Instantaneous Ambiguity Resolved GLONASS FDMA Attitude Determination

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G1 – Geodetic Theory and Algorithms --- G1.3 High-precision GNSS: methods, open problems and Geoscience applications

Theorem 1 (Integer-Estimability) *Let* $\mathsf{E}(y) = Az + Bb$ *be a mixed-integer model, where* $y \in \mathbb{R}^m$ $z \in \mathbb{Z}^n$ *and* $b \in \mathbb{R}^\nu$. *Then, the necessary and sufficient conditions for p linearly independent functions* $\tilde{z} = F^T z$ *to be estimable or integer-estimable are as follows*:

1. $\tilde{z} = F^T z$ *is estimable* iff $F = A^T B^\perp X$ *for some* X, *where* $B^\perp$ *is a basis matrix of the orthogonal complement of the range space of* B.

2. $\tilde{z} = F^T z$ *is integer-estimable* iff $F = A^T B^\perp X$ *for some* X *and* $F^T \mathcal{Z} = [I_p, 0]$ *for some admissible ambiguity transformation* $\mathcal{Z}$.

EXAMPLE

integer combination that is estimable is

$$\eta_{,j} = a_{12} z_{1r,j}^{13} - a_{13} z_{1r,j}^{12}$$

It is an integer combination of GLONASS DD ambiguities, which can be written in terms of the undifferenced ambiguities as $\eta_{,j} = F^T z$, where $F^T = [a_{23}, -a_{13}, a_{12}]$ and $z = [z_{1r,j}^1, z_{1r,j}^2, z_{1r,j}^3]^T$. As we have the decomposition

$$\underbrace{[a_{23}, -a_{13}, a_{12}]}_{F^T} \underbrace{\begin{bmatrix} \alpha & a_{13}/g & 1 \\ \beta & a_{23}/g & 1 \\ 0 & 0 & 1 \end{bmatrix}}_{\mathcal{Z}} = [g, 0, 0]$$

with $\alpha a_{23} - \beta a_{13} = g$ and $g = \mathrm{GCD}(a_{23}, a_{13})$, it directly follows that $\eta_{,j}$ is not integer-estimable in general. It is integer-estimable if $a_{23} = 1$, $a_{13} = 1$ or $a_{12} = 1$, since then $g = 1$. Note that $\mathrm{GCD}(a_{23}, a_{13}) = \mathrm{GCD}(a_{23}, a_{12})$.

Integer-estimable GLONASS DD model

The new GLONASS FDMA DD model of (Teunissen 2019) is given as

$$\mathsf{E}\begin{bmatrix} p \\ \phi \end{bmatrix} = \begin{bmatrix} e \otimes G & 0 \\ e \otimes G & \Lambda \otimes L \end{bmatrix} \begin{bmatrix} b \\ a \end{bmatrix} \quad (1)$$

in which $p \in \mathbb{R}^{2(m-1)}$ and $\phi \in \mathbb{R}^{2(m-1)}$ denote the DD pseudorange (code) and phase observables, m is the number of tracked satellites, $e = (1, 1)^T$, $\otimes$ denotes the Kronecker product, $G \in \mathbb{R}^{(m-1)\times\nu}$ is the relative receiver-satellite geometry matrix, $\Lambda = \mathrm{diag}(\lambda_1, \lambda_2)$ is the diagonal matrix of wavelengths; $L \in \mathbb{R}^{(m-1)\times(m-1)}$ is a full-rank, easy-to-compute lower-triangular matrix, $b \in \mathbb{R}^{\nu}$ is the baseline vector ($\nu = 3$ in the absence of a Zenith Tropospheric Delay, otherwise $\nu = 4$) and $a \in \mathbb{Z}^{2(m-1)}$ is the newly defined GLONASS integer ambiguity vector.

$$\begin{aligned} L_{ii} &= 2848 \times \frac{g_{i+1}}{a_{i+1} g_i} \quad \text{for } i = 1, \ldots, m-1 \\ L_{ij} &= -\,2848 \times \frac{\alpha_j a_{1(i+1)}}{a_{i+1} g_j} \quad \text{for } i = j+1, \ldots, m-1 \end{aligned} \quad (2)$$

where $a_{1(i+1)} = a_{i+1} - a_1$. The integers α_i and β_i are given by

$$-\alpha_i a_{i+1} + \beta_i g_i = g_{i+1} \quad (3)$$

in which $a_i = 2848 + \kappa^i$, $\kappa^i \in [-7, +6]$ are the channel numbers, $g_1 = a_1$ and $g_i = \mathrm{GCD}(a_1, \ldots, a_i)$ $(1 < i \le m)$, with GCD denoting the Greatest Common Divisor. Software pseudo-code for computing the entries of matrix L is given in Fig. 1. We have also provided a MATLAB routine 'GLONASS_L.m' for the L-matrix computation. The routine can be accessed and downloaded via the GPS Toolbox website at http://www.ngs.noaa.gov/gps-toolbox.

