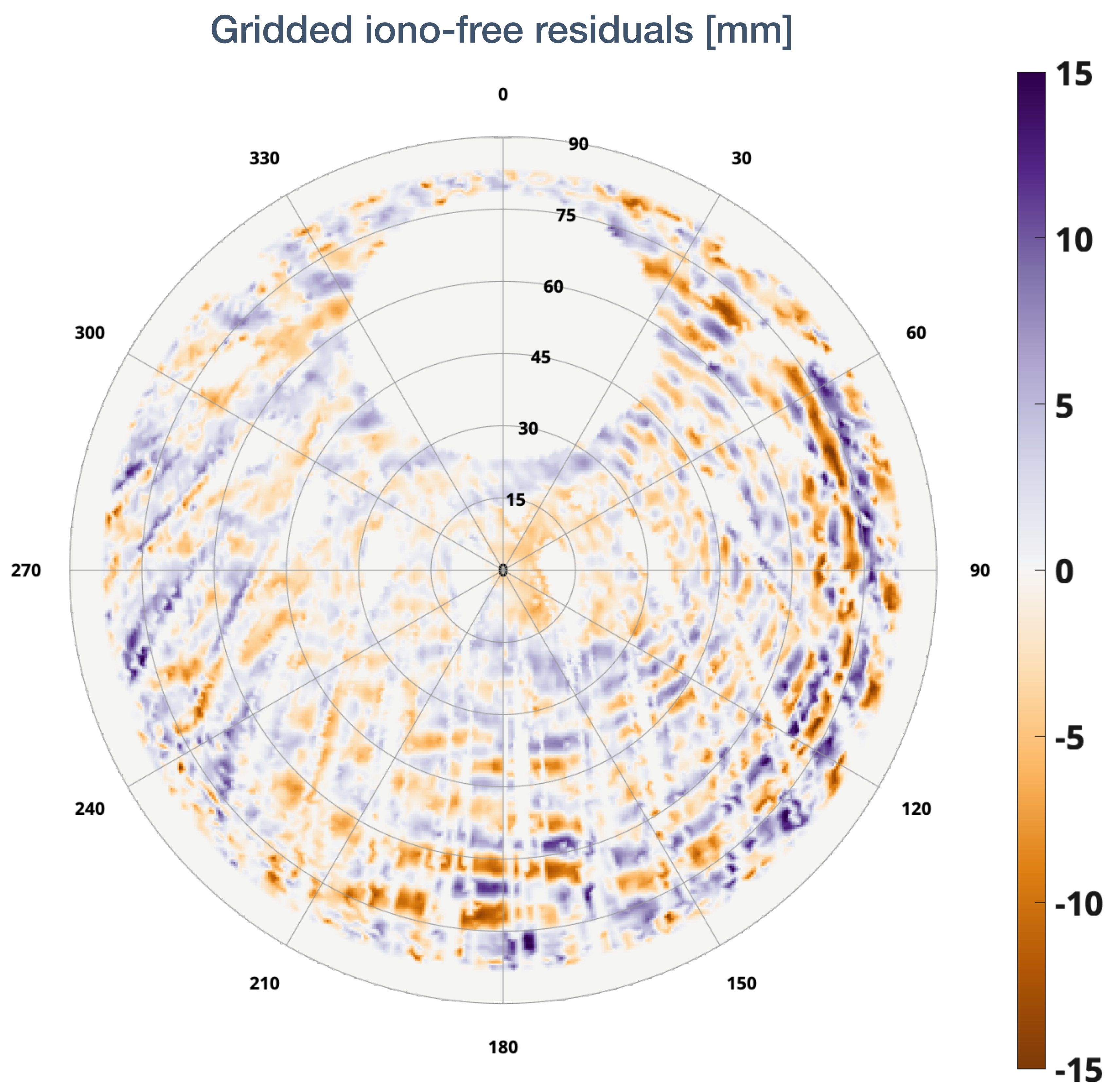
Grid with regular size of 1 x1



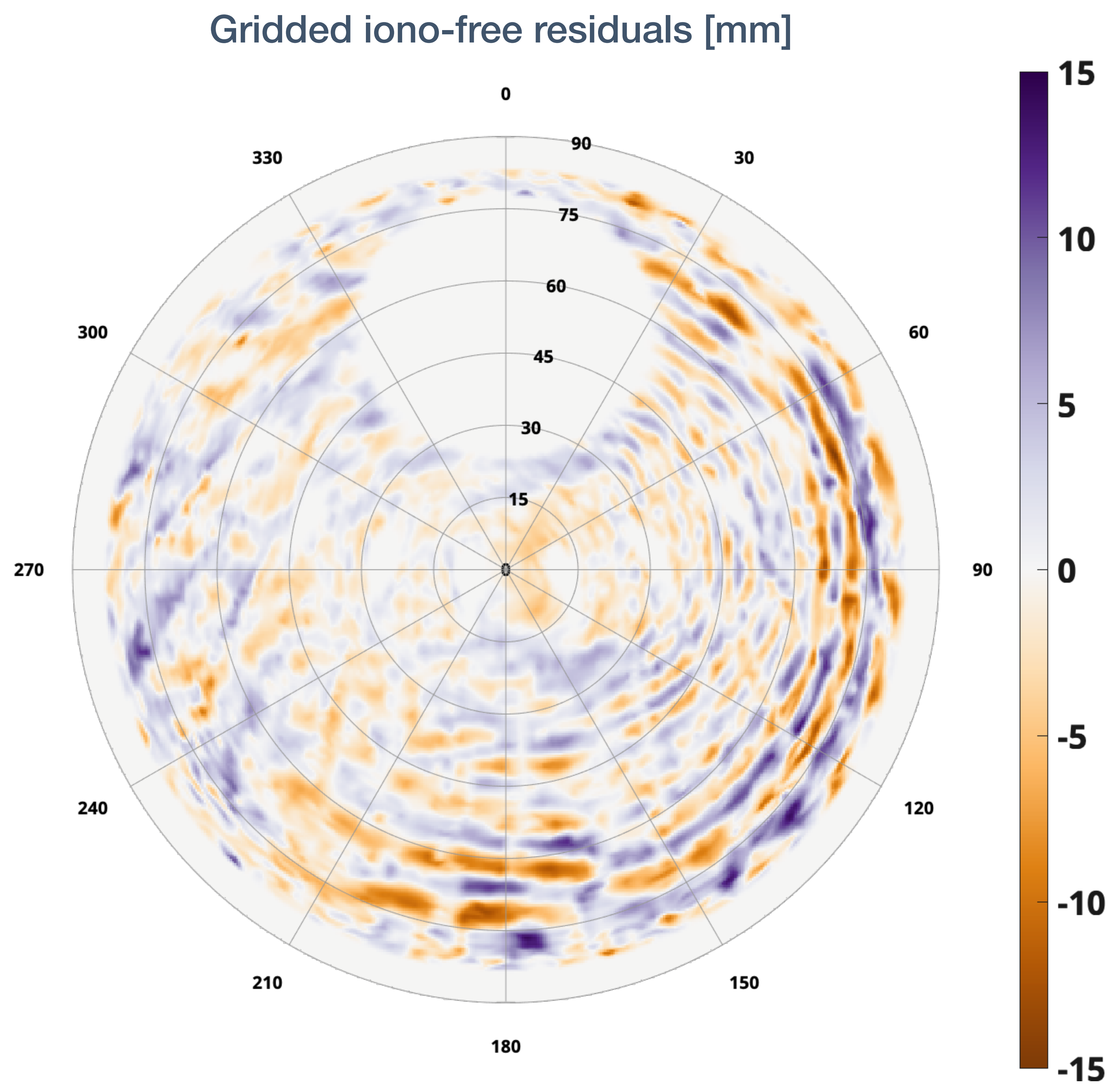
Grid with regular size of 2 x1
linearly upscaled at 0.5x0.5

Gridding using congruent cells

Similar to the previous case the grids are produced taking the **average** of the residuals falling in a cell, this time the resolution is fixed in elevation while in azimuth the number of cells decrease increasing the elevation



