



Constraints on the Rheology of the Earth's Deep Mantle from Decadal Observations of the Earth's Figure Axis and Rotation Pole

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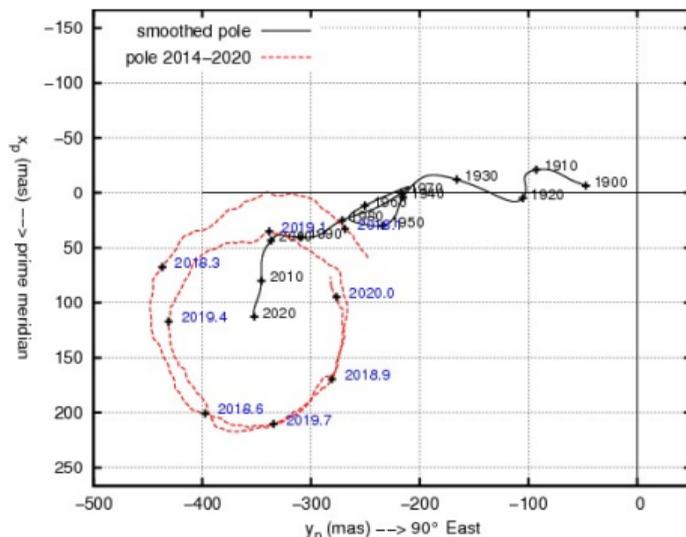


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 - 3. Modeling the Earth viscoelastic response
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1. Background
 2. Mean figure and rotation axes
 3. Modeling the Earth viscoelastic response
 4. Derived model versus previous theories
 5. Conclusions

❖ Definitions

- *Polar motion* : displacements (> 2 days) of the rotation pole from the geographic North pole.



Beating Chandler wobble (435 days) and annual oscillations superimposed on the low-frequency (> 10 years) polar motion [<https://hpiers.obspm.fr>].

- *Figure axis* : mean axis of maximum inertia, whose terrestrial coordinates can be derived from the degree-2 order-1 geopotential terms : $x_I = -\sqrt{\frac{5}{3}} \frac{MR_e^2}{C-A} \bar{C}_{2,1}$ & $y_I = -\sqrt{\frac{5}{3}} \frac{MR_e^2}{C-A} \bar{S}_{2,1}$.

❖ Three independent data sources

➤ Space geodetic *measurements*

1. (x, y) Earth orientation parameters (mainly from GPS & VLBI) : IERS 14C04 (Bizouard et al., 2019).
2. SLR observations to geodetic spheres (Starlette/Stella, LARES, LAGEOS-1/2, Ajisai) makes it possible to measure the long-term displacement (34-year period 1984-2017) of the figure axis,

$$\begin{cases} \bar{C}_{21}^*(t) = \bar{C}_{21}(t_0) + \Delta\bar{C}_{21}^{\text{solid Earth tide}} + \Delta\bar{C}_{21}^{\text{ocean tide}} + \Delta\bar{C}_{21}^{\text{solid Earth/ocean pole tide}} + \Delta\bar{C}_{21}^{\text{dealiasing}} + \Delta\bar{C}_{21}^{\text{load}} \\ \bar{S}_{21}^*(t) = \bar{S}_{21}(t_0) + \dots, \end{cases} \quad (1)$$

where **blue terms** are left unmodeled to observe the full GIA, changes in the continental hydrology/ice sheets, pole tide and non-tidal hydro-atmospheric mass-related long-term excitations of polar motion.