Algorithm 1 L-matrix

input: vector of channel numbers $\kappa \in \mathbb{Z}^m$

output: Lower-triangular matrix $L \in \mathbb{Z}^{(m-1)\times(m-1)}$

function $L \leftarrow \textsc{comptL}(\kappa)$

  $z_\epsilon \leftarrow 2848;$

  $m \leftarrow \mathrm{length}(\kappa);$

  for $i \leftarrow 1$ to m **do**

    $a[i] \leftarrow z_\epsilon + \kappa[i];$

  end for

  $L \leftarrow \mathrm{zeros}(m-1);$ ▷ a matrix of zeros

  $g[1] \leftarrow a[1];$

  for $i \leftarrow 1$ to $m-1$ **do**

    $[g[i+1], \alpha, \beta] \leftarrow \textsc{ExEuclid}(a[i+1], g[i]);$

    $L[i,i] \leftarrow (z_\epsilon\, g[i+1])/(g[i]\, a[i+1]);$

    for $j \leftarrow i+1$ to $m-1$ **do**

      $L[j,i] \leftarrow (z_\epsilon\, \alpha)\,(a[j+1]-a[1])/(g[i]\, a[j+1]);$

    end for

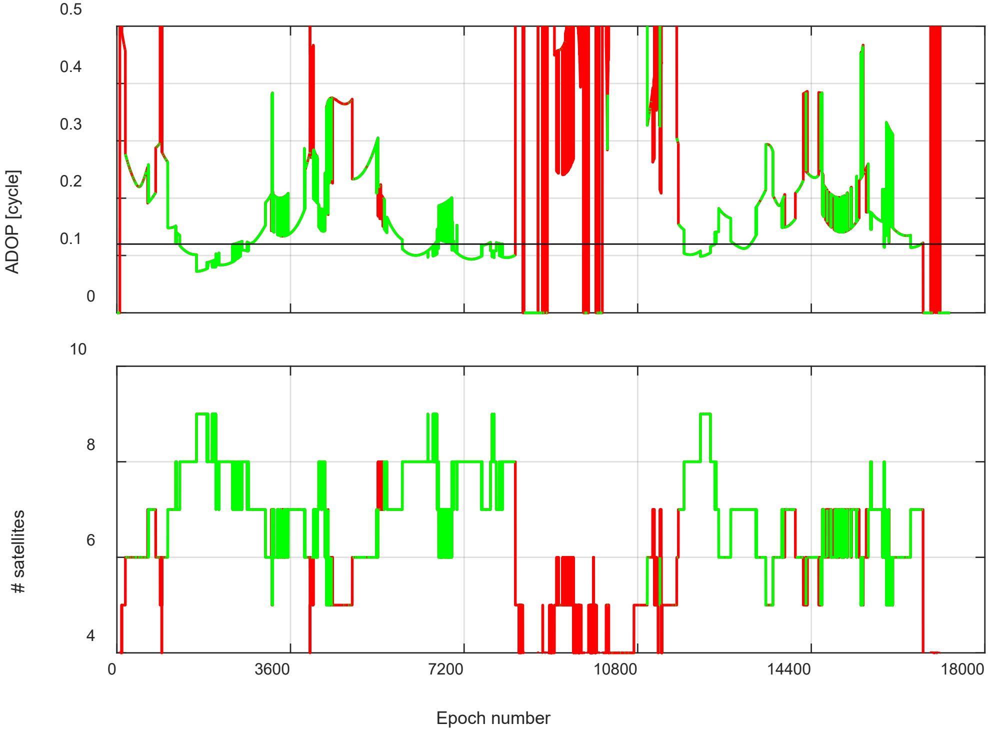
  end for

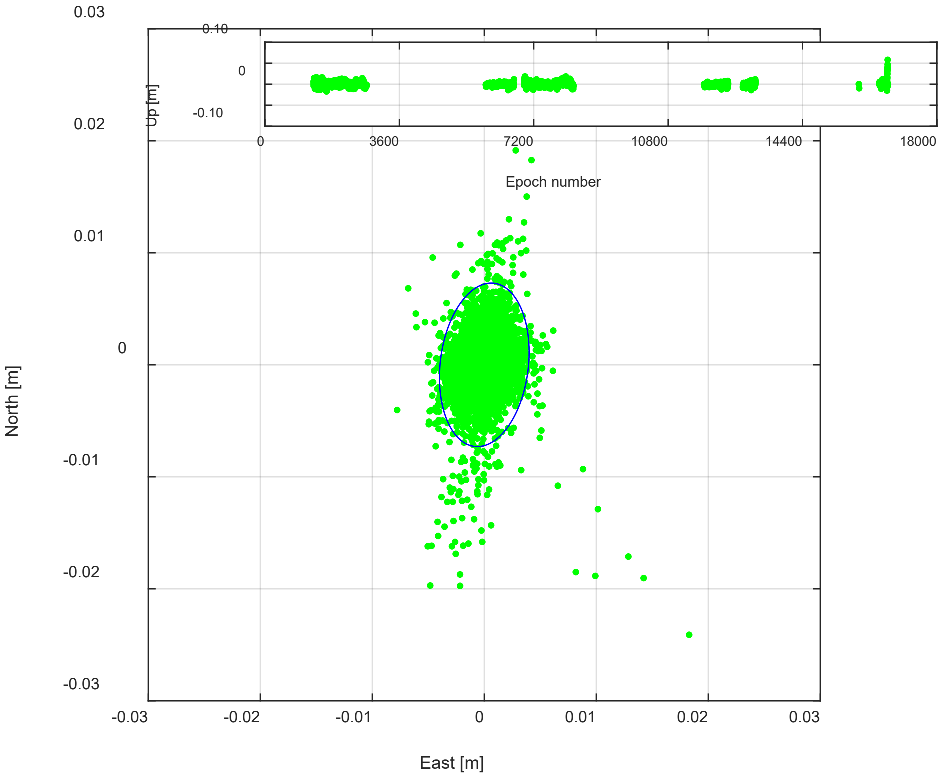
end function

Algorithm 2 Extended Euclidean

input: Two integers $a \in \mathbb{Z}$ and $b \in \mathbb{Z}$
output: GCD g and coefficients $\alpha \in \mathbb{Z}$ and $\beta \in \mathbb{Z}$
function $[g, \alpha, \beta] \leftarrow \textsc{ExEuclid}(a, b)$
    $\alpha \leftarrow 0,\ \beta \leftarrow 1;$
    $\alpha_o \leftarrow 1,\ \beta_o \leftarrow 0;$
    while $a \neq 0$ **do**
        $q \leftarrow \text{floor}(\frac{b}{a});$
        $r \leftarrow (b - q\,a);$
        $r_\alpha \leftarrow (\alpha - q\,\alpha_o);$
        $r_\beta \leftarrow (\beta - q\,\beta_o);$
        $b \leftarrow a,\quad a \leftarrow r;$
        $\alpha \leftarrow \alpha_o,\quad \beta \leftarrow \beta_o;$
        $\alpha_o \leftarrow r_\alpha,\quad \beta_o \leftarrow r_\beta;$
    end while
    $g \leftarrow b;$
    if $g < 0$ **then** ▷ negate the coefficients when $g < 0$
        $g \leftarrow -g,\ \alpha \leftarrow -\alpha,\ \beta \leftarrow -\beta;$
    end if
end function

