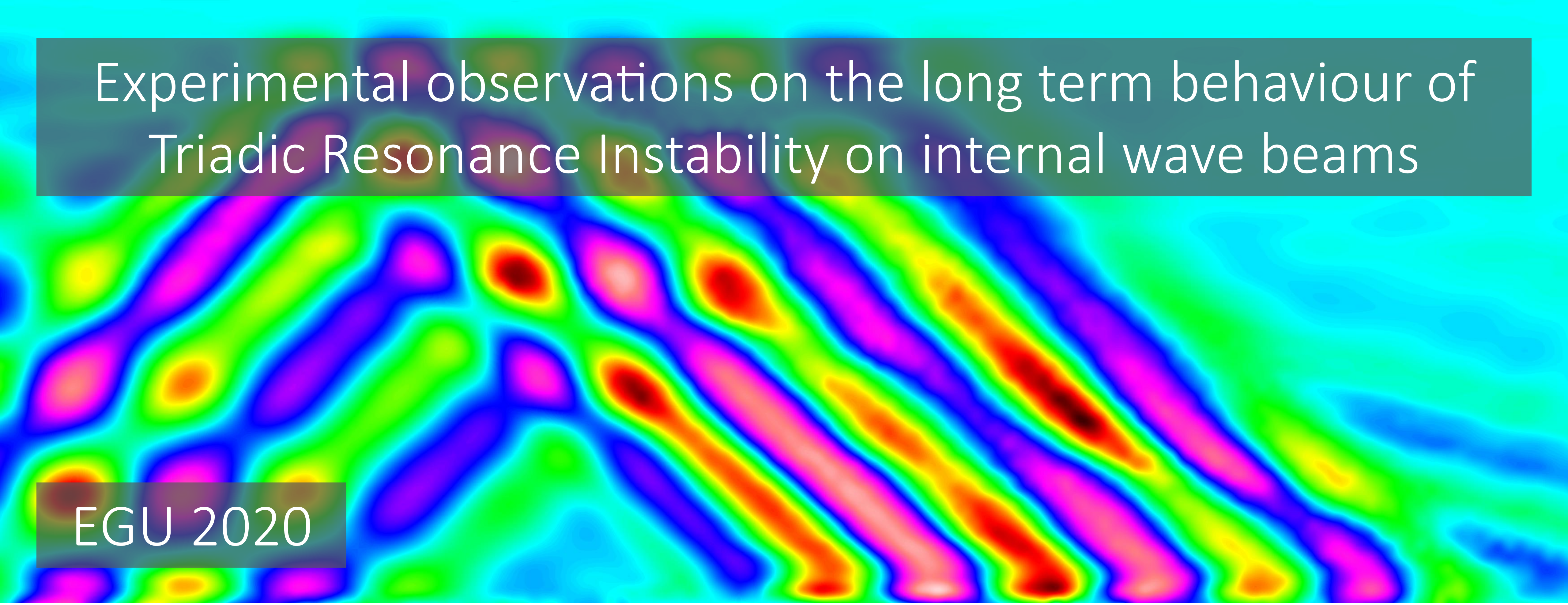
Experimental observations on the long term behaviour of Triadic Resonance Instability on internal wave beams



EGU 2020

DAMTP (Department of Applied Mathematics and Theoretical Physics)
Katherine Grayson

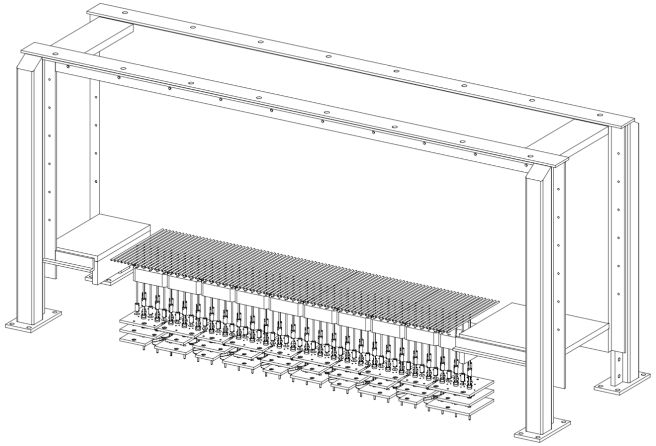
Supervised by:
Stuart Dalziel *(University of Cambridge)*
Andrew Lawrie *(University of Bristol)*