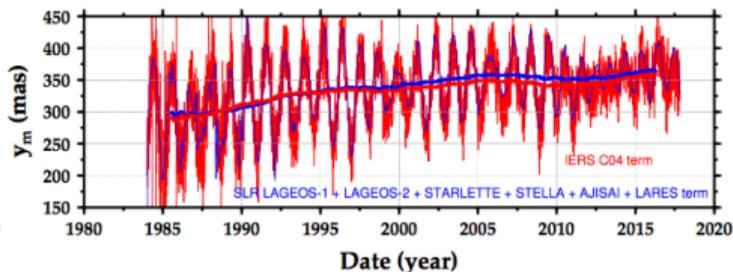
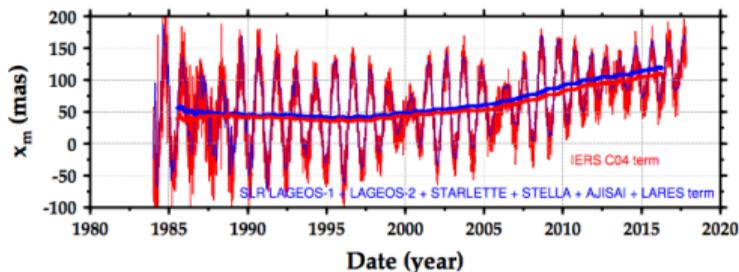
Starlette/Stella, LARES, LAGEOS-1/2, Ajisai (the proportion of the scale is kept).

➤ Geophysical excitation *models* of polar motion

3. Hydro-atmospheric angular momentum functions ($\chi_1^{\text{ma+mo}}$, $\chi_2^{\text{ma+mo}}$) from Dobslaw and Dill (2018).

❖ Long-term behavior of the independent time series

- The **rotational axis** has a circular motion around the **figure axis** at seasonal time scales.
 - *What happens at decadal time periods and how well the two axes are aligned ?*



Rotation pole and mean figure axis coordinates x_m (left) and y_m (right), in mas.

- The well-known Euler-Liouville equations enable to relate oscillations of the Earth's rotation pole to those of its modeled excitation functions.
 - *Could the elastic Earth approximation be revised to explain the discrepancies between the observed terrestrial path of the rotation pole and our current knowledge of its surface mass-related excitation ?*

❖ Euler-Liouville equations

➤ Applying the principle of conservation of angular momentum to the Earth system, we have

$$\begin{cases} x + \frac{A_m}{\Omega(C-A)} \dot{y} - \frac{1}{k_s} \int_{-\infty}^t k(t-\tau)x(\tau)d\tau = \chi_1^{\text{ma+mo}} + \int_{-\infty}^t k'_2(t-\tau)\chi_1^{\text{ma}}(\tau)d\tau \\ -y + \frac{A_m}{\Omega(C-A)} \dot{x} + \frac{1}{k_s} \int_{-\infty}^t k(t-\tau)y(\tau)d\tau = \chi_2^{\text{ma+mo}} + \int_{-\infty}^t k'_2(t-\tau)\chi_2^{\text{ma}}(\tau)d\tau. \end{cases} \quad (2)$$

- C and A are the two principal inertia moments of the Earth (composed of the fluid core, mantle, lithosphere and surface fluids), Ω its mean angular velocity, R_e and M its equatorial radius and mass.
- $k(t)$, $k'_2(t)$, and k_s are the degree-2 effective pole tide (solid Earth and equilibrium ocean), load, and secular Love numbers, respectively.

➤ Hypotheses

- The convolution integral of the degree-2 pole tide and load Love numbers (in place of the common multiplication by constant Love numbers) accounts for the **viscoelastic deformation of the solid Earth**.
- The introduction of A_m (*extended mantle* without the core) assumes (Sasao and Wahr, 1981) that the Earth core has the same polar motion as the mantle (valid over periods larger than a few days).
- Other potential sources of decadal motion-related excitations of polar motion, such as climatic oscillation events (e.g., El Niño/La Niña) or the electromagnetic core-mantle interaction are lacking.

❖ A rheology encompassing two different characteristic times

- The response of the Earth's mantle to the *present-day melting of the polar ice sheets*
 - Considering the Earth as incompressible, the short-term (transient) viscoelastic time domain Love numbers $k(t)$ and $k'_2(t)$ can be expressed as an "apparent" normal mode expansion of the form

$$\begin{cases} k(t) = k\delta(t) + qe^{-st}\mathcal{H}(t) \\ k'_2(t) = k'_2\delta(t) + q'e^{-st}\mathcal{H}(t), \end{cases} \quad (3)$$

in which s is an inverse relaxation time, q and q' viscoelastic amplitude coefficients, k and k'_2 the corresponding elastic Love numbers, $\delta(t)$ and $\mathcal{H}(t)$ are the Dirac and Heaviside step functions.