Dual-frequency, single-epoch GLONASS positioning

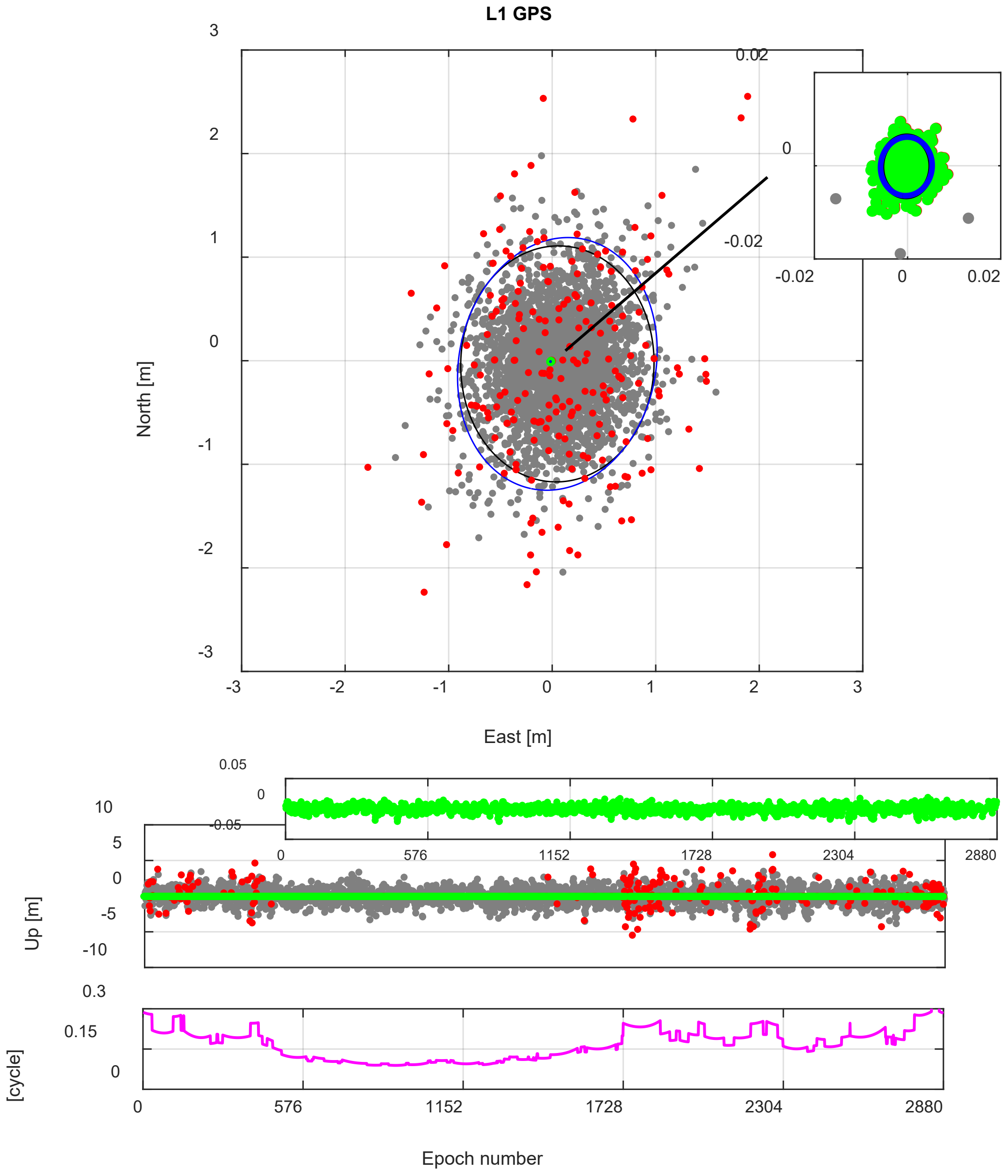




Instantaneous Integer-Fixed GLONASS Positioning

