Towards non-linear inverse problem for atmospheric source term determination

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Problem formulation

- we assume linear model of atmospheric dispersion using a source-receptor sensitivity (SRS) matrix $\mathbf{M}$ as

$$\mathbf{y} = \mathbf{M}\mathbf{x} + \mathbf{e}, \tag{1}$$

$\mathbf{y} \in \Re^{p}$ is a vector aggregating measurements
$\mathbf{M} \in \Re^{p \times n}$ is the SRS matrix
$\mathbf{x} \in \Re^{n}$ is a vector of the unknown release to be estimated
$\mathbf{e} \in \Re^{p}$ is error model

Atmospheric model error

- SRS matrix $\mathbf{M}$ is traditionally assumed to be correct, which may be misleading
- here, we consider (in general) bi-linear model of the source term estimation problem in the form

$$\mathbf{y} = (\mathbf{M} + \Delta_{\mathbf{M}})\mathbf{x} + \mathbf{e}, \tag{2}$$

  where $\Delta_{\mathbf{M}}$ is the deviation of $\mathbf{M}$ from the "correct" SRS fields.
- the deviation $\Delta_{\mathbf{M}}$ can express, e.g., temporal shift and/or spatial shift

Bi-linear formulation

- bi-linear formulation of the problem

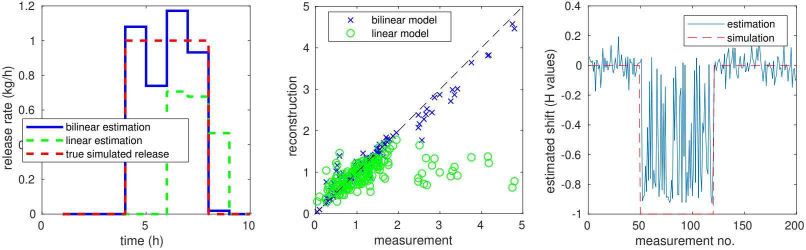
$$\mathbf{y} = \Bigg(\mathbf{M} + \underbrace{\operatorname{diag}(\mathbf{h}_t)}_{\mathbf{H}_t} \underbrace{(\mathbf{M}_{\text{t-shift+}} - \mathbf{M}_{\text{t-shift-}})}_{\mathbf{S}_t} \Bigg) \mathbf{x} + \mathbf{e}, \qquad (3)$$

  - $\mathbf{h}_t \in [-1; +1]$ are (unknown) coefficients
  - $\mathbf{M}_{\text{t-shift+}}$ and $\mathbf{M}_{\text{t-shift-}}$ are shifted SRS matrices

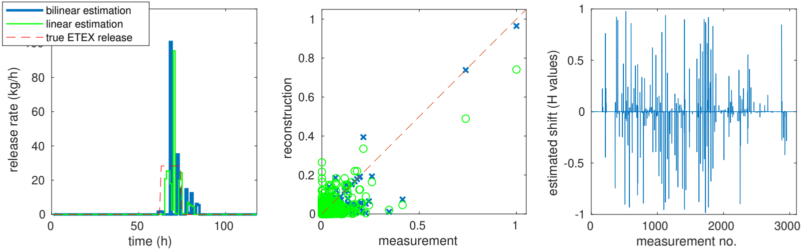
Variational Bayes solution (in short)

- prior $p(\mathbf{y})$ is modeled as Gaussian with estimated scalar precession
- $p(\mathbf{H}_t)$ is modeled according to the sparse Bayesian learning
  [Tipping, M. E. Sparse Bayesian learning and the relevance vector machine. Journal of machine learning research, 1, 211-244, 2001.]
- $p(\mathbf{x})$ is modeled as the LS-APC prior
  [O. Tichý, V. Šmídl, R. Hofman, and A. Stohl. LS-APC v1.0: a tuning-free method for the linear inverse problem and its application to source-term determination. Geoscientific Model Development, 9(11):4297–4311, 2016.]

Synthetic example



ETEX example



Preliminary conclusions

- it is possible to estimate parametric corruptions the SRS fields and correct them
- better measurements fit is observed (indeed, also overfiting in specific cases)

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