- This single relaxation time primarily reflects the **lower mantle rheological properties**, due to its large volume and depth range, as the Earth's rotation depends on the global deformation of the planet.
- The response of the Earth's mantle to the *GIA signal following the last deglaciation event*
 - The long-term viscoelastic behavior of the Earth is introduced through bias and drift terms (D_i, E_i) , corresponding to the effects of GIA observed in (x, y) and $(\overline{C}_{2,1}^*, \overline{S}_{2,1}^*)$, but lacking in the modeled excitation functions $(\chi_1^{\text{ma+mo}}, \chi_2^{\text{ma+mo}})$.

❖ Problem to be solved

- Adding the previously derived gravity coefficients (mirroring the viscoelastic mass-related excitation of polar motion) yields

$$\left\{ \begin{array}{l} \frac{k_s - k}{k_s} x + \frac{A_m}{\Omega(C - A)} \dot{y} - \frac{q}{k_s} \int_{1984}^t e^{-s(t-\tau)} x(\tau) d\tau = D_1 + E_1 t + \chi_1^{\text{mo}} + (1 + k'_2) \chi_1^{\text{ma}} + q' \int_{1984}^t e^{-s(t-\tau)} \chi_1^{\text{ma}}(\tau) d\tau + B_1 e^{-st} \\ \frac{k - k_s}{k_s} y + \frac{A_m}{\Omega(C - A)} \dot{x} + \frac{q}{k_s} \int_{1984}^t e^{-s(t-\tau)} y(\tau) d\tau = D_2 + E_2 t + \chi_2^{\text{mo}} + (1 + k'_2) \chi_2^{\text{ma}} + q' \int_{1984}^t e^{-s(t-\tau)} \chi_2^{\text{ma}}(\tau) d\tau + B_2 e^{-st} \\ -\sqrt{\frac{5}{3}} \frac{MR_e^2}{C - A} \bar{C}_{2,1}^* - \frac{q}{k_s} \int_{1984}^t e^{-s(t-\tau)} x(\tau) d\tau - \frac{k}{k_s} x = D_1 + E_1 t + (1 + k'_2) \chi_1^{\text{ma}} + q' \int_{1984}^t e^{-s(t-\tau)} \chi_1^{\text{ma}}(\tau) d\tau + B_1 e^{-st} \\ -\sqrt{\frac{5}{3}} \frac{MR_e^2}{C - A} \bar{S}_{2,1}^* + \frac{q}{k_s} \int_{1984}^t e^{-s(t-\tau)} y(\tau) d\tau + \frac{k}{k_s} y = D_2 + E_2 t + (1 + k'_2) \chi_2^{\text{ma}} + q' \int_{1984}^t e^{-s(t-\tau)} \chi_2^{\text{ma}}(\tau) d\tau + B_2 e^{-st} \end{array} \right. \quad (4)$$

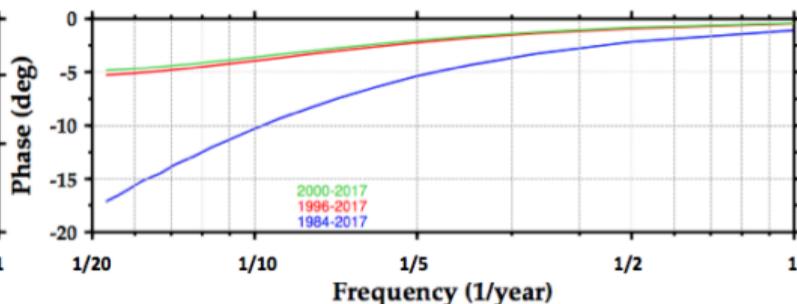
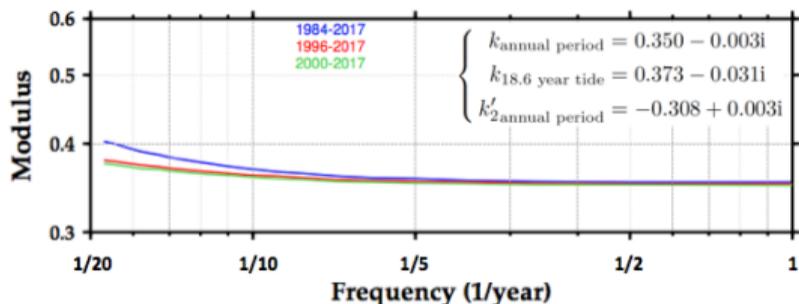
- B_1 and B_2 denote constant viscoelastic initial condition parameters that will be determined with the other *historical terms* (D_i, E_i), as well as the *contemporary terms* of the model (k, s, q, q'), within the constraint that the residuals of the Euler-Liouville equation system are minimized (using least square adjustment).