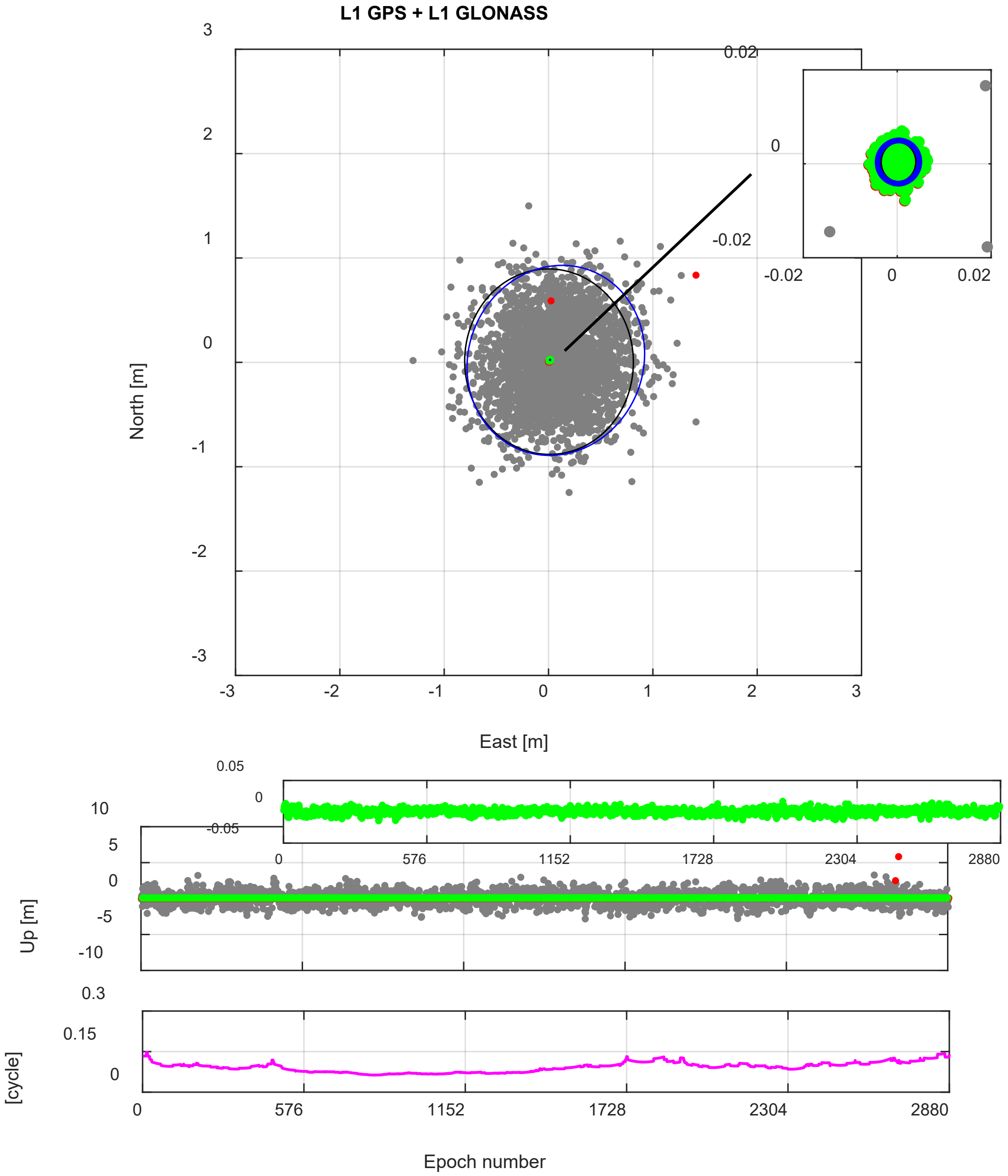
Single-frequency, Single-System, GPS Single-Epoch positioning
(Perth, Australia):