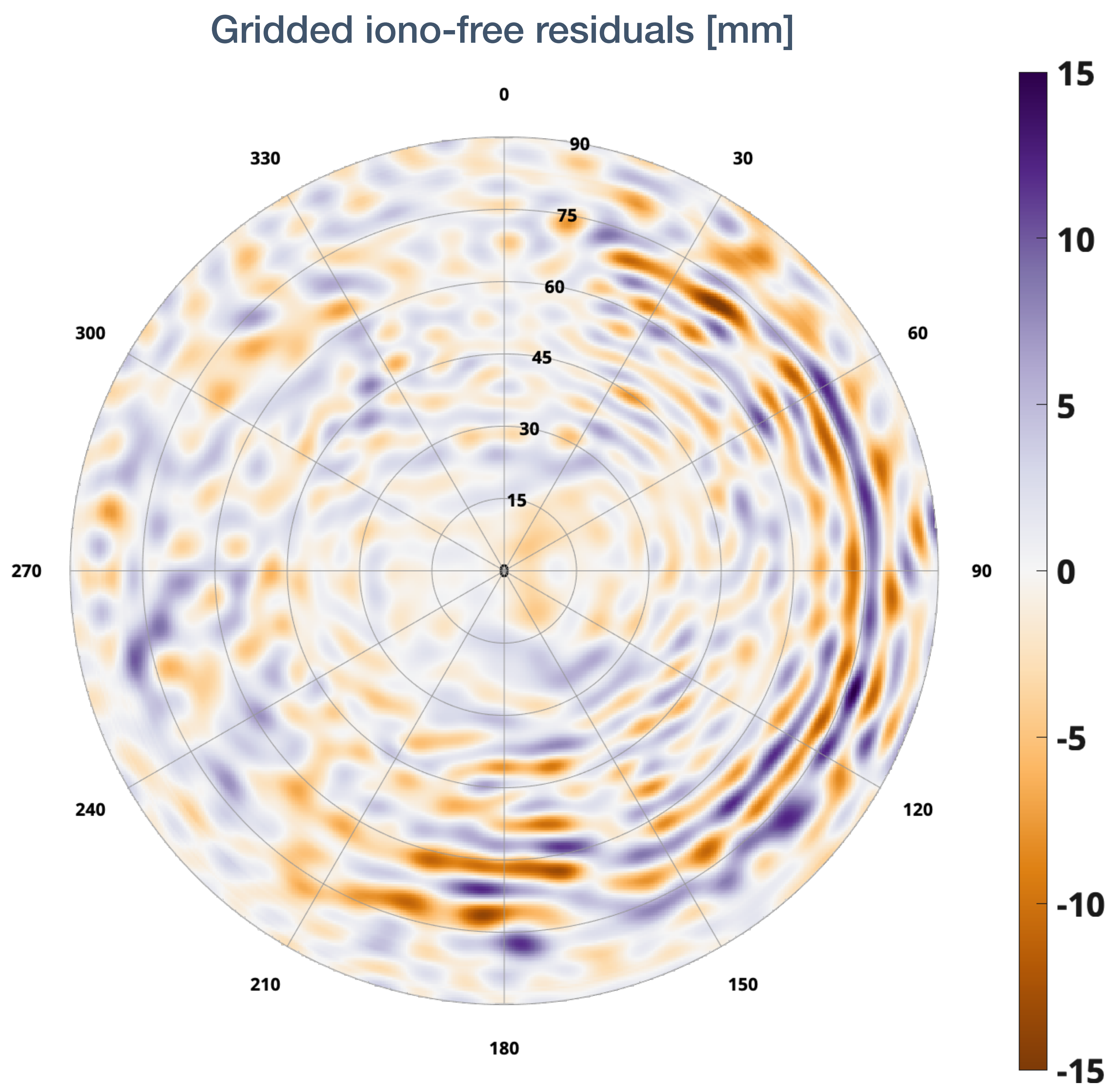
Grid with regular size of 1 x1



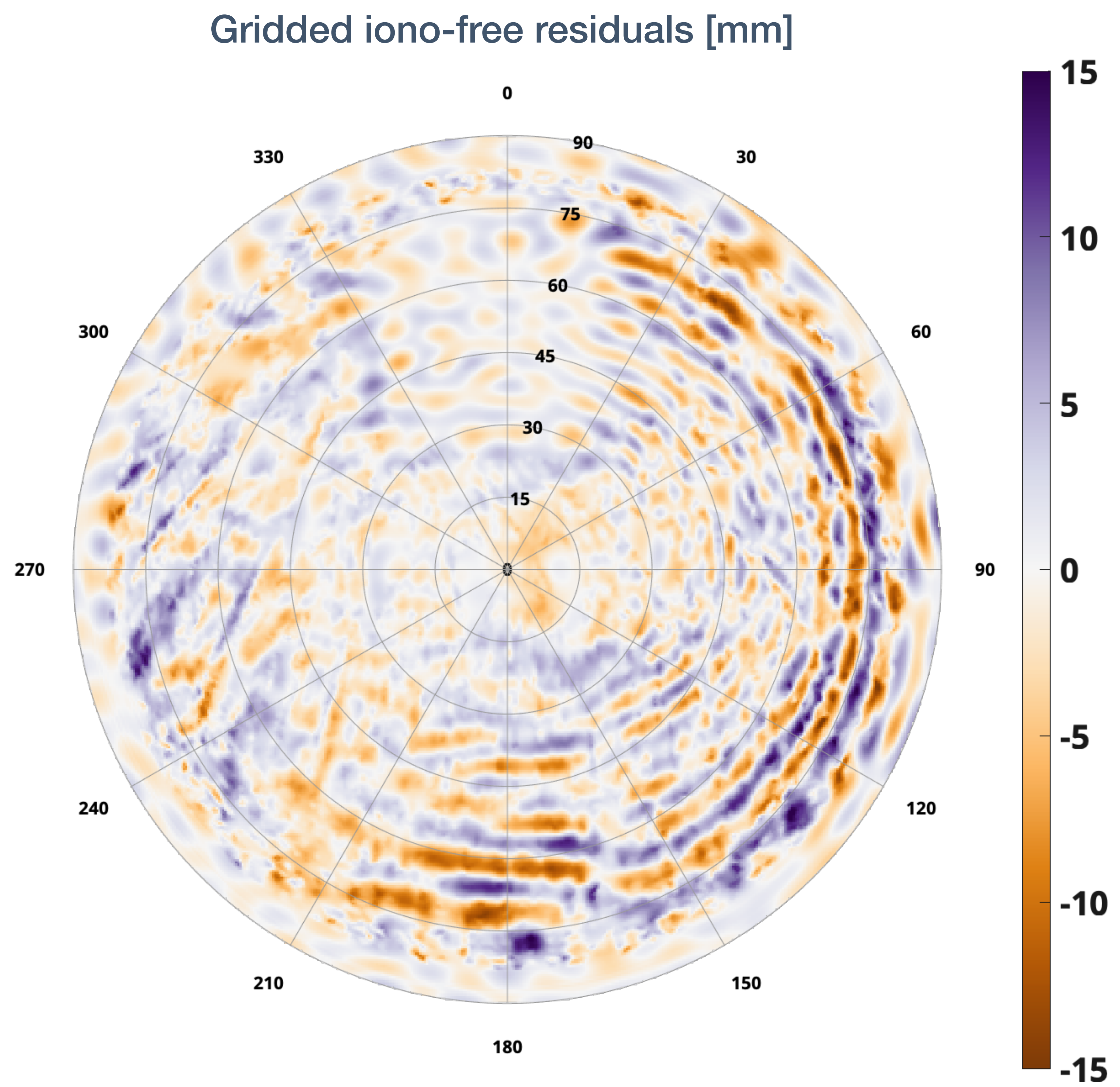
Grid with regular size of 2 x1
linearly upscaled at 0.5x0.5

Gridding using goGPS approach

Grids by Zernike interpolation are **synthesised** from the coefficient computed by a least-squares adjustment, on the residuals of it a gridding is computed and summed



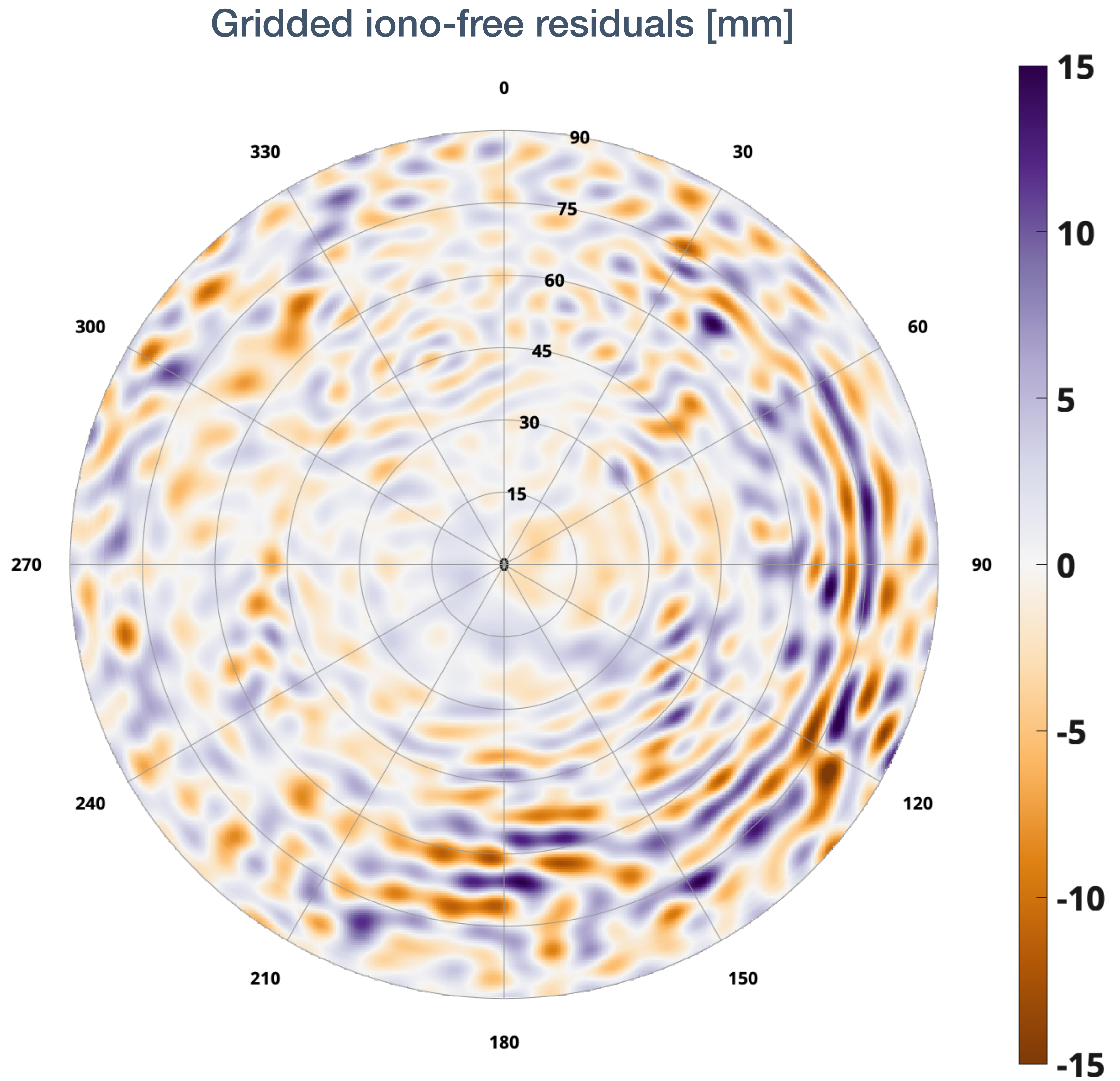
Zernike synthesized grid at 0.5 x 0.5



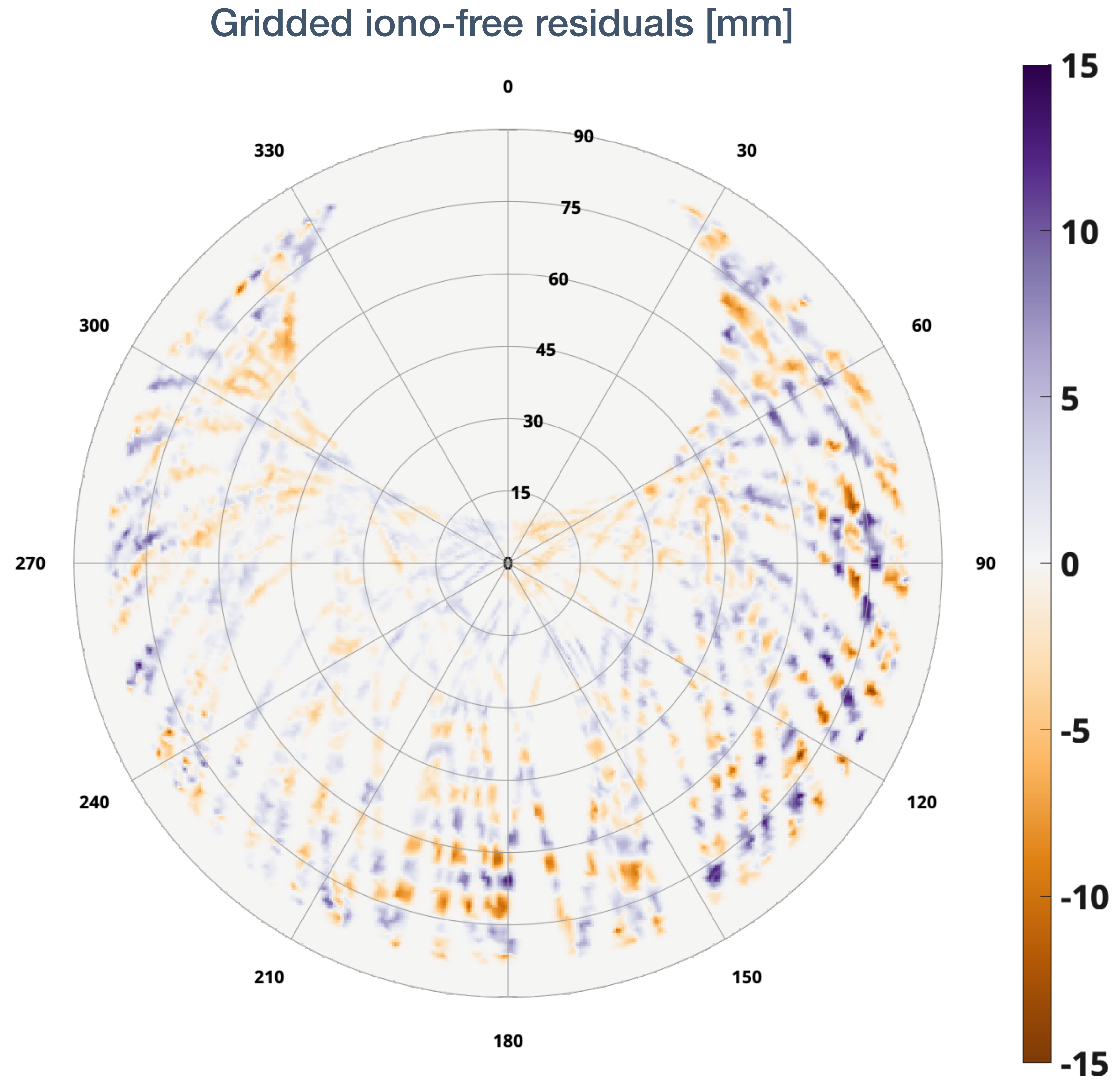
Zernike synthesized grid at 0.5 x 0.5
Plus his residuals gridded with
congruent cells

Zernike vs Gridding

In case of fewer tracks of data available the map estimated with a Zernike expansion fill the gaps with an interpolation, however to avoid instabilities the computation of the coefficients is regularised



Zernike up to degree 43



Congruent cell gridding 1x1

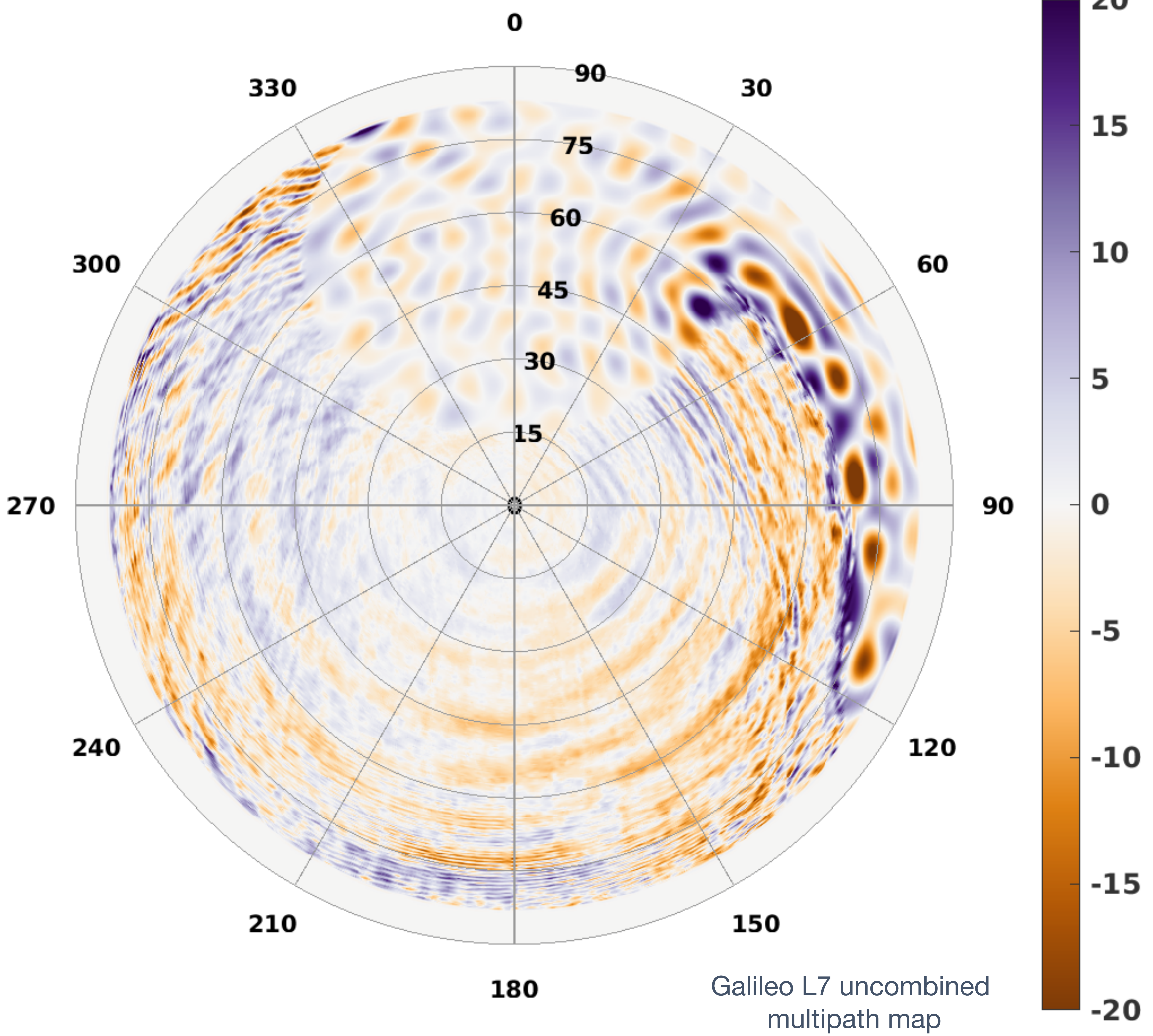
ZERNIKE + GRIDDING FOR HIGH RESOLUTION GRIDS

The techniques implemented in goGPS can create a high resolution grid with a multi-step procedure:

1. Zernike polynomials able to describe the lower frequencies of the multipath effect present in the residuals.
2. Congruent cells grid (1.5 x 0.5 degrees) are added to describe higher frequencies.
3. A final step with regular grid of 0.75 x 0.1 degree is then estimated.

The value of the cell is assigned only if a minimum number of good observations are present (e.g. 15).

This multi-step procedure allows to recover even higher frequencies compensating the greater limit of the multipath maps vs sidereal filtering. The synthesis of the corrections is performed from the grid using bicubic spline interpolation.



Galileo L7 uncombined multipath map of a low cost receiver

Testing the maps

A test is performed on a location with **close stations**, all in a radius of less than 100 meters.

The PPP for all the stations is computed with the same parameters among all the receivers but using 7 different configurations for the multipath mitigation:

- 1 as a baseline reference with **no maps** applied
- 2 using the 2 maps from the **regular gridding** (●●)
- 2 using the 2 maps from the **congruent gridding** (●●)
- 2 using the 2 maps using **Zernike interpolation** (●●)

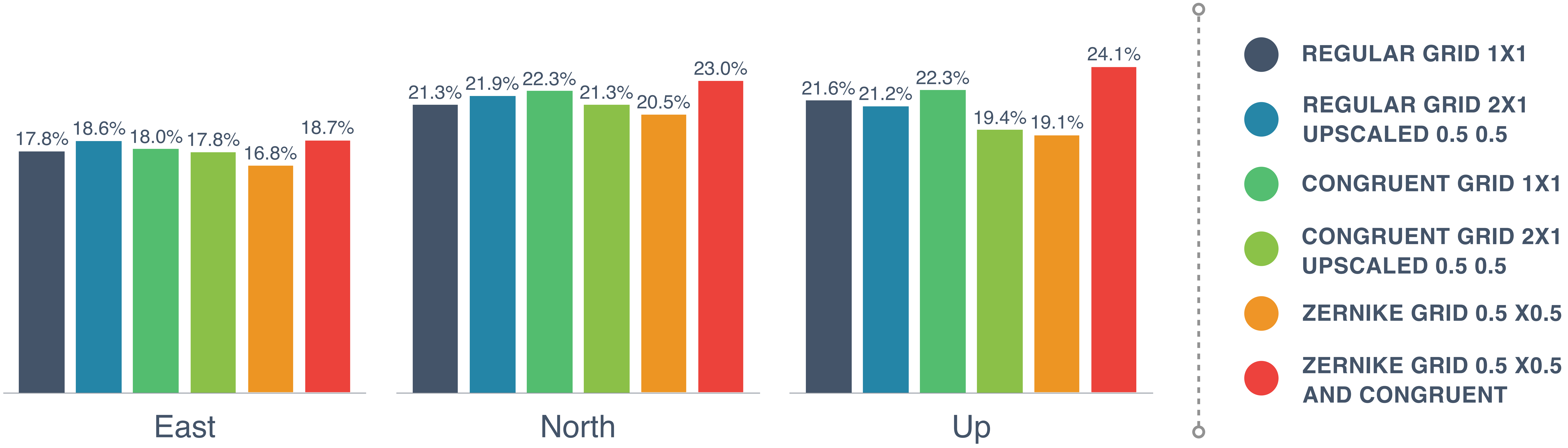
The following parameters are evaluated:

1. Standard deviation of the **residuals** for the station with higher multi-path
2. Cross validation of the **ZWD** and of the **gradients** among all the stations
3. Standard deviation of the station **coordinates** computed every quarter of hour for the station with higher multipath

Testing the maps - Wettzell

6 Geodetic stations with calibrated antennas

The multipath maps have been estimated on the entire October 2019 and applied to the data of November 2019



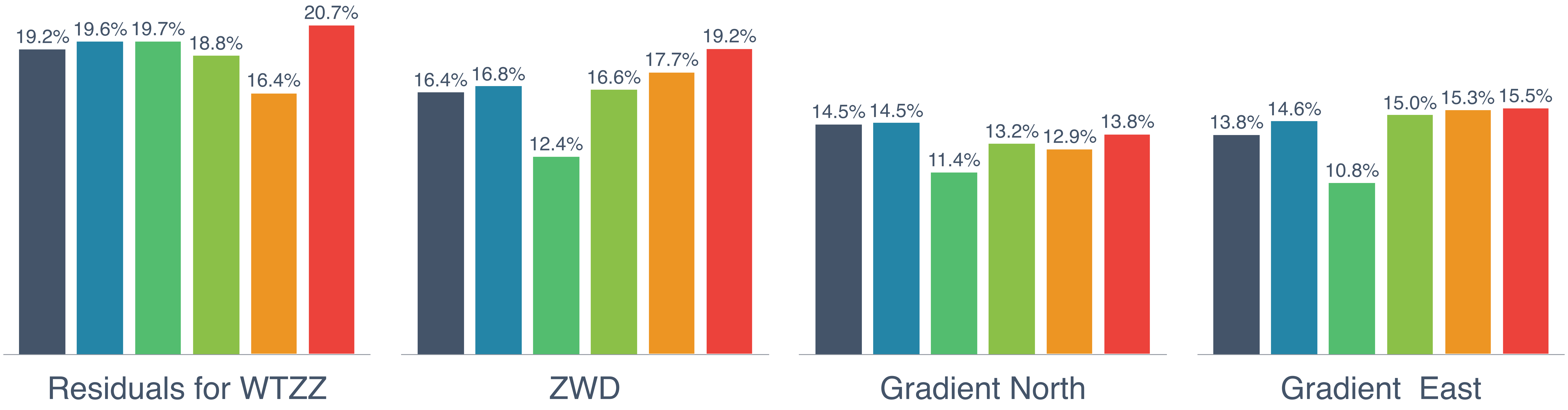
WTZZ is the only station with strong multipath effects, other stations have an improvement on the STD of the coordinates of about 12% in up and 6 % in planar

All the values are expressed as a percentage of improvements vs the reference PPP with no maps, higher is better

Testing the maps - Wettzell

5 Geodetic stations with calibrated antennas

The multipath maps have been estimated on the entire October 2019 and applied to the data of November 2019



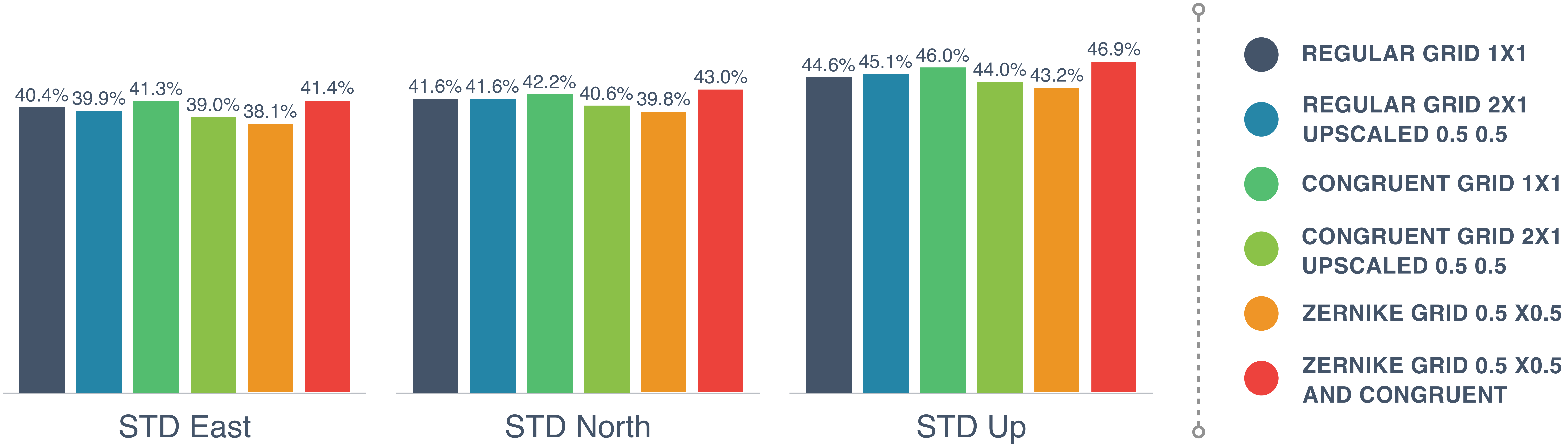
WTZZ is the only station with strong multipath effects, other stations have an improvement on the STD of the residuals of about 2.5 %

All the values are expressed as a percentage of improvements vs the reference PPP with no maps, higher is better

Testing the maps - low-cost

1 month of 2 low-cost receivers colocated on a bridge

The multipath maps have been estimated on the entire October 2019 and applied to the data of November 2019



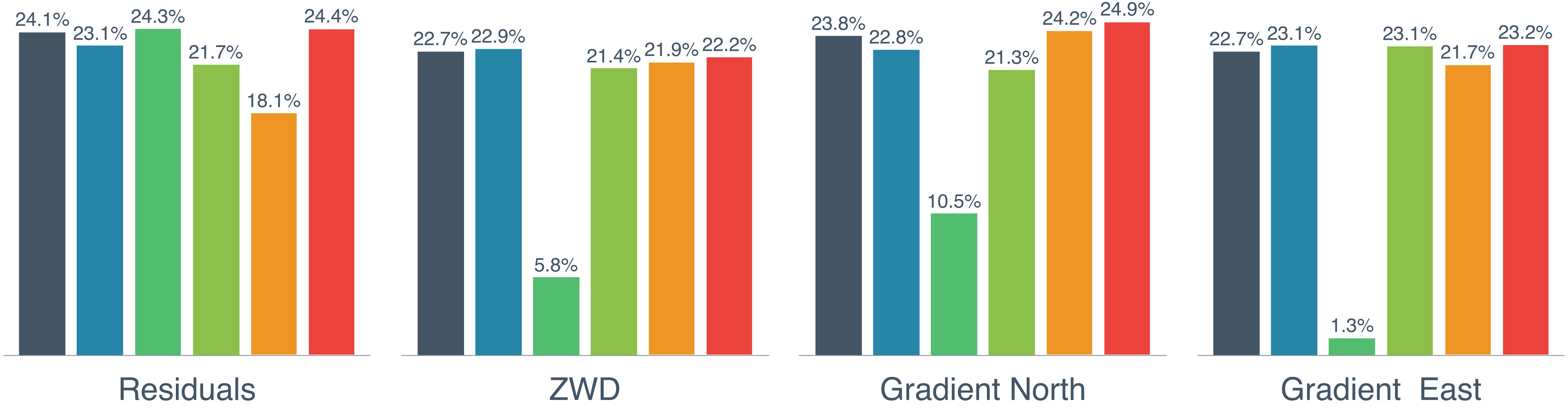
The two receivers are UBX-9T processed using PPP in multi constellation mode G + R + E with an uncalibrated UBLOX patch antennas

All the values are expressed as a percentage of improvements vs the reference PPP with no maps, higher is better

Testing the maps - low-cost

1 month of 2 low-cost receivers colocated on a bridge

The multipath maps have been estimated on the entire October 2019 and applied to the data of November 2019



The standard deviation of the difference between the two ZWD is 2.3 mm (it was 2.9 mm without maps).

All the values are expressed as a percentage of improvements vs the reference PPP with no maps, higher is better

goGPS integration

what is there and what is missing

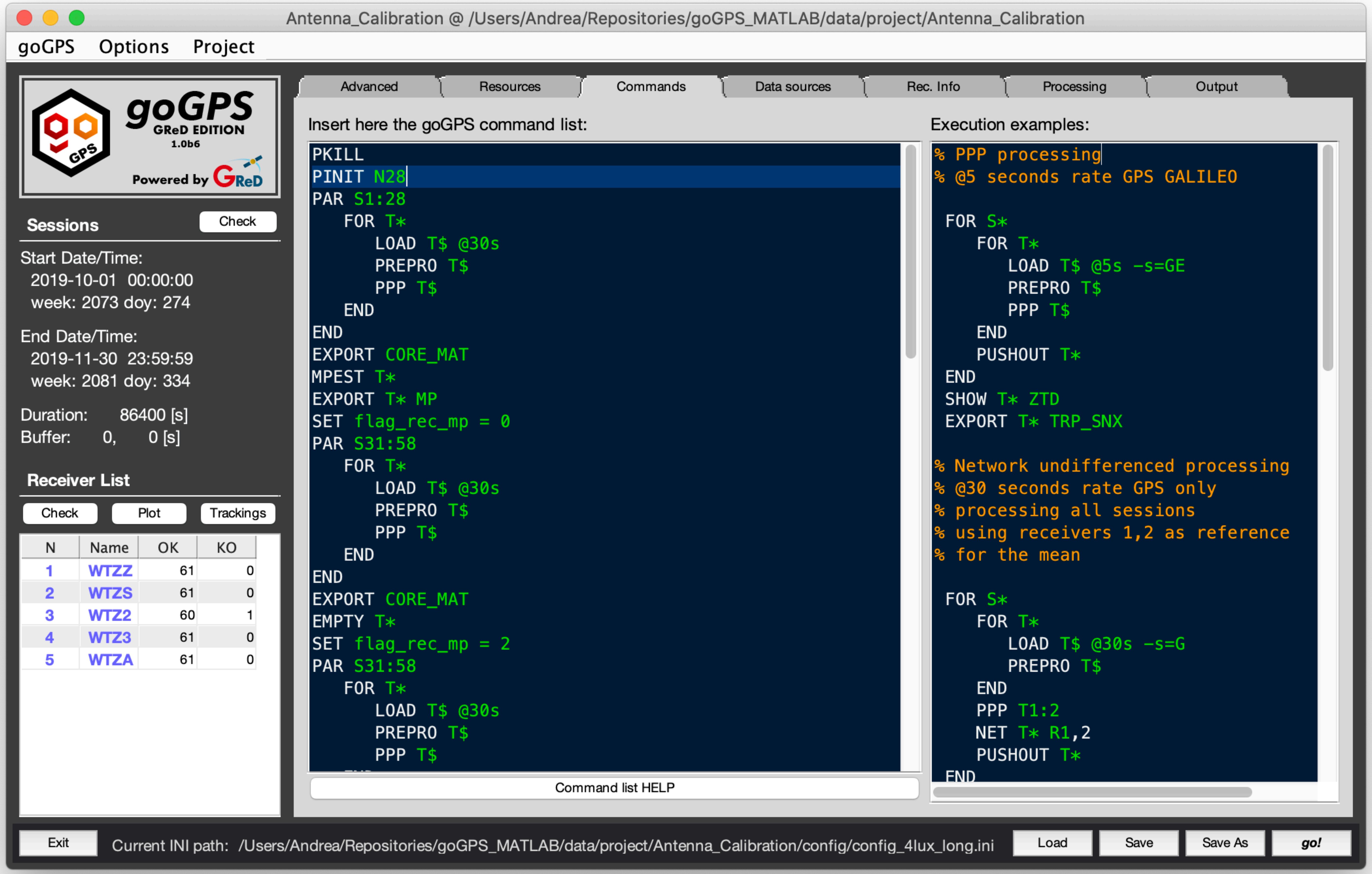
Every test performed for this presentation have been executed in a unique run of goGPS

The software is already capable of producing and using all the multipath maps here presented.

Multipath maps are available also for the uncombined engine provided with goGPS but more testing is required to evaluate their quality and reliability

Feel free to download and test goGPS from:

https://gogps-project.github.io



Conclusions

Multipath maps are an old but **valid instrument** to **improve multi-constellation PPP** solutions for both positioning and tropospheric studies

- Zernike polynomials can be used to interpolate the residuals but they are **limited** by the **maximum** estimated **degree**, increasing the degree the generation of the grids became slower and instabilities may decrease the performance of the interpolation
- When **coupled with** an higher resolution **gridding** they **perform better** in most of the cases tested
- **goGPS integrates** all the techniques showed here and can be used for testing different approaches, some minor improvements are ongoing but the code is close to its completion
- During the test of these products the gain percentage of the multipath mitigation changed by modifying the various processing parameters
- A deep study relative on the optimal time span for the creation of the maps can improve the performance of the maps in fast changing scenarios
- The advantage of the maps depends on the level of multipath influencing the observations of the station

Future works

- The usage of Zernike polynomials can be extended in geodetic studies when hemispherical maps are required
- In goGPS (experimental) Zernike can already be used to model tropospheric delays, but further test are required to assess the quality of the estimated Slant Total Delays
- The paper on which this presentation is based is almost ready, some other numerical test are ongoing to better access the performances of the method, but it is almost ready for submission