4. DERIVED MODEL VERSUS PREVIOUS THEORIES

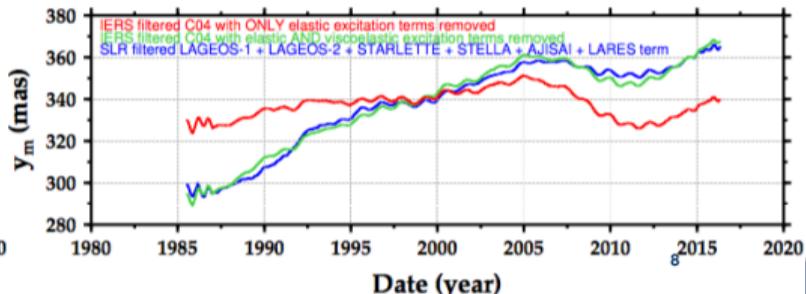
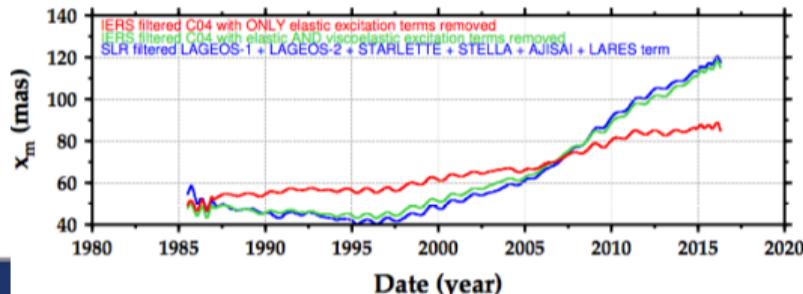
❖ Results

⇒ When starting the estimation after 1996, the derived frequency-dependent Love number k agrees well with independent values (e.g., $k_{\text{annual/Chandler period}} = 0.353 - 0.003i$ in Seitz et al. (2012),

$k_{im\ 18.6\ \text{year tide}} = -0.028i$ in Benjamin et al. (2006), or the R3/R4 models of Nakada and Karato (2012)).



⇒ Agreement between the filtered motion of the Earth's figure axis and the traditional elastic modeling of the mass-related excitation of polar motion and the Earth's viscoelastic response derived from our model.



❖ Summary

➤ Findings

- Polar motion observations (with geophysical excitation models) were used with those of the figure axis variations to constrain Earth's inelasticity at periods till 18.6 years. For this purpose, the *full Stokes coefficients* (including the pole tide effect) were previously derived, not applying corrective models based upon the quasi-elastic approximation of the mantle rheology.
- Nakada and Okuno (2013) showed that the long-term evolution of the polar wander is significantly sensitive to the viscosity of the *D'' layer of the deepest Earth's mantle*, essentially through rotational potential viscoelastic perturbations of the polar motion. Also, our results should mostly be interpreted as constraints on the viscosity of the *D'' deeper part of the lower mantle at the decadal time scale*.

➤ Recommendations

- Reproducing this type of analysis in the future would enable us to benefit from possibly improved geophysical excitation models and accurate polar motion observations over a long enough time span (at least 1996–2022), to *underpin the relaxation time, while validating the derived Love number k_2 value for the 18.6 year tide*, in order to infer the viscosity structure of the deep lower mantle, including the *D'' layer, a crucial quantity in discussing mantle dynamics*.
- As it is clear that the Earth behaves differently over different time scales, *the viscoelastic contribution to the pole tide perturbation should be further investigated*. This is, among other things, *a necessary step to estimate accurately recent ice melting*, and sharpen our knowledge of the Earth's response to present day climate change.