JAVAD TRE-G3T DELTA receivers and Trimble TRM59800.00 antennae



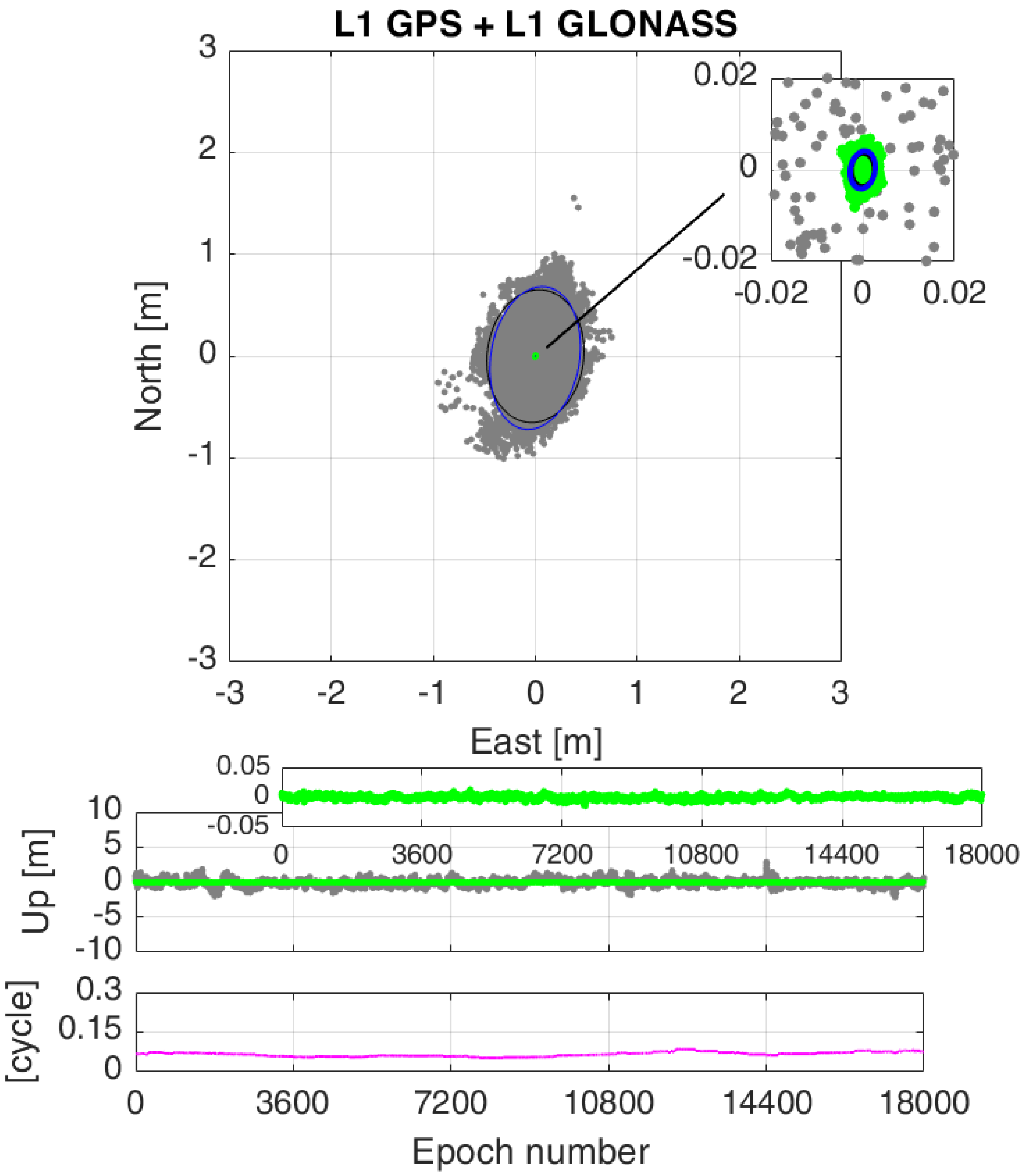
Single-frequency, Dual-System, Single-Epoch
positioning
(Perth, Australia):

JAVAD TRE-G3T DELTA receivers and Trimble TRM59800.00 antennae



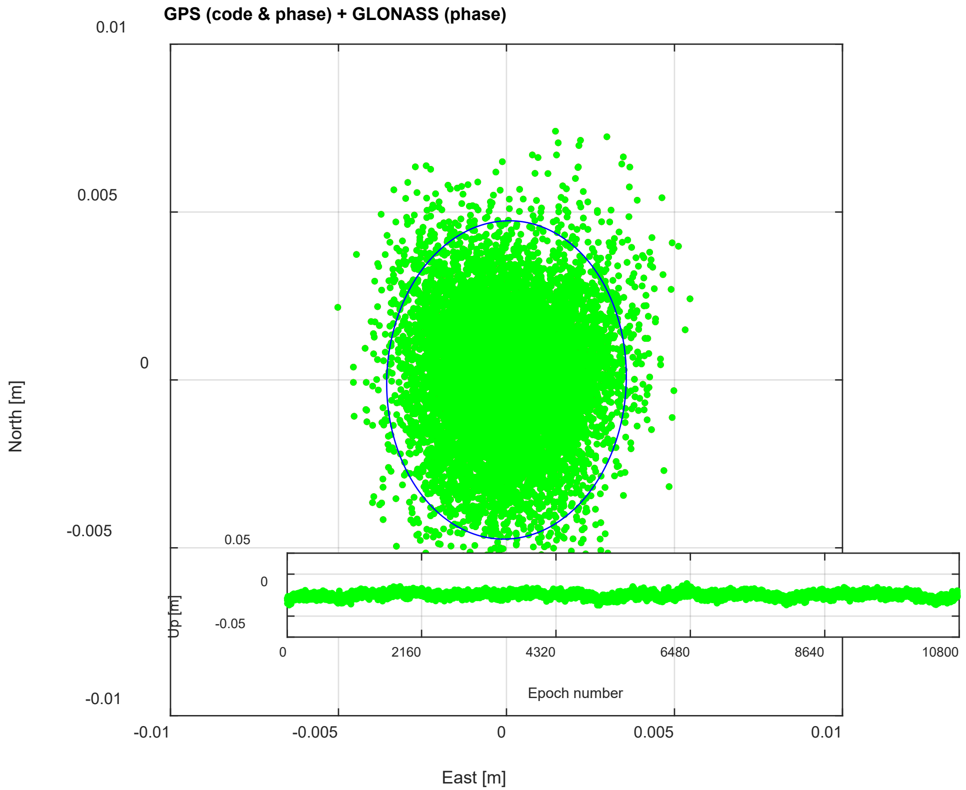
Single-frequency, **Dual-System, Low-Cost, Single-Epoch** positioning
(Dunedin, New Zealand):

