TiPES Kick-off Meeeting

IHP, Paris,
16 October 2019

Bifurcations, Global Change, Tipping Points and All That

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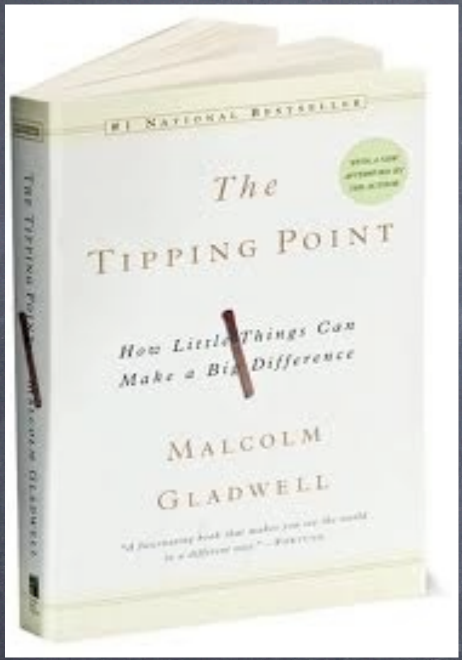
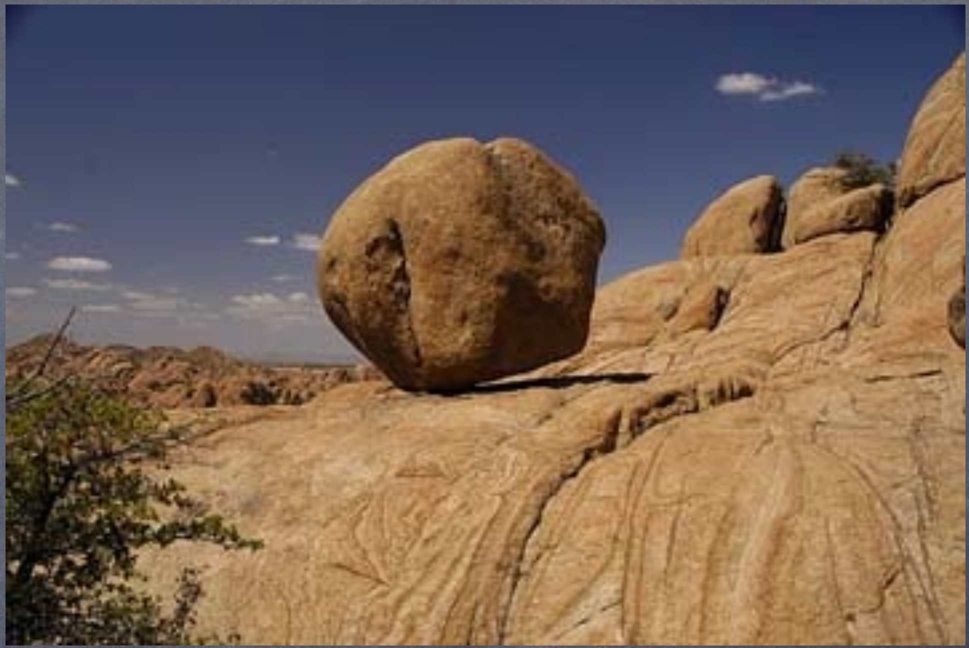
Please visit these sites for more info. on the talk

http://www.atmos.ucla.edu/tcd/, http://www.environnement.ens.fr/,
and https://www.researchgate.net/profile/Michael_Ghil

SC1/NP1.5:
Tipping Points in the Geosciences





Michael Ghil, Peter Ditlevsen and Henk Dijkstra
NP division, EGU-GA 2012

Outline

- **Intrinsic** vs. **forced** variability
  - short-, intermediate, & long-term prediction
  - multiple scales of motion
  - **IPCC** & the **uncertainties**
- Time-dependent forcing
  - **pullback** and **random** attractors (**PBAs** & **RAs**)
  - **tipping points** (TPs)
- An illustrative example
  - the **Lorenz** convection **model** with **stochastic** forcing – **LORA**
  - its topological analysis (**BraMaH**)
  - "grand unification" = **deterministic** + **stochastic**
- Conclusions and references
  - what **do we** & **don't we** know?
  - selected bibliography

Motivation

- There's a lot of talk about "tipping points."
- It sounds threatening, like falling off a cliff: that's why we care!
- But what are they, and what do **we know** about them?
- Here's a **disambiguation page** (cf. Wikipedia), first.
- *Sociology*: "the moment of critical mass, the threshold, the boiling point" (Gladwell, 2000); a previously rare phenomenon becomes rapidly and dramatically more common.
- *Physics*: the point at which a system changes from a stable equilibrium into a new, qualitatively dissimilar equilibrium (throwing a switch, tilting a plank, boiling water, etc.).
- *Climatology*: "A climate tipping point is a somewhat ill-defined concept […]"— so we'll try to actually define it better.