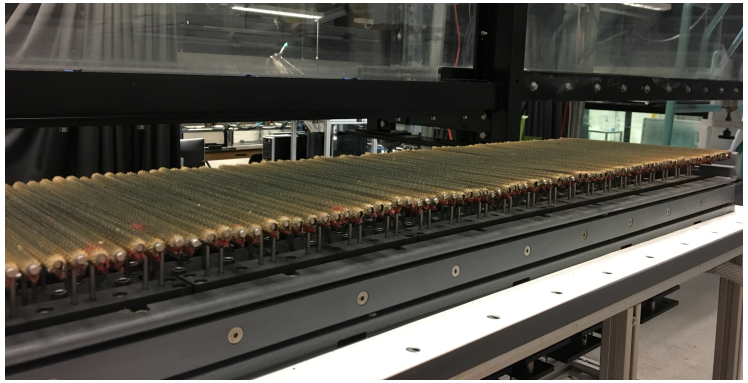
UNIVERSITY OF CAMBRIDGE

NERC DOCTORAL TRAINING

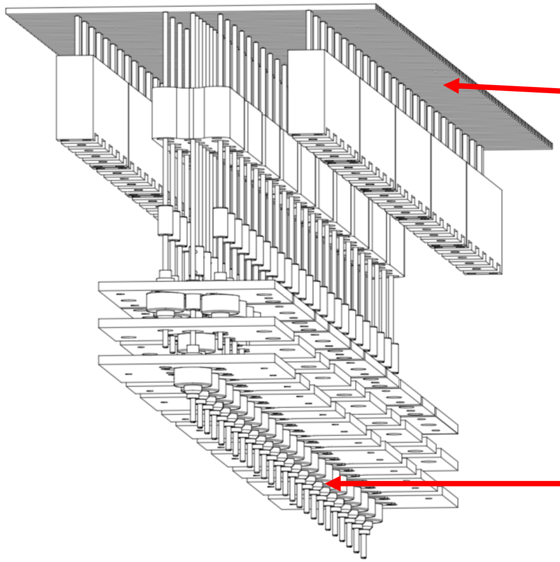
Experimental Setup

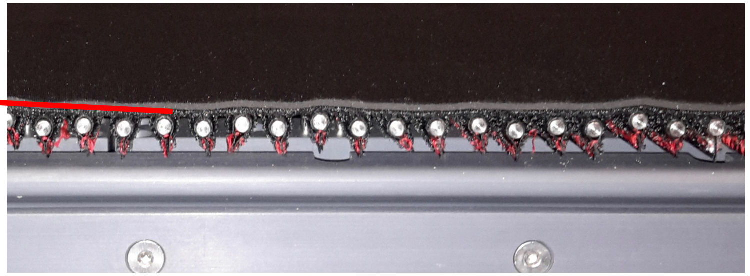
- Perspex Tank
  - 255 mm wide
  - 625 mm deep
  - 11 m long

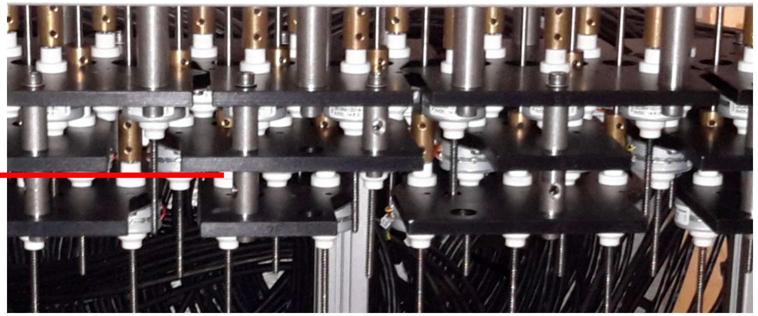




- Arbitrary Spectrum Wave Maker (ASWaM)
  - 96 independently controlled horizontal rods



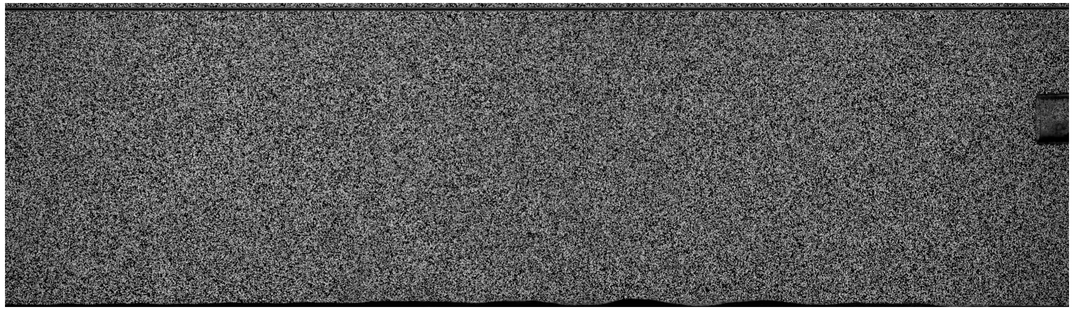




Dobra, T., Lawrie, A., & Dalziel, S. (2019). The magic carpet: an arbitrary spectrum wave maker for internal waves. *Experiments in Fluids*.

Synthetic Schlieren

Raw Image



Here the tank is filled with a **linear stratification.**

N = 1.69 rad s^{-1}

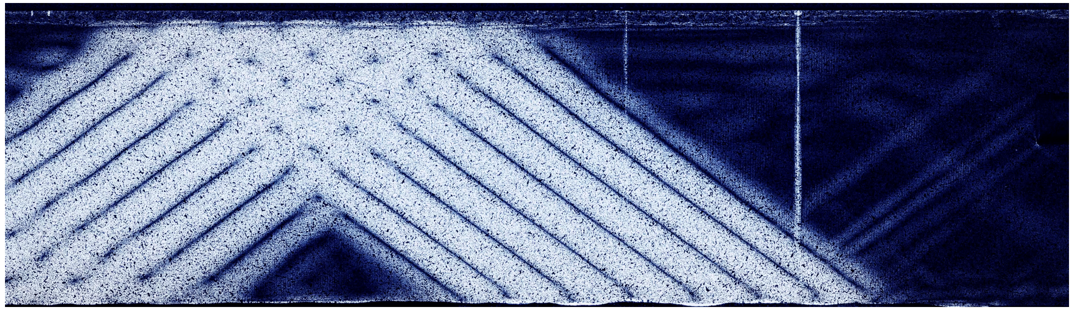
Forcing parameters of wave from wavemaker are:

l = 0.05 mm^{-1}

ω = 1 rad s^{-1} $\frac{\omega}{N}$ = 0.6

η_o = 4 mm $\frac{k_x}{\eta_o}$ = 0.0125

Qualitative Preview



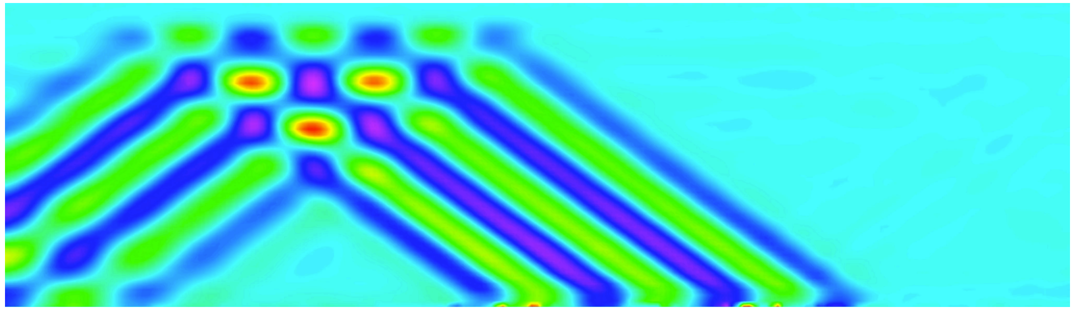
Output = $\alpha \left| Im - Im_{ref} \right|$

Once processed Synthetic Schlieren measures the gradient of the density perturbation in both x and z

$$\beta_z = \frac{1}{\rho_0}\frac{\partial \rho'}{\partial z}$$

$$\beta_x = \frac{1}{\rho_0}\frac{\partial \rho'}{\partial x}$$

Processed Output



Output = $\beta_z = \frac{1}{\rho_0}\frac{\partial \rho'}{\partial z}$

What is Triadic Resonance Instability?

- Weakly non-linear resonant mechanism, also known as Parametric Subharmonic Instability (PSI)
- Primary wave becomes unstable to infinitesimal perturbations and generates the growth of two secondary waves
- Two resonant waves must satisfy:

$\omega_0 = \omega_1 + \omega_2$ (temporal resonant condition)

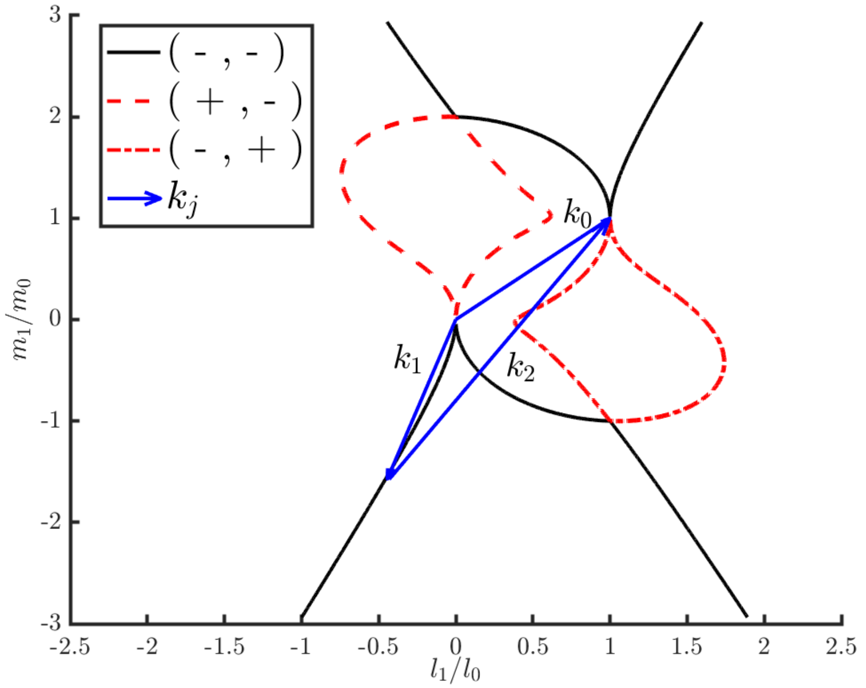
$\boldsymbol{k}_0 = \boldsymbol{k}_1 + \boldsymbol{k}_2$ (spatial resonant condition)

The interaction must also satisfy the dispersion relationship

$$\frac{\omega_j}{N} = \frac{|l_i|}{\sqrt{l_i^2+m_i^2}} \text{ for i = 0, 1 or 2.}$$

Here $k = (l, m)$ Solving the above provides the relationship between the wavenumber vectors in the Triadic Resonance

$$s_0 \frac{|l_0|}{\sqrt{l_0^2+m_0^2}} = s_1 \frac{|l_1|}{\sqrt{l_1^2+m_1^2}} + s_2 \frac{|l_0 - l_1|}{\sqrt{(m_0 - m_1)^2+(l_0 - l_1)^2}}$$



Resonant loci curve showing all possible solutions of $\boldsymbol{k}_1$ given $\boldsymbol{k}_0$. Blue arrows indicate experimental wavenumber vectors.

Where $k = (l, m)$. In these experiments s_0 is negative. That leaves combinations of (+,+), (-,+), (+,-) and (-,-) for s_1 and s_2. Only (-,-) provides resonantly growing solutions. Solving this resonant condition with a known input wave vector k_0 provides all the possible values of the first resonant wave k_1. Shown as the black curve.

Current Theory

Using weakly nonlinear theory the temporal development for the amplitude of the wave beams is given as

$$\frac{d\Psi_0}{dt} = -|I_0|\Psi_1\Psi_2 - \frac{\nu}{2}\kappa_0^2\Psi_0 + c_{g,0}\frac{(\Psi_{in} - \Psi_0)}{2L}$$

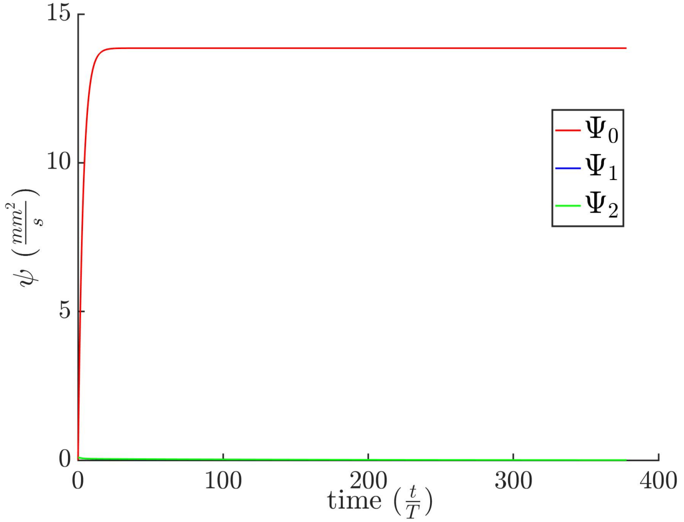
$$\frac{d\Psi_1}{dt} = +|I_1|\Psi_0\Psi_2^* - \left(\frac{\nu}{2}\kappa_1^2 + \frac{|\boldsymbol{c}_{g,1} \cdot \boldsymbol{e}_{k_0}|}{2W}\right)\Psi_1$$

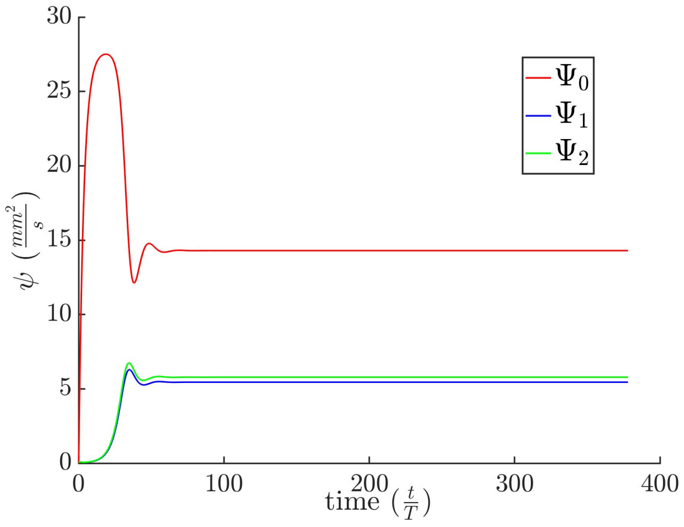
$$\frac{d\Psi_2}{dt} = +|I_2|\Psi_0\Psi_1^* - \left(\frac{\nu}{2}\kappa_2^2 + \frac{|\boldsymbol{c}_{g,2} \cdot \boldsymbol{e}_{k_0}|}{2W}\right)\Psi_2$$

Where

$$I_i = \frac{l_j m_r - m_j l_r}{2\omega_i\kappa_i^2}\left[\omega_i\left(\kappa_j^2 - \kappa_r^2\right) + l_i N^2\left(\frac{l_j}{\omega_j} - \frac{l_r}{\omega_r}\right)\right]$$ *for i, j, r = 0, 1, 2*

Solving this set of ODE's gives the amplitude of all three waves (Ψ_0 Ψ_1 and Ψ_2) against time. These are shown by these two graphs for two different initial forcing amplitudes. The top graph has an initial forcing amplitude $\Psi_0 = 15\, mm^2\, s^{-1}$ and the bottom graph the initial forcing amplitude $\Psi_0 = 30\, mm^2\, s^{-1}$





T is the period of the primary wave beam = 6.61 s

Amplitude of the resonant waves are therefore a function of the characteristics of the primary wave vector and frequency (I), Amplitude (Ψ_0) and viscosity (ν)

Bourget, B., Scolan, H., Dauxois, T., Le Bars, M., Odier, P., & Joubaud, S. (2014). Finite-size effects in parametric subharmonic instability. *Journal of Fluid Mechanics*, *759*, 739–750. https://doi.org/10.1017/jfm.2014.550

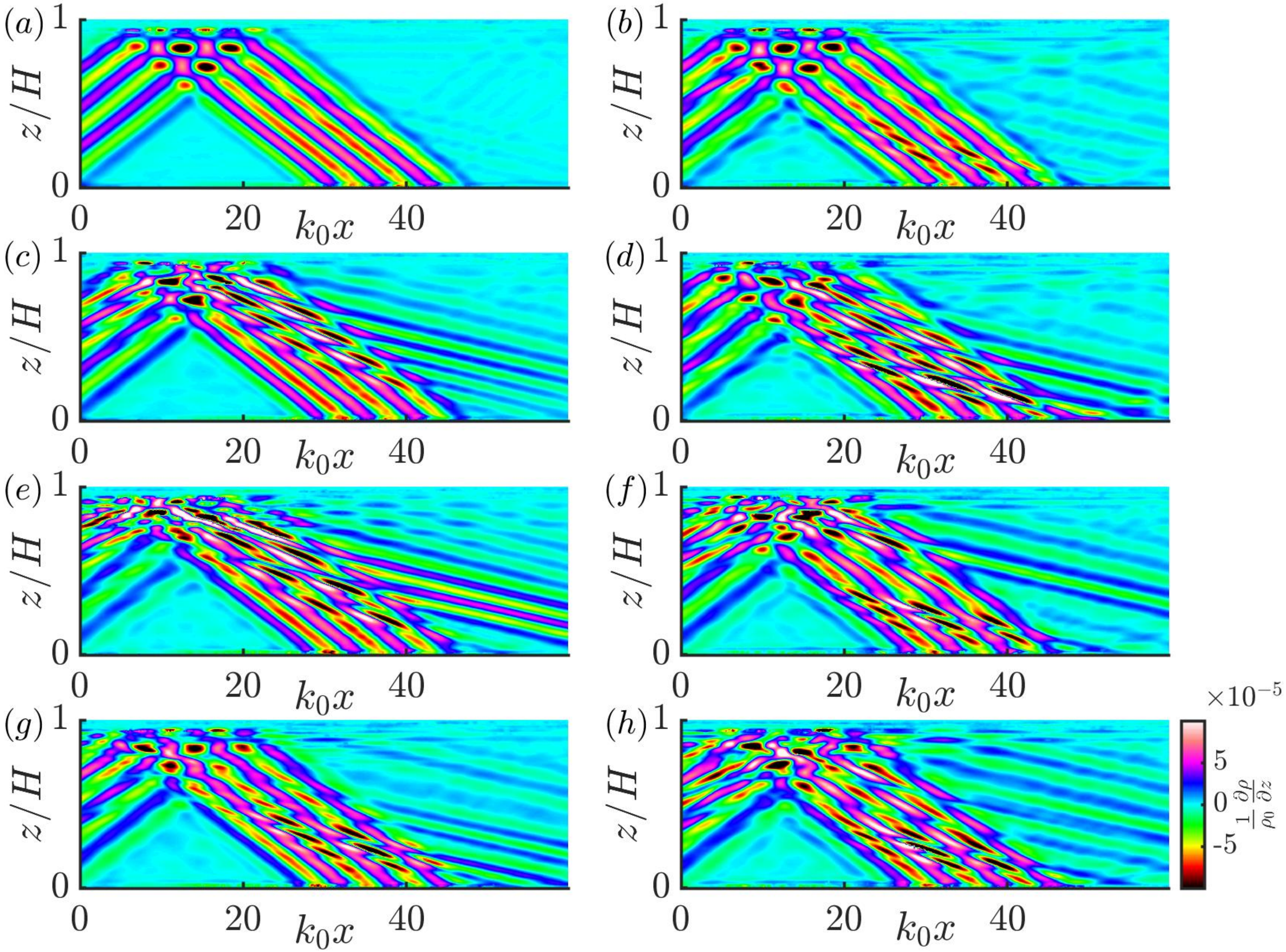
Snapshots of Experiments

We are interested in the slow evolution of the Triadic Resonance Instability

Forcing parameters of wave from ASWaM are:

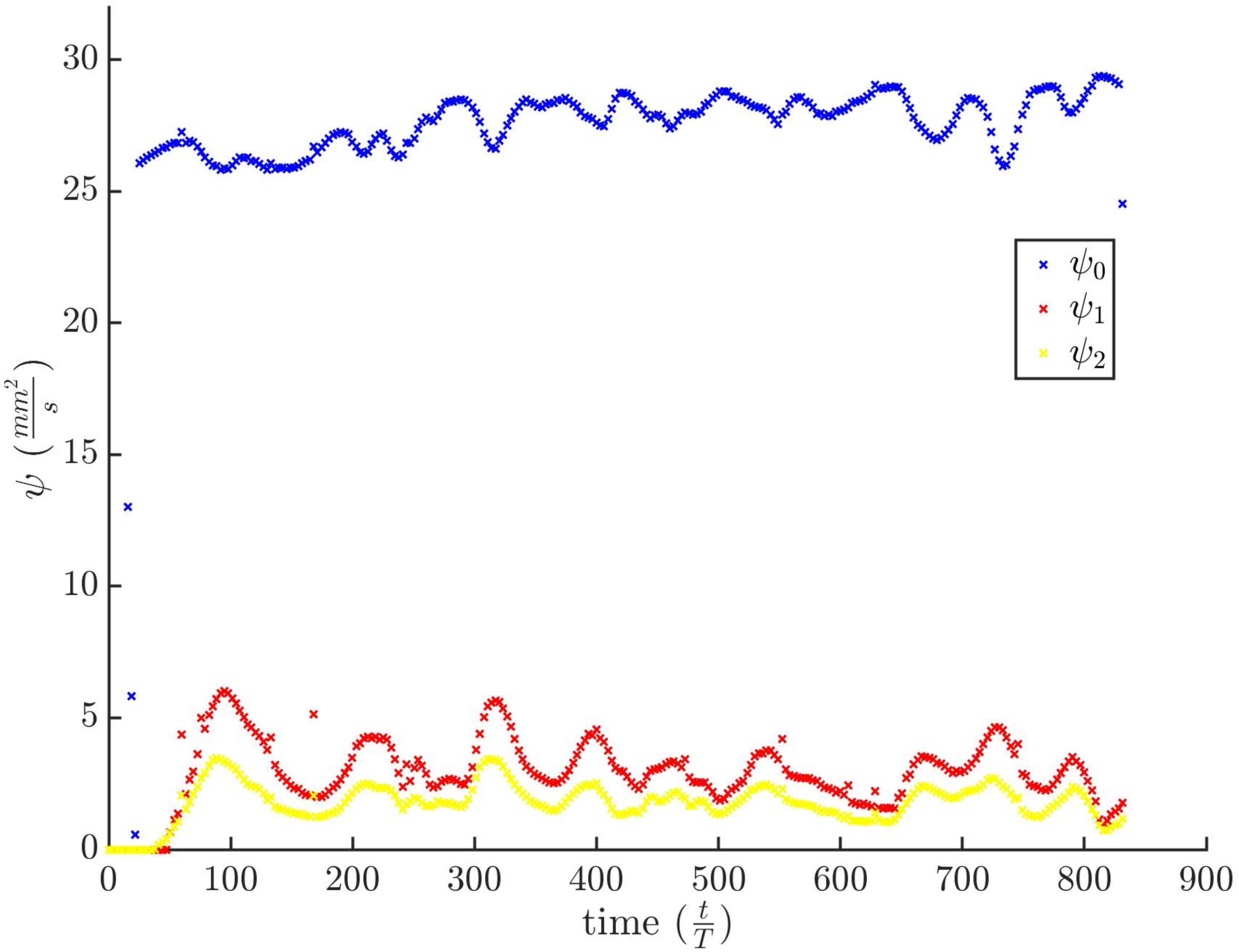
l = 0.05 mm^{-1}
Beam width = 4 λ
$\frac{\omega}{N}$ = 0.6
η_o = 4.5 mm (forcing amplitude)

Where $k = (l, m)$. Snapshots are shown 200 s apart. These experiments show both varying location and amplitude of the resonant wave beams



Experimental Results

- Amplitude of the Streamfunction
- Results from this experiment (η_o = 4.5 mm forcing amplitude) shown for the duration of 1.5 hours.
- Continuous oscillation of amplitude of all three beams.
- 36 experiments showed the same behavior.
- Some experiments run for 3 hours did **not** reach a steady amplitude.



Conclusions

- Current theory predicts that after an initial instability period the energy flux between the three waves in the triad should remain constant.

- Experimental evidence suggests that there is a continuous oscillation to the energy between the three waves.
    - Some experiments run for 3 hours did **not** reach a steady amplitude.
    - The location and frequency of the resonant waves was also seen to oscillate.
    - Shown for a range of amplitudes.

Further work:

- Develop a two-dimensional numerical theory to capture this energy transfer.

- Preform further experiments exploring the role of beam width, stratification and input frequency to the instability.