U-blox ZED F9P receivers and Trimble Zephyr 2 antennae

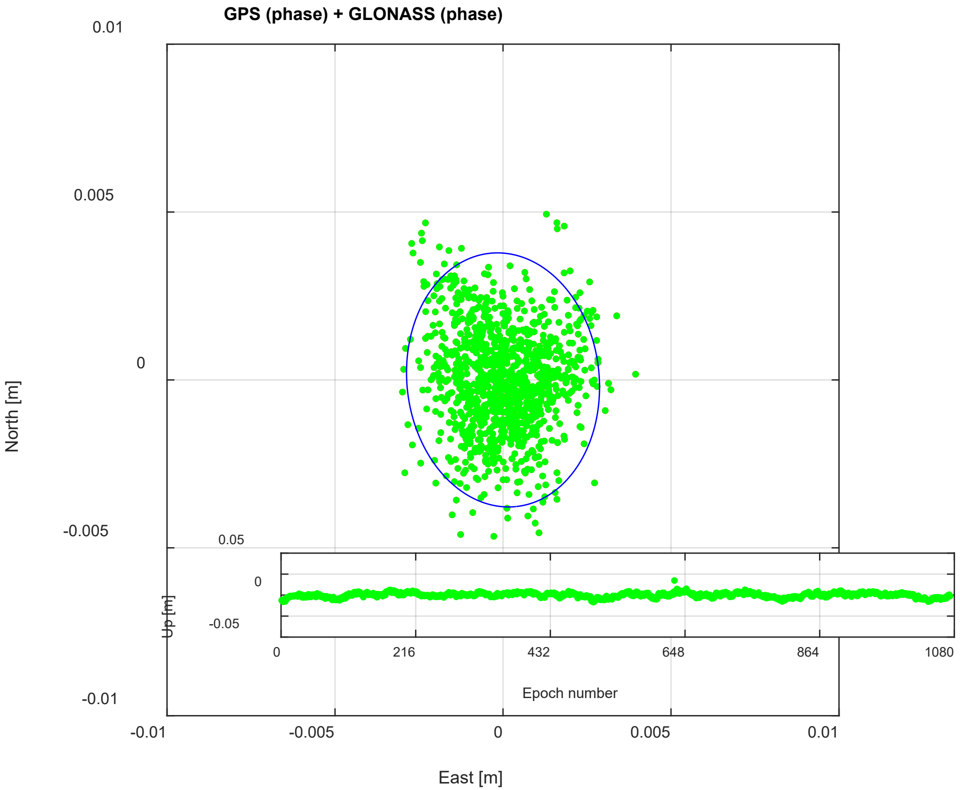


Mixed-receivers GLONASS Phase-only IAR:
Trimble NetR9 and U-blox M8T
(Delft, The Netherlands)

Single-epoch



Multi-epoch (10 epochs)



The GNSS Compass Model for Direction-Finding

$$\mathrm{E}(y) = Aa + Bb \ , \ ||b|| = l \ , \ a \in \mathbb{Z}^n, b \in \mathbb{R}^p,$$
$$\mathrm{D}(y) = Q_{yy}$$

$$\min_{a \in \mathbb{Z}^n, b \in \mathbb{S}_l} \|y - Aa - Bb\|^2_{Q_{yy}} = ||\hat{e}||^2_{Q_{yy}}$$
$$+ \min_{a \in \mathbb{Z}^n} \left(\|\hat{a} - a\|^2_{Q_{\hat{a}\hat{a}}} + \min_{b \in \mathbb{S}_l} \|\hat{b}(a) - b\|^2_{Q_{\hat{b}(a)\hat{b}(a)}} \right)$$

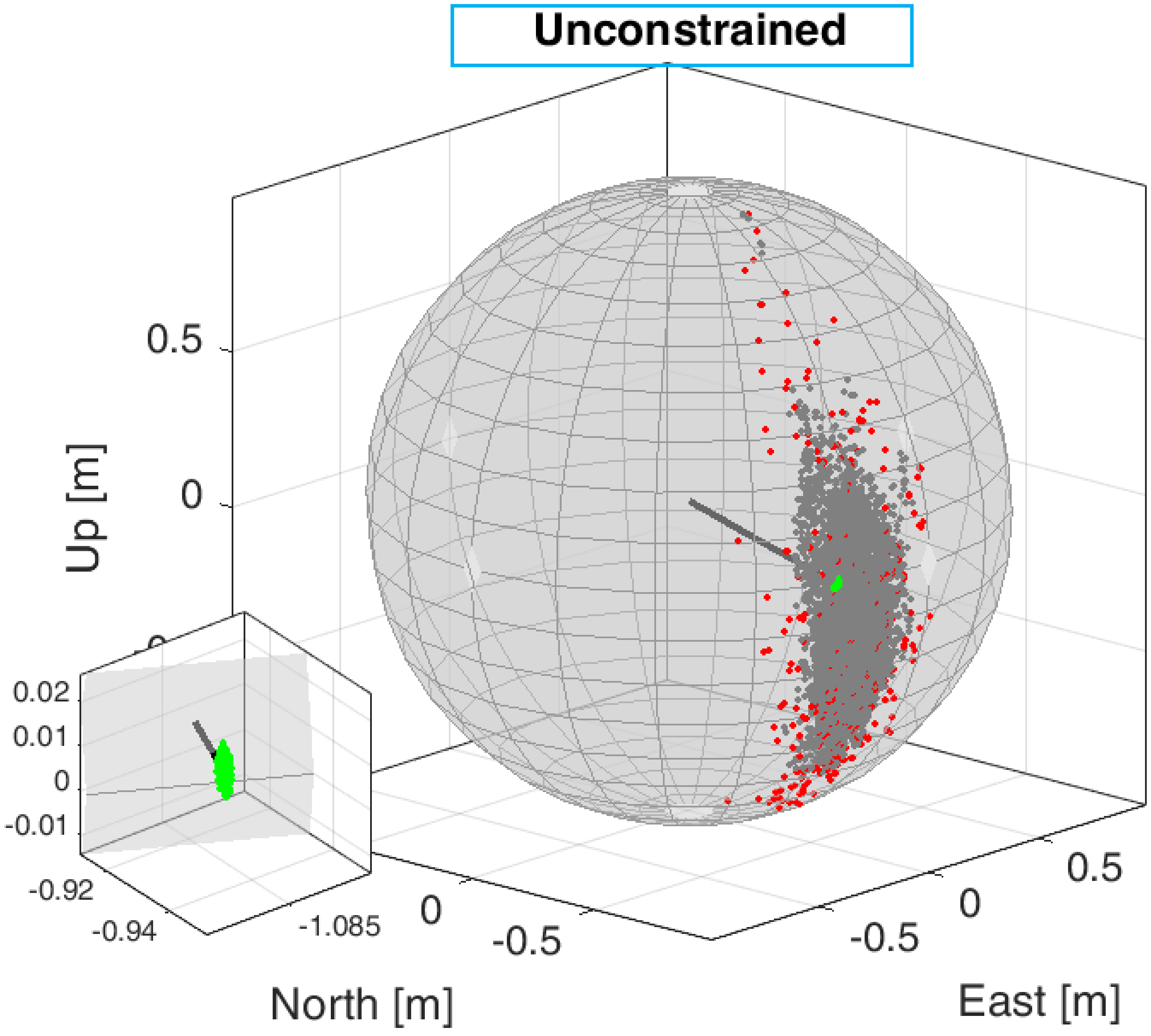
Solved with C-LAMBDA method
(Teunissen, 2010)

$$\check{b}(a) = \arg\min_{b \in \mathbb{S}_l} \|\hat{b}(a) - b\|^2_{Q_{\hat{b}(a)\hat{b}(a)}}$$

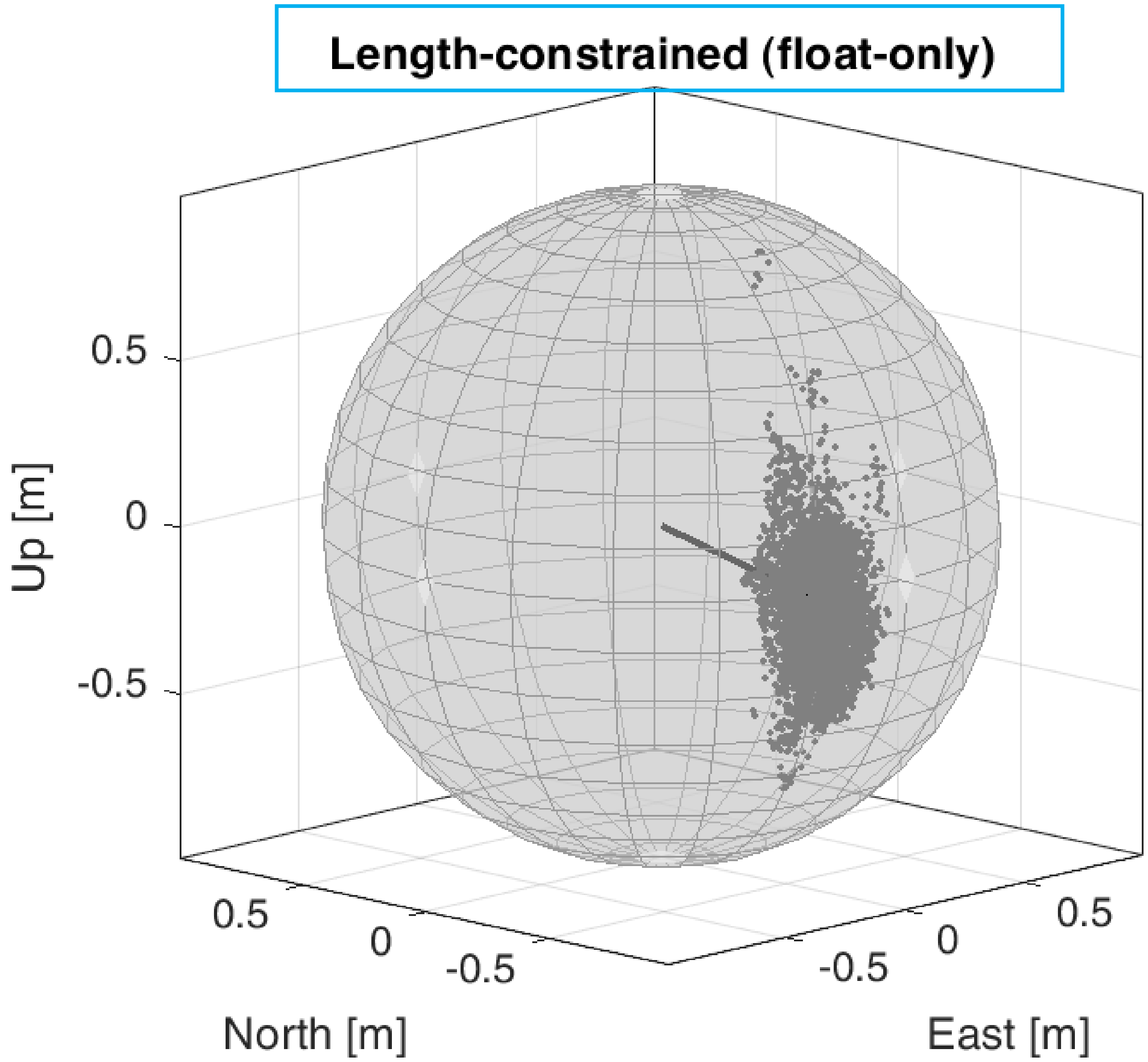
$$\check{a} = \arg\min_{a \in \mathbb{Z}^n} \left(||\hat{a} - a||^2_{Q_{\hat{a}\hat{a}}} + ||\hat{b}(a) - \check{b}(a)||^2_{Q_{\hat{b}(a)\hat{b}(a)}} \right) \quad \text{and}$$
$$\check{b} = \check{b}(\check{a})$$

GLONASS **single-frequency, single-epoch** direction finding

Grey: ambiguity float solution
Red: wrong integer fixes
Green: correct integer fixes



GLONASS **single-frequency, single-epoch** direction finding



GLONASS **single-frequency, single-epoch** direction finding

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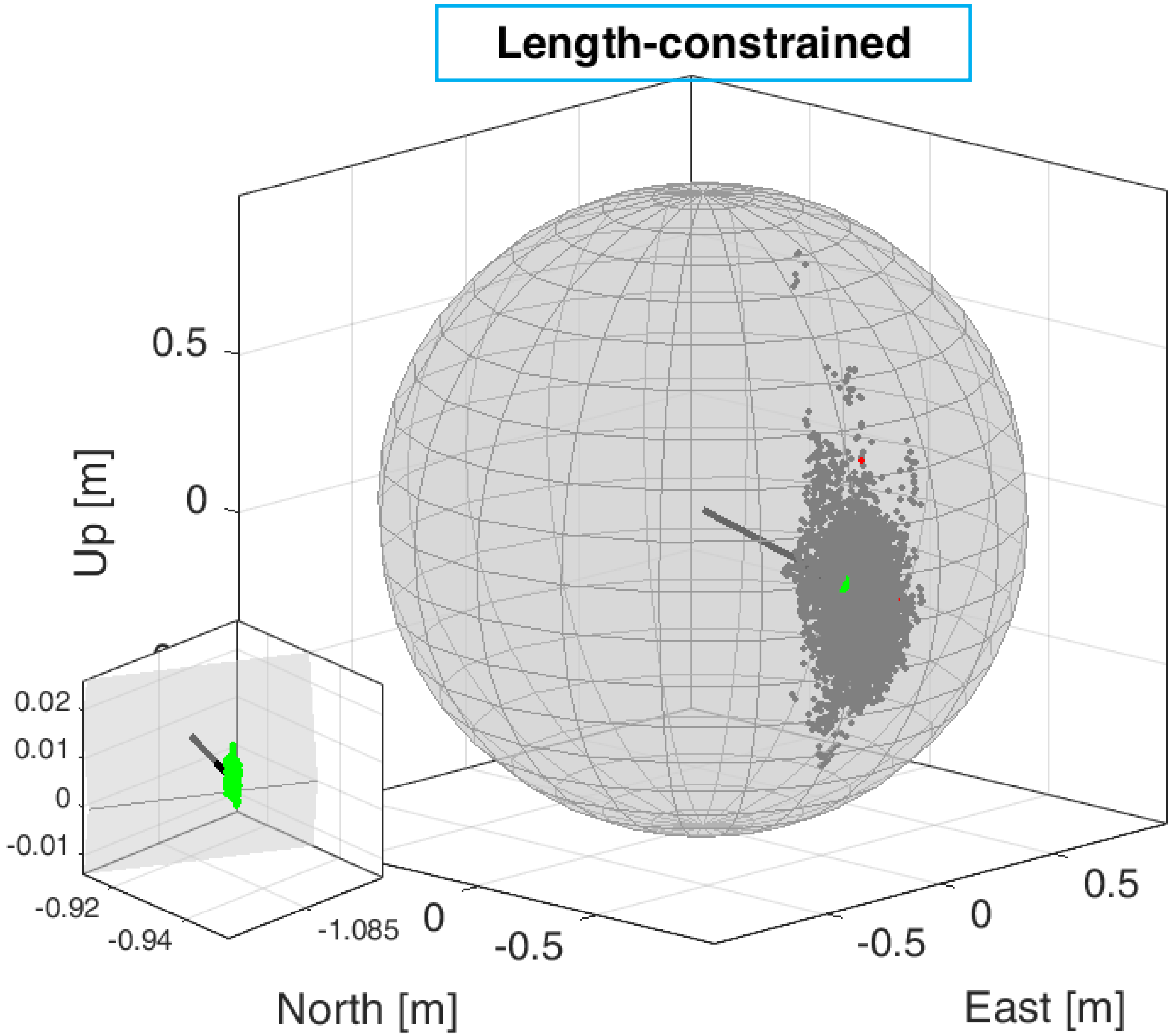

Conclusions:

With the above performance studies, we have only presented a few of the many possible applications of the new GLONASS FDMA model (1).

Many more can be considered, such as those for long baselines, network-RTK, PPP-RTK and even for the direct combination of current GLONASS FDMA with its near-future GLONASS CDMA that is currently under development.

We therefore believe that the flexibility of the model and its close resemblance to CDMA models open up a whole variety of carrier-phase-based GNSS applications that have hitherto been a challenge for GLONASS ambiguity resolution.

Selected References

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