- *Catastrophe theory*: branch of bifurcation theory in the study of dynamical systems; here, a tipping point is "a parameter value at which the set of equilibria abruptly change." ➔ **Let's see!**

M. Gladwell (2000) *The Tipping Point: How Little Things Can Make a Big Difference.*
T. M. Lenton *et al*. (2008) Tipping elements in the Earth's climate system, *PNAS,* v. **105.**

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Dynamical systems and predictability

- The initial-value problem → *numerical weather prediction* (*NWP*)

  — **easiest**!
- The asymptotic problem → long-term climate

  — **a little harder**
- The intermediate problem → low-frequency variability (LFV) –

  – multiple equilibria, long-periodic oscillations, intermittency, slow transients, "tipping points"

  — **hardest!!**

Paraphrasing John von Neumann, in
R. L. Pfeffer (ed.), *Dynamics of Climate* (Pergamon, 1960)
now re-edited as an Elsevier E-book

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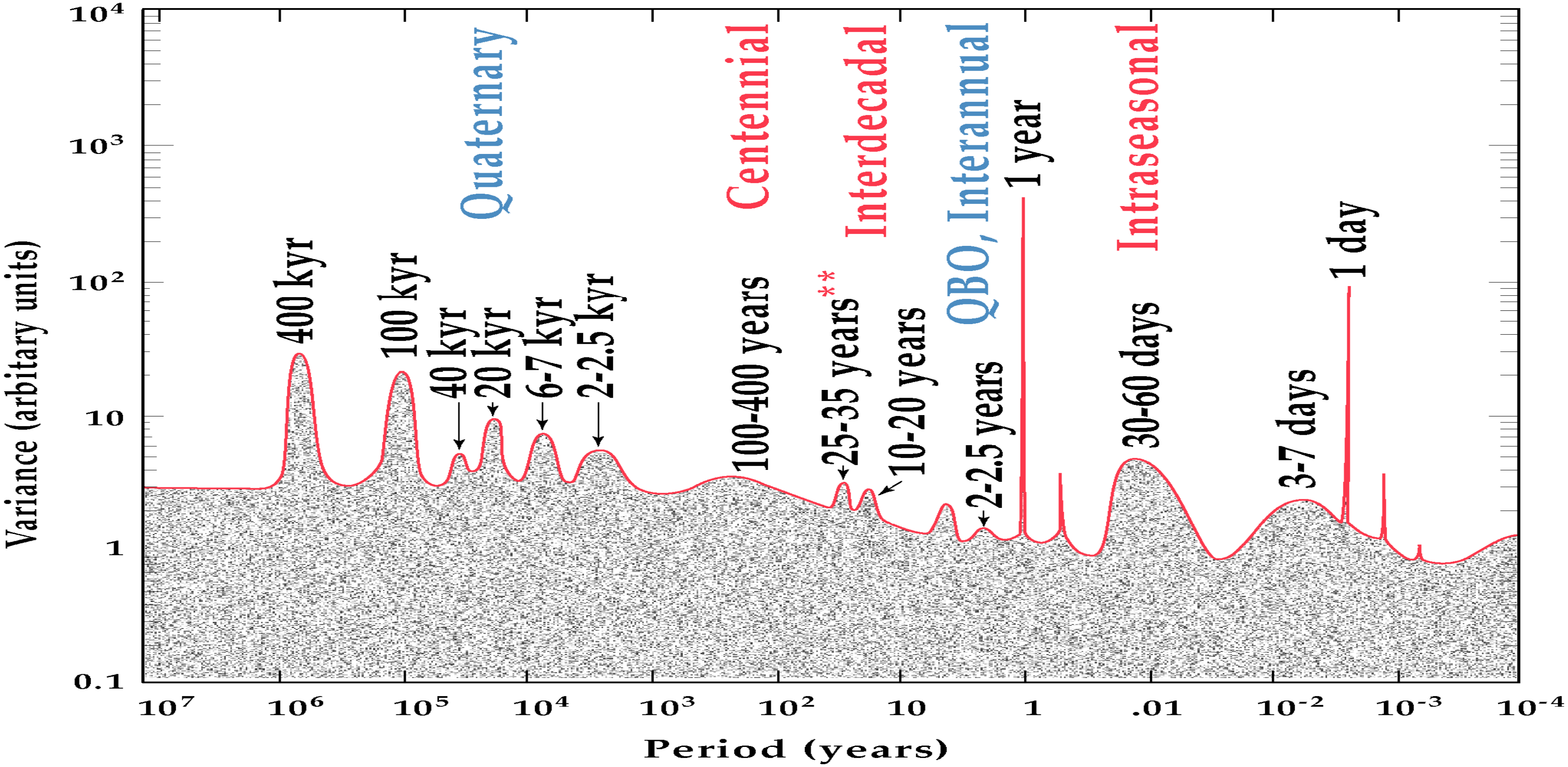
Composite spectrum of climate variability

Standard treatement of frequency bands:

1. Higher frequencies – white (or "warm-colored") noise

2. Lower frequencies – slow ("adiabatic") evolution of parameters



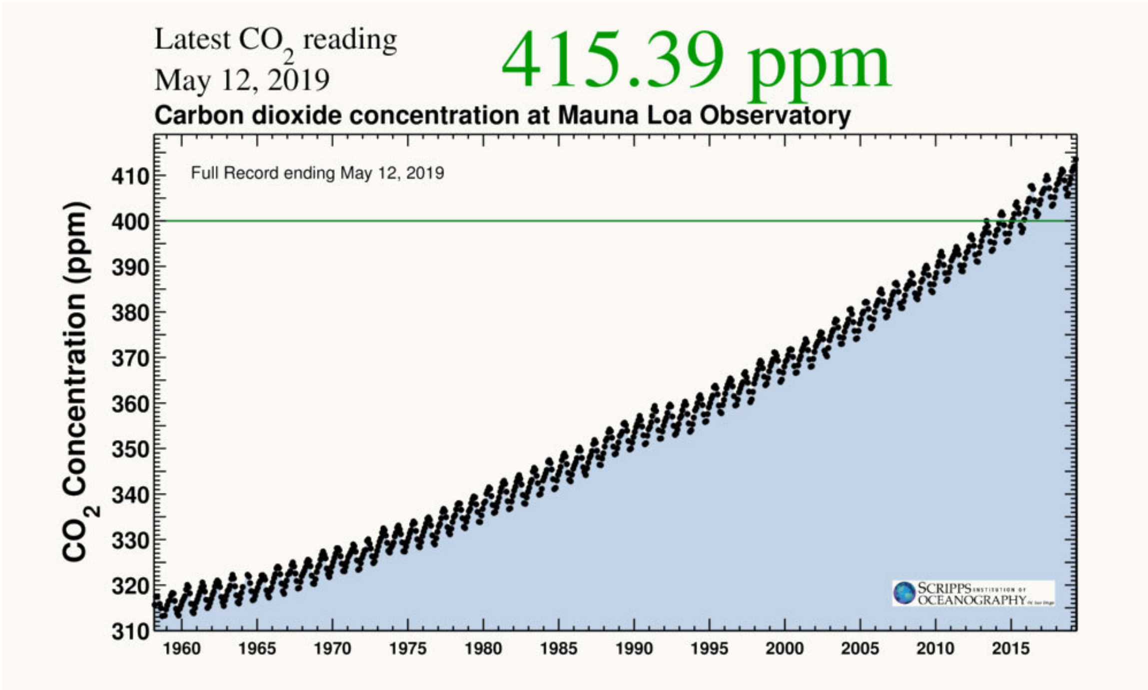
From **Ghil (2001, *EGEC*)**, after **Mitchell* (1976)**

*** "No known source of deterministic internal variability"**

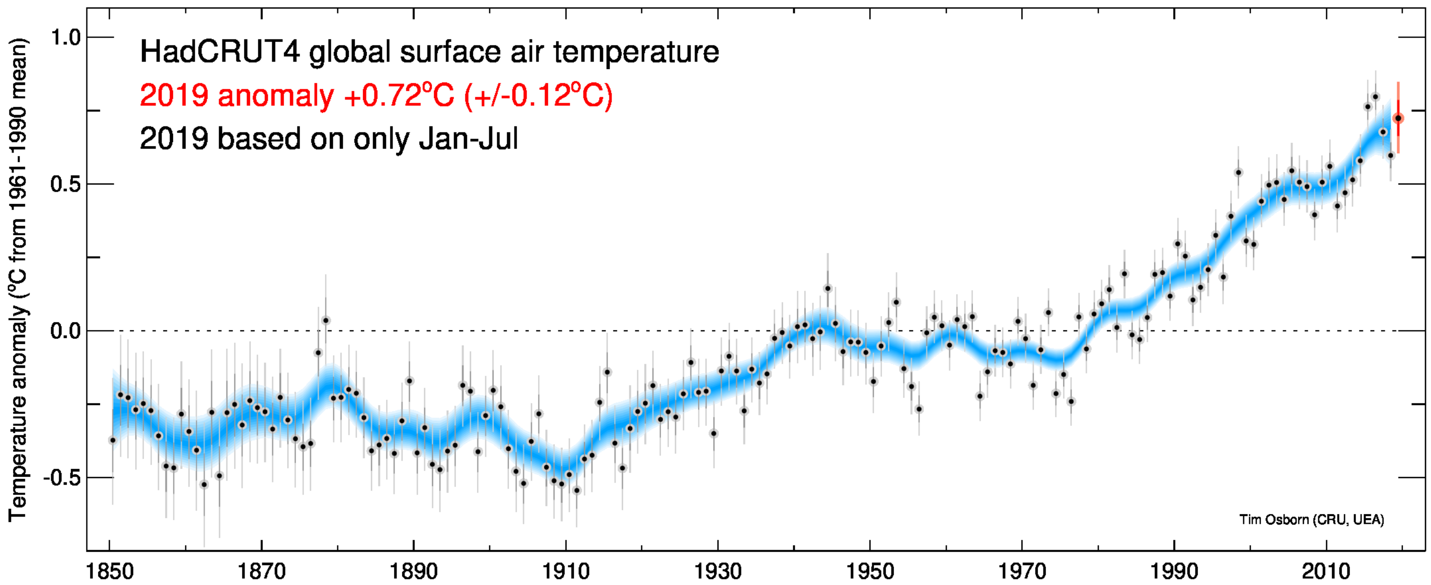
** **27 years – Brier (1968, *Rev. Geophys.*)**

Climate Change: CO_2 & Temp. Observations

Exponential increase in CO2 should result in linear increase in temperatures.
Why is that not so?



Courtesy of Henk Dijkstra



Unfortunately, things aren't all that easy!

What to do?

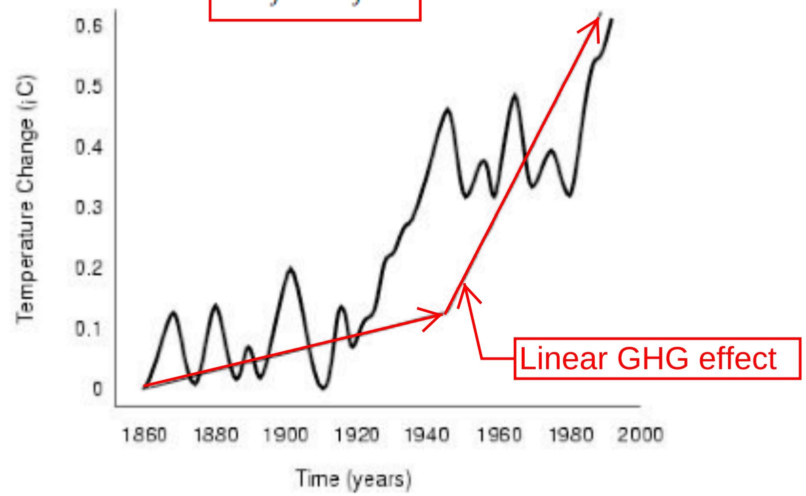
Try to achieve better interpretation of, and agreement between, models …

Ghil, M., 2002: Natural climate variability, in *Encyclopedia of Global Environmental Change*, T. Munn (Ed.), Vol. 1, Wiley

Natural variability introduces additional complexity into the anthropogenic climate change problem

The most common interpretation of observations and GCM simulations of climate change is still in terms of a scalar, linear Ordinary Differential Equation (ODE)

$$\boxed{c\frac{dT}{dt} = -kT + Q}\qquad \boxed{\begin{aligned} k &= \sum k_i \\ Q &= \sum Q_j \\ Q_j &= Q_j(t) \end{aligned}}\begin{aligned} &-\text{feedbacks (+ve and -ve)} \\ &-\text{sources \& sinks} \\ & \end{aligned}$$



Linear response to CO_2 vs. observed change in T

Hence, we need to consider instead a system of nonlinear Partial Differential Equations (PDEs), with parameters and multiplicative, as well as additive forcing (deterministic + stochastic)

$$\boxed{\frac{d\boldsymbol{X}}{dt} = \boldsymbol{N}(\boldsymbol{X}, t, \mu, \beta)}$$

Global warming and its socio-economic impacts

Temperatures rise:

- What about impacts?
- How to adapt?

The answer, my friend, is blowing in the wind, *i.e.*, it depends on the accuracy and reliability of the forecast …

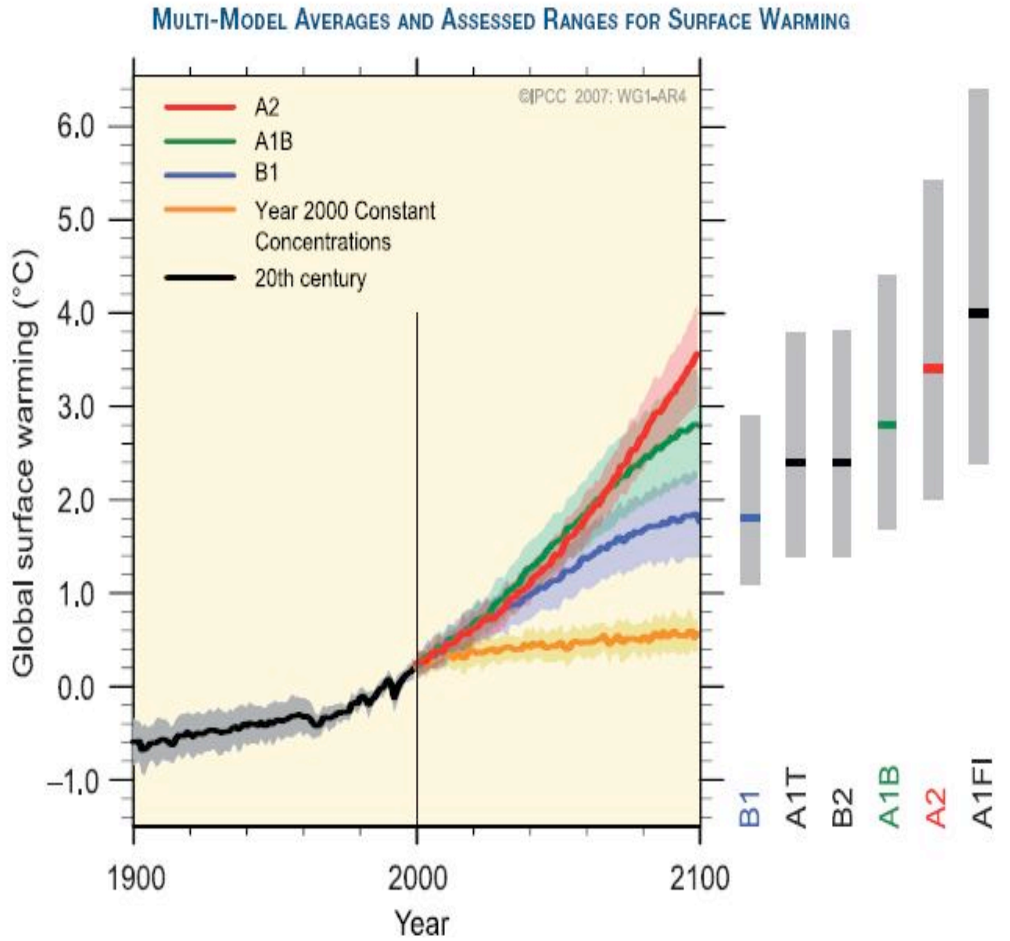


Figure SPM.5. *Solid lines are multi-model global averages of surface warming (relative to 1980–1999) for the scenarios A2, A1B and B1, shown as continuations of the 20th century simulations. Shading denotes the ±1 standard deviation range of individual model annual averages. The orange line is for the experiment where concentrations were held constant at year 2000 values. The grey bars at right indicate the best estimate (solid line within each bar) and the* ***likely*** *range assessed for the six SRES marker scenarios. The assessment of the best estimate and* ***likely*** *ranges in the grey bars includes the AOGCMs in the left part of the figure, as well as results from a hierarchy of independent models and observational constraints. {Figures 10.4 and 10.29}*

Source : IPCC (2007), *AR4, WGI, SPM*

Climate and Its Sