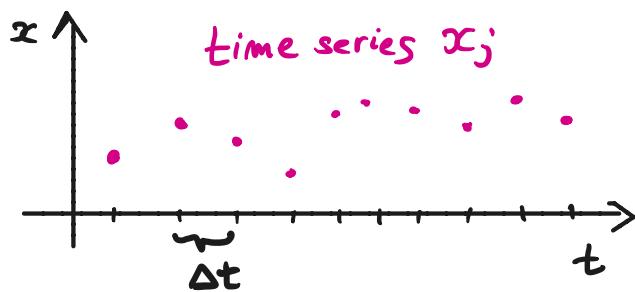


Introduction into Dynamic Mode Decomposition (DMD)



trajectory-wise picture : $\dot{x} = f(x) \quad x(0) = x_0$

- + finite dimensional
- non linear

Koopman-operator: observable $\vec{\varphi}(\vec{x})$ $X_t \vec{\varphi}(x) = \vec{\varphi}(x(t))$

- infinite dimensional
- + linear

X_t : Koopman operator

Dynamic Mode Decomposition

$$X = \begin{pmatrix} | & | & | & & | \\ \vec{\varphi}(x_{t_0}) & \vec{\varphi}(x_{t_1}) & \cdots & \cdots & \vec{\varphi}(x_{t_m}) \\ | & | & & & | \end{pmatrix}$$

$$\overset{'}{X} = \begin{pmatrix} | & | & | & & | \\ \vec{\varphi}(x_{t_0+\delta}) & \vec{\varphi}(x_{t_1+\delta}) & \cdots & \cdots & \vec{\varphi}(x_{t_m+\delta}) \\ | & | & & & | \end{pmatrix}$$

$$\overset{'}{X} = K^{(s)} X \rightsquigarrow K^{(s)} = \overset{'}{X} X^+$$

$K^{(s)}$ is a good approximation of the Koopman operator X_s if

- i) data sufficiently sparse
- ii) data sufficiently rich
(eigenfunctions of $\overset{'}{X}$ lie in span of $\{\vec{\varphi}\}$)

ii) is hardly ever satisfied and also not a priori verifiable

Reconstruction of dynamics

$$x(t_{k+1}) \approx \sum_{j=1}^r \phi_j e^{\omega_j k \Delta t} b_j + \text{c.c.} \quad K^{(s)} \phi_j = \lambda_j \phi_j$$

$$\omega_j = \ln \lambda_j / \Delta t$$

- good for short times ("linear")
- needs good truncation r
- Δt needs to be adapted to characteristic timescale

Main Message:



Transitions typically occur on time scales faster than the timescale of the equilibrium dynamics.

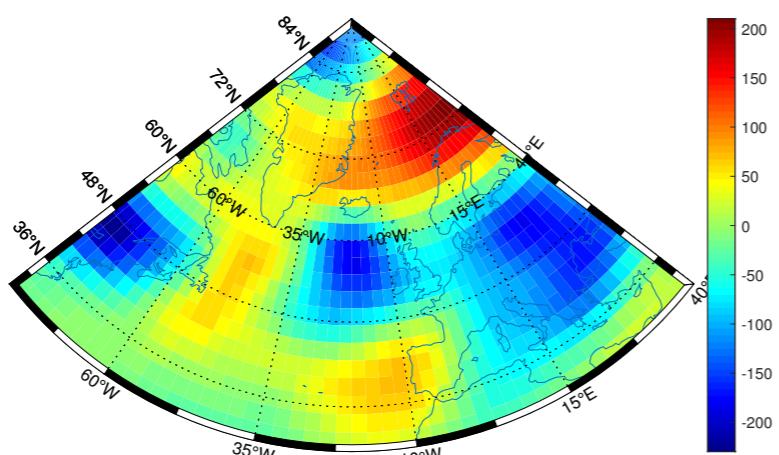
A sampling Δt chosen to compute $K^{(\delta t)}$ may not be sufficient to resolve the transient dynamics and hence DMD-reconstruction may be bad.

(Gottwald & Gugole, J Stat Phys. 2019)

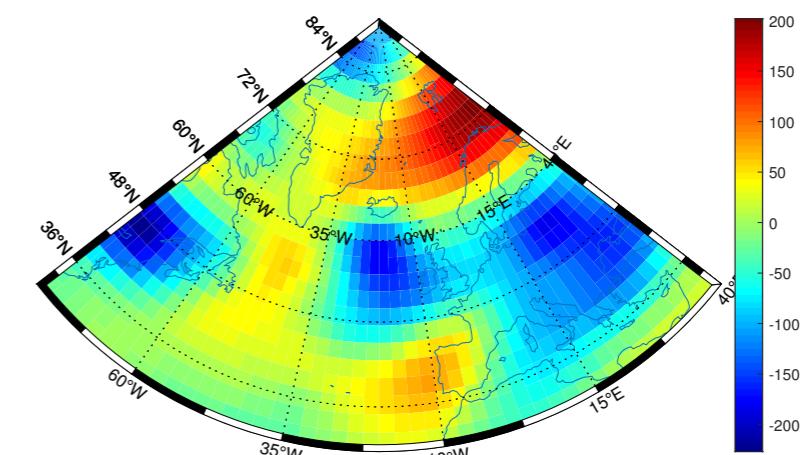
tested for artificial data from the Kuramoto-Sivashinsky equation, and for NCEP reanalysis data

Reconstruction error: $\mathcal{E}(t_k, r) = \frac{1}{m} \sum_{l=0}^{m-1} \|x(t_{k+l}) - \Phi \exp(\Omega l \Delta t) b_k - c.c\|$

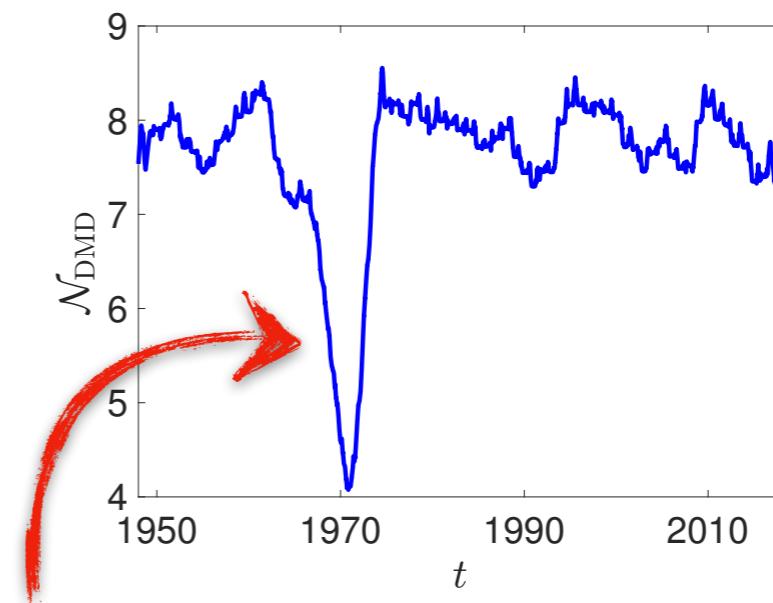
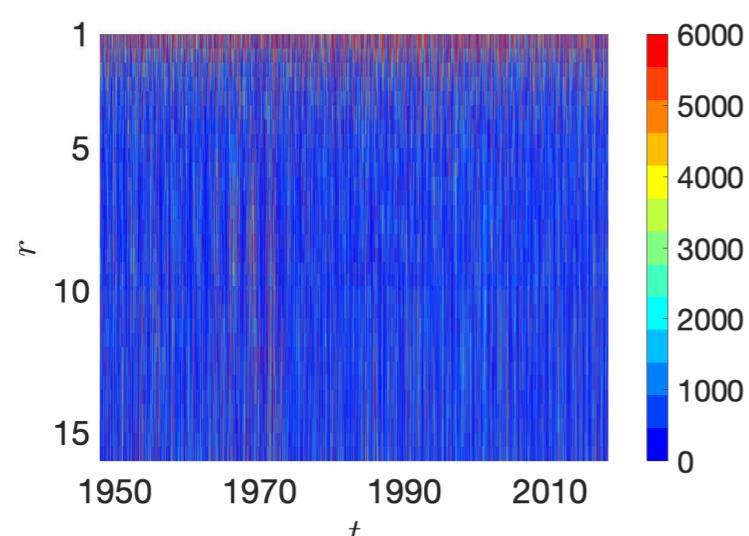
NCEP reanalysis data: Northern Hemisphere



reanalysis



DMD reconstruction



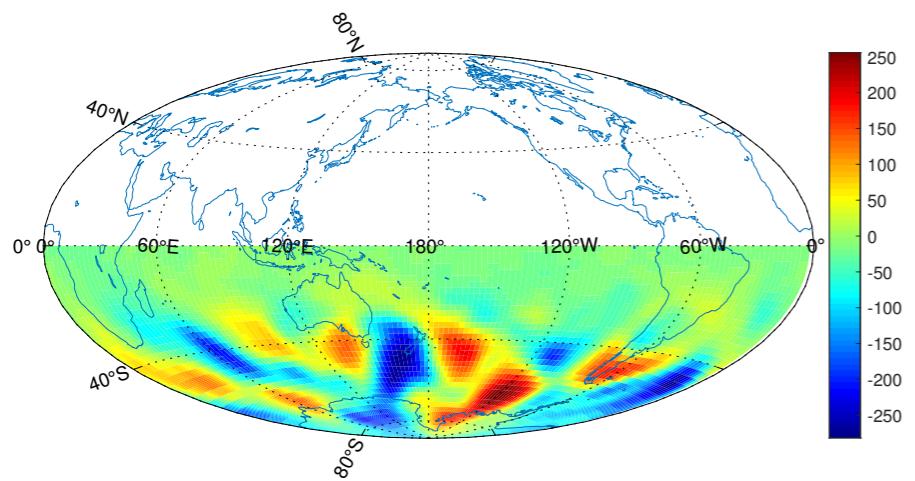
large-scale regime change in the 1970s

transition from a predominantly negative NAO phase to a predominantly positive NAO phase

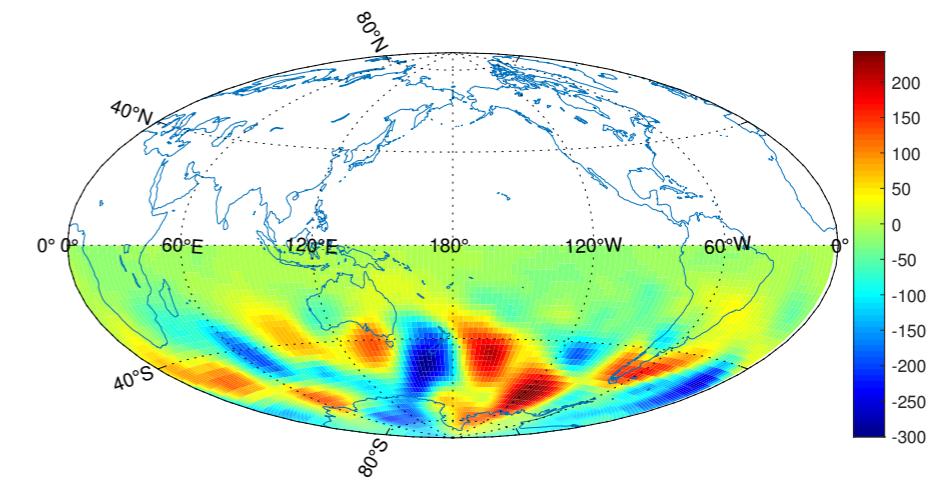
Reconstruction error:

$$\mathcal{E}(t_k, r) = \frac{1}{m} \sum_{l=0}^{m-1} \|x(t_{k+l}) - \Phi \exp(\Omega l \Delta t) b_k - c.c\|$$

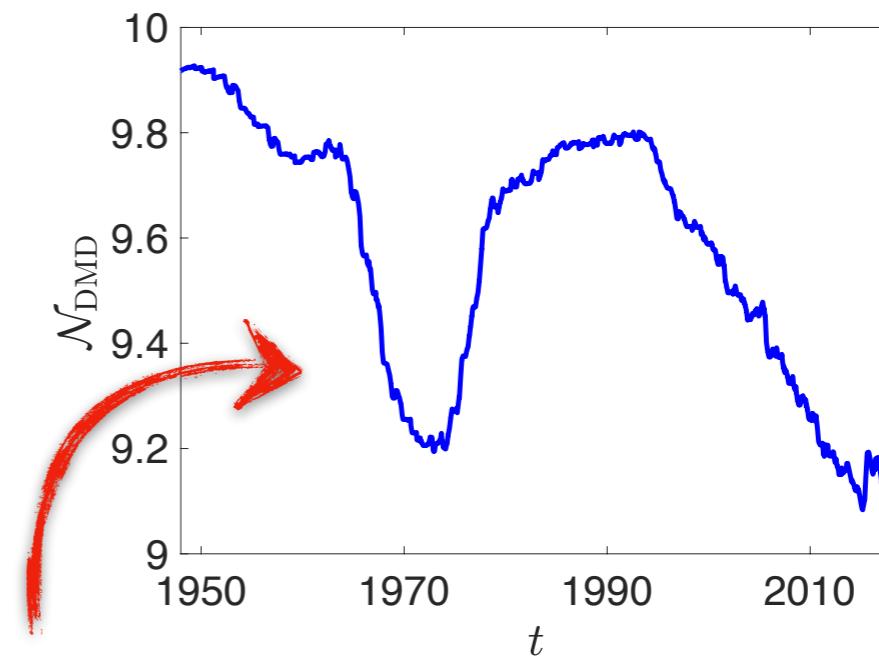
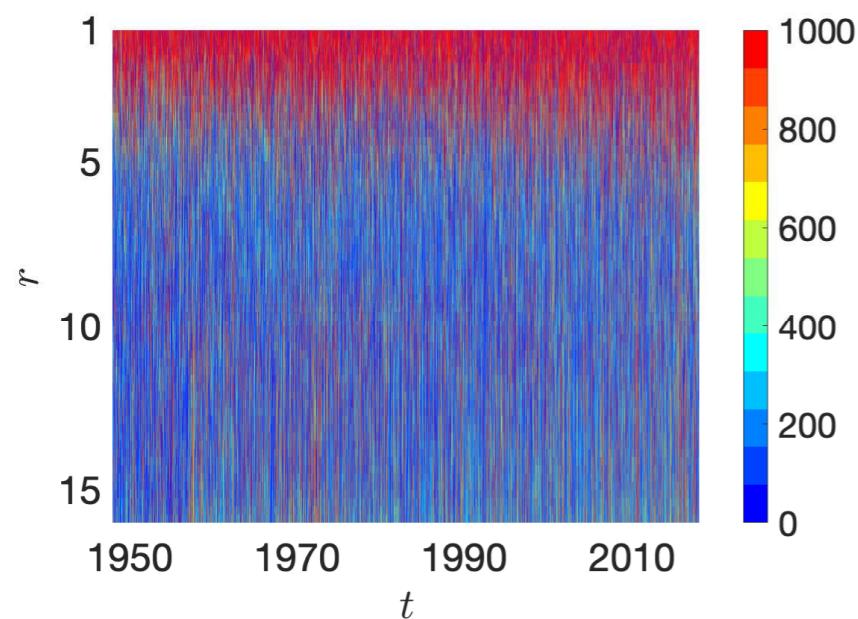
NCEP reanalysis data: Southern Hemisphere



reanalysis



DMD reconstruction



large-scale regime change in the 1970s

Significant decrease of frequency of blocking events around the mid 1970s and more zonal flow