Fusion Applied to Phase Altimetry in a GNSS Buoy System

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Outlines

- **Altimetry measurements**

  Approaches and associated geometries

  Observable

  Linear-Circular Regression

- **Experimentations**

  - Synthetic data

    Noise influence
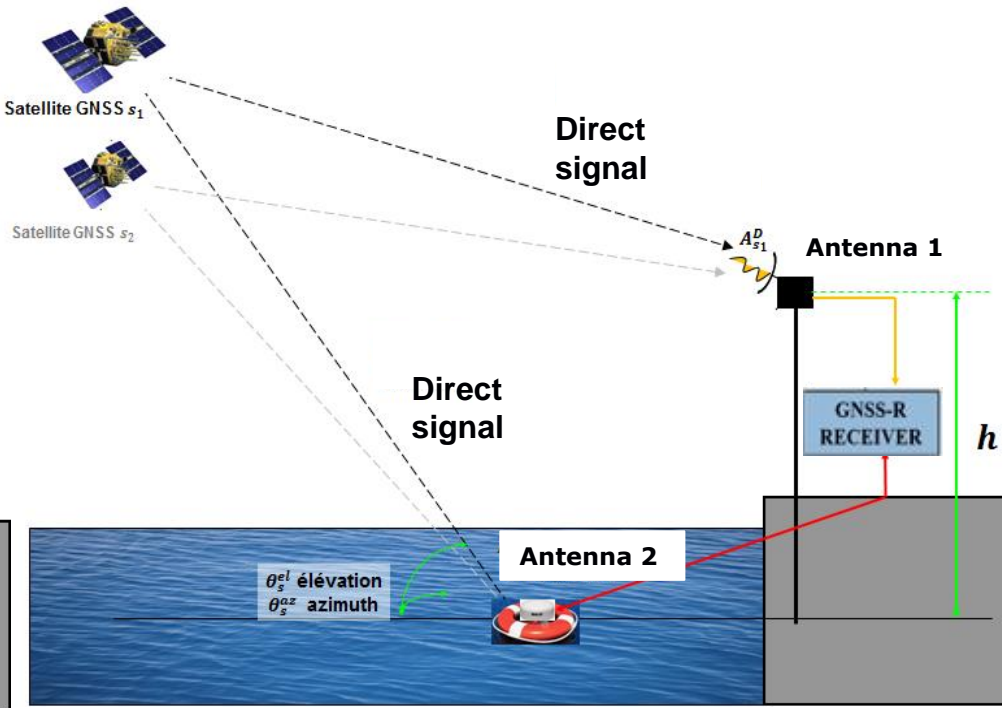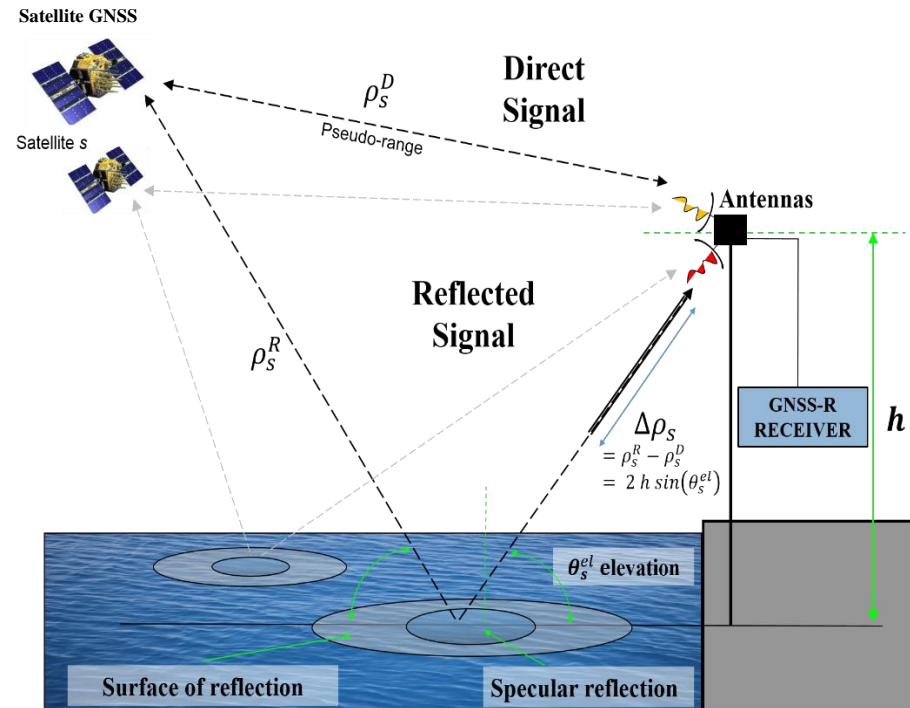
    Geometry influence

  - Real data

    Fusion application

- **Conclusion**

Outlines

Altimetry measurement systems

Reflectometry approach

Buoy approach

Reflectometry approach:

Satellite GNSS

Satellite s

ρ_s^D

Pseudo-range

Direct Signal

Antennas

Reflected Signal

ρ_s^R

$\Delta\rho_s$
$= \rho_s^R - \rho_s^D$
$= 2\,h\,sin(\theta_s^{el})$

GNSS-R RECEIVER

h

θ_s^{el} elevation

Surface of reflection

Specular reflection

Buoy approach:

Satellite GNSS s_1

Satellite GNSS s_2

Direct signal

$A_{s_1}^D$

Antenna 1

Direct signal

GNSS-R RECEIVER

h

Antenna 2

θ_s^{el} élévation
θ_s^{az} azimuth

Reflectometry approach's geometry

Path difference: Phase difference:
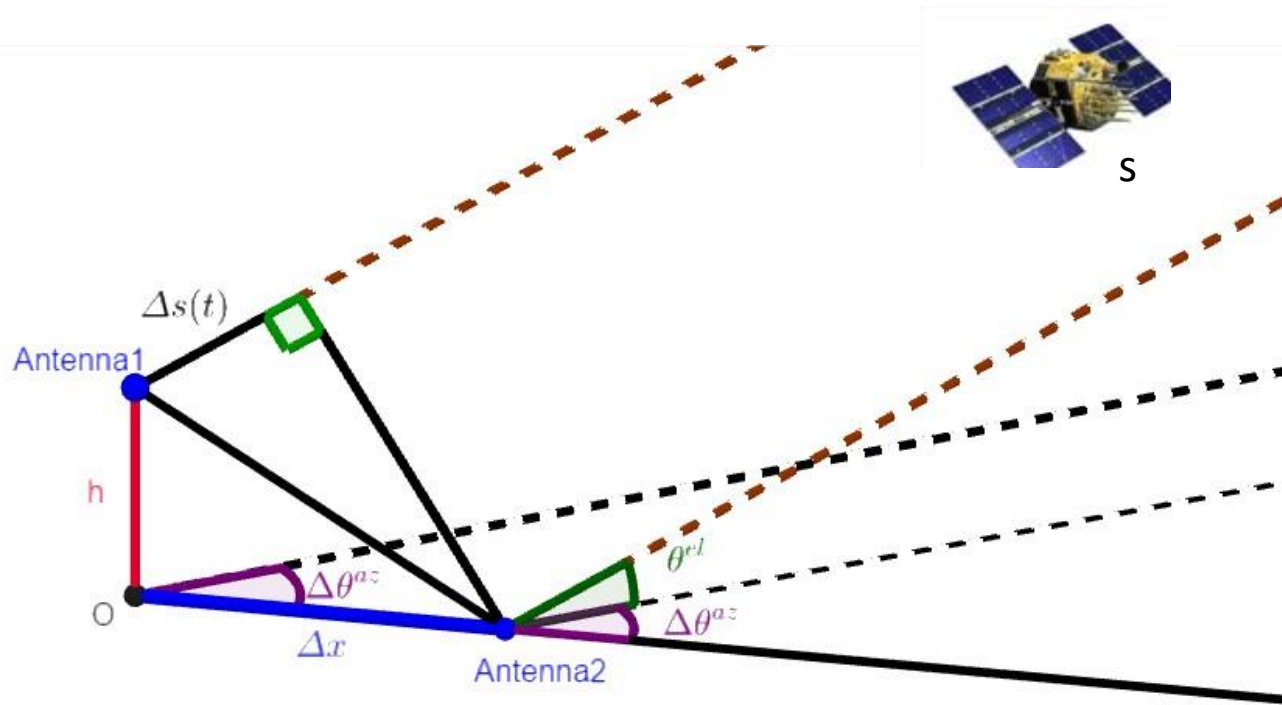
$$\Delta_s(t) = 2h \sin\left(\theta_s^{el}(t)\right)$$

$$\Psi_s(t) = \left(\tfrac{2\pi}{\lambda}\Delta_s(t)\right) mod[2\pi]$$

$$\Psi_s(t) = \left(\tfrac{4\pi}{\lambda} h sin(\theta_s^{el}(t))\right) mod[2\pi]$$

This a geometry for a reception system close to the ground. Thus, the incoming signals are supposed to be respectively parallel and we assume the reflection surface to be flat.

Buoy approach's geometry

Path difference:

$$\Delta_s(t) = \Delta x cos(\theta_s^{el}) cos(\Delta\theta_s^{az}) - h sin(\theta_s^{el})$$

Phase difference:

$$\Psi_s(t) = \left(\frac{2\pi\Delta_s(t)}{\lambda}\right) mod(2\pi)$$

$$\Psi_s(t) = \left(\frac{2\pi}{\lambda}(\Delta x cos(\theta_s^{el}) cos(\Delta\theta_s^{az}) - h sin(\theta_s^{el}))\right) mod(2\pi)$$

Observable

Model:
$$\Omega_s(t) = \left(\tfrac{2\pi}{\lambda}(\Delta x cos(\theta_s^{el})cos(\Delta\theta_s^{az}) - hsin(\theta_s^{el}))\right)$$

Measures:
$$\Psi_s(t) = \left(\tfrac{2\pi}{\lambda}(\Delta x cos(\theta_s^{el})cos(\Delta\theta_s^{az}) - hsin(\theta_s^{el})) + \nu_s(t)\right) mod(2\pi)$$

$\nu_s(t)$ is a zero-mean additive noise following a Circular Normal distribution

Assuming that the path difference Δx, the elevation and azimuth angles are known using GNSS positioning and satellites ephemeris, we have:

$$\gamma(t) = \frac{2\pi}{\lambda}\Delta x cos(\Delta\theta^{az}(t)) \cos(\theta^{el}(t)))$$

- Known part in [-π, π] :

$$\gamma 1(t) = \left[\frac{2\pi}{\lambda}\Delta x cos(\Delta\theta^{az}(t)) \cos(\theta^{el}(t)) + \pi\right] mod(2\pi) - \pi$$

- Obtained model in [-π, π] :
$$\Omega_{pi}(t) = \left[\gamma(t) - \frac{2\pi}{\lambda}h\sin(\theta^{el}(t)) + \pi\right] mod(2\pi) - \pi$$

Observable

- New Phase difference in [-π, π]
  after removing γ1 :

$$\Psi_{lcr}(t) = \left[\Psi(t) - \gamma1(t) + \pi\right] mod(2\pi) - \pi$$

$$\Psi_{lcr}(t) = \left[-\frac{2\pi}{\lambda}h\sin(\theta^{el}(t)) + \pi + \nu(t)\right] mod(2\pi) - \pi$$

➢ This new phase observable is always a circular data depending on h and the satellite elevation sine. Thus we can apply a **linear-circular regression** approach in order to estimate h.

Linear-circular regression

➢ We Consider n measurements following a linear-circular model:

$$\Psi_n = (\alpha + \beta x_n + \nu_s(t)) \, mod(2\pi)$$

Where X_n is a known vector and α and β are the parameters to estimate.

- The Von Mises distribution of the measurements is:

$$f(\Psi_n; \alpha, \beta, \kappa) = \frac{1}{2\pi I_0(\kappa_s)} exp(\kappa \cos(\Psi_n - (\alpha + \beta \, x_n)))$$

- The measurements likelihood is :

$$L(\Psi) = f(\Psi_1; \alpha, \beta; \kappa), ..., f(\Psi_N; \alpha, \beta; \kappa)$$

$$= \prod_{n=1}^{N} \frac{1}{2\pi I_0(\kappa_s)} \exp\left(\kappa \cos\left(\Psi_n - (\alpha + \beta x_n)\right)\right)$$

$$= \frac{1}{(2\pi I_0(\kappa))^N} \exp\left(\kappa \sum_{n=1}^{N} \cos\left(\Psi_n - \alpha - \beta x_n\right)\right)$$

Linear-circular regression

- The log-likelihood:

$$\ln L(\Psi) = -N \ln \left((2\pi I_0(\kappa)) \right) + \kappa \sum_{n=1}^{N} \cos \left(\Psi_n - \alpha - \beta x_n \right)$$

- Maximizing the log-likelihood, we derive an estimation of α :

$$\frac{\partial L(\Psi)}{\partial \alpha} = 0 \qquad \Longrightarrow \qquad \hat{\alpha} = \arctan^* \left(\frac{\sum_{n=1}^{N} \sin \left(\Psi_n - \beta\, x_n \right)}{\sum_{n=1}^{N} \cos \left(\Psi_n - \beta\, x_n \right)} \right)$$

- The maximum likelihood estimates of α and β can be obtained recursively using the Newton-Raphson Algorithm:

$$\hat{\alpha}^i = \arctan^* \left(\frac{\sum_{n=1}^{N} sin(\Psi_n - \hat{\beta}^i\, x_n)}{\sum_{n=1}^{N} cos(\Psi_n - \hat{\beta}^i\, x_n)} \right)$$

$$\hat{\beta}^{i+1} = \hat{\beta}^i + g \left(\frac{\partial^2 L(\Psi)}{\partial \beta^2} \right)^{-1} \left(\frac{\partial L(\Psi)}{\partial \beta} \right) \qquad \Longrightarrow \qquad \hat{\beta}^{i+1} = \hat{\beta}^i + g \left(\frac{\sum_{n=1}^{N} x_n \sin(\Psi - (\hat{\alpha}^i + \hat{\beta}^i\, x_n))}{\sum_{n=1}^{N} (x_n)^2 \cos(\Psi - (\hat{\alpha}^i + \hat{\beta}^i\, x_n))} \right)$$

Linear-circular regression

In our application:

$$\Psi_n = (\alpha + \beta x_n + \nu_s(t)) \, mod(2\pi)$$ With :

- $x_n = \sin\theta_n^{el}, n \in 1..N$

Where n is the sampling instant

- $\alpha = 0$

Obtained directly from the expression of Ψ_{lcr}

- $\hat{\beta} = \frac{-2\pi}{\lambda}\hat{h}$

Thus $\hat{h}$ can be estimated from the expression of $\hat{\beta}$

Outlines

- Altimetry measurements

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    Observable

    Linear-Circular Regression

- Experimentations

    - Synthetic data

        Noise influence

        Geometry influence

    - Real data

        Fusion application

- Conclusion

Evaluations on synthetic data

Noise level influence:

▪ Simulations parameters:

- $\Delta x = 20$ m
- Height to estimate: h=20 m
- Number of realizations: 100
- Carrier frequency: 1,57542 GHz
- 3s of signal
- Mean of elevation: 30°
- Mean of $\partial\theta(t)/\partial t$: 0,005 °/s

$C/N_0(\mathrm{dB.Hz})$	50	40	30	20
RMSE (cm)	0,0402	0,0675	0,1884	0,2832

▪ **Conclusion**: We notice that the accuracy obtained on the estimated height depends on the noise level. The lower the noise is the better is the precision of the estimator. However, the Linear-Circular regression method is robust to higher noise levels

Evaluations on synthetic data

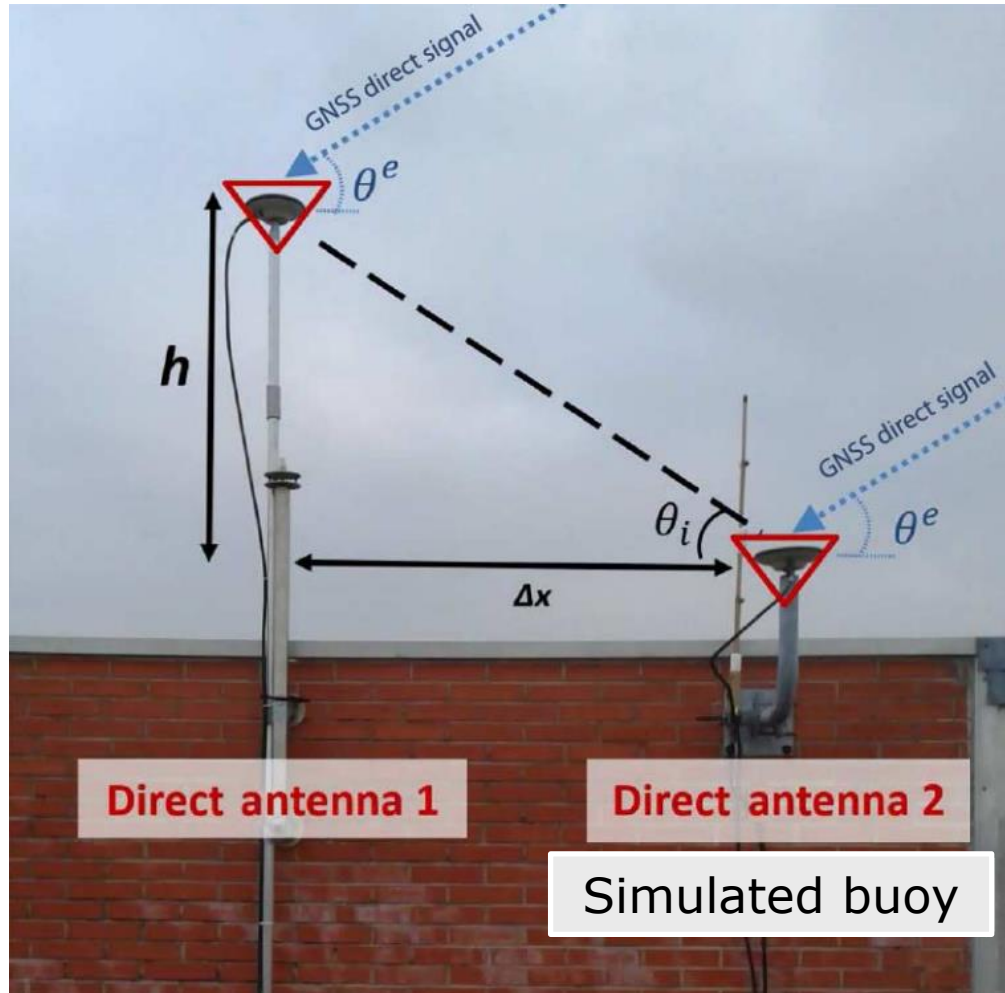### *Geometry Influence (C/N0 = 30dB.Hz):*

30° average elevation sine

$\frac{\partial(\theta^{el}(t))}{\partial(t)}(°/s)$	8.10^{-3}	5.10^{-3}	2.10^{-3}
$\frac{\partial(\sin\theta^{el}(t))}{\partial(t)}(s^{-1})$	$1,0300.10^{-4}$	$6,9033.10^{-5}$	$2,9261.10^{-5}$
Estimated RMSE (cm)	$0,1140$	$0,1884$	$0,4053$

70° average elevation sine

$\frac{\partial(\theta^{el}(t))}{\partial(t)}(°/s)$	8.10^{-3}	5.10^{-3}	2.10^{-3}
$\frac{\partial(\sin\theta^{el}(t))}{\partial(t)}(s^{-1})$	$1,9281.10^{-5}$	$1,8830.10^{-5}$	$1,0200.10^{-5}$
Estimated RMSE (cm)	$0,6089$	$0,6859$	$1,2727$

Conclusion: For a visible satellite, the variation of its elevation sine over the time must be fast for a better precision on the estimate. The best satellites in this case are those for which the elevation sine is varies in an interval which is on the increasing part of the representative sinus curve. For satellites whose elevation sine varies over a given interval which is close to the top of the representative sinus curve, their elevation sine varies slowly and the accuracy is worst.

Fusion Application: Real data

GNSS direct signal

θ^e

h

GNSS direct signal

θ_i θ^e

Δx

Direct antenna 1   **Direct antenna 2**

Simulated buoy

 In our implementation on real data, we use an experimental setup that allows recording of two direct signals. This setup is presented in this figure.

In the proposed setup, the GNSS signal are received using two GPS antennas. The processing are done on a software receiver. For this experimentation, we record 18s of data for a distance and a height between the two antennas of Δx=144cm and h=51,30 cm.

Fusion Application: Real data
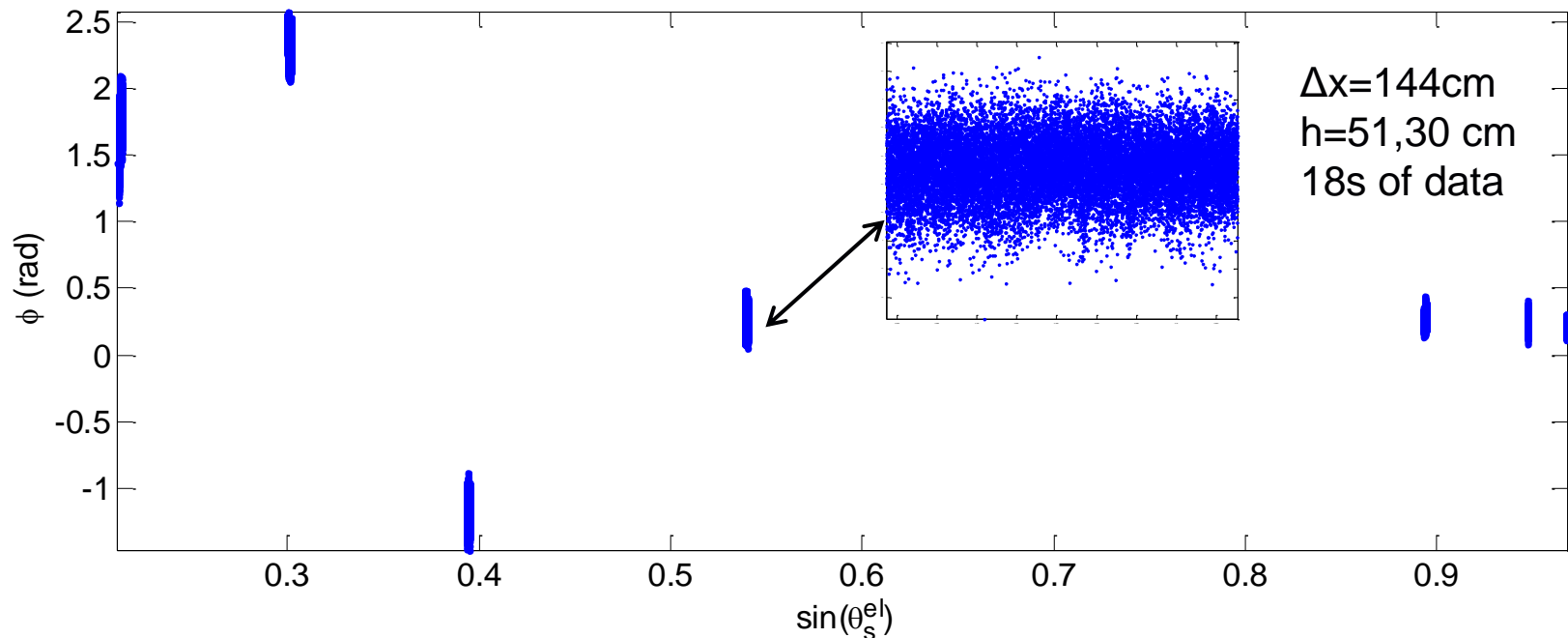
In the experimentation using synthetic data, we considered N phase measurements. In the case of the fusion, the amount of data is extended to N*S where S is the number of satellites. Thus, the advantage here, is to increase the amount of data and the elevation sine span like in this figure. Then, we show (in the next table) that we can reach a centimeter precision using this technic.

Fusion Application: Real data

$\Delta x = 144$ cm
h=51,30 cm
18s of data
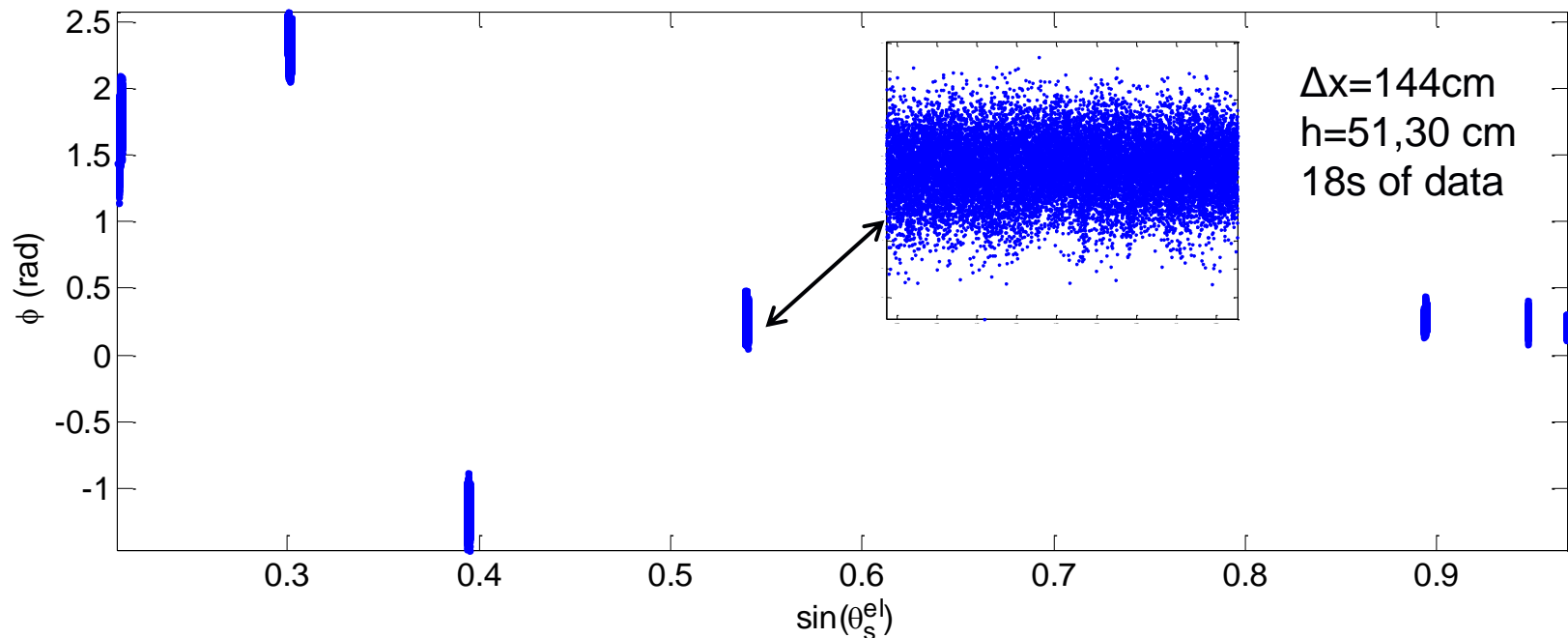
Satellite PRN	16	20	21	26	27	29	31
Estimated mean error(cm)	87.85	-44.15	299.8	82.35	-127.6	-63.58	502.0
Estimated mean error after fusion (cm)	3.33						

Conclusion

- measures in z axis in GPS are less accurate than x and y
- Use of altimetry for height measurements
- Linear-Circular Regression: Robust to high noise levels
- High influence of the elevation sine on the accuracy
- Centimeter precision
- Better results using data fusion over several satellites

Perspectives:

- Real setting with a buoy
- Joint estimation of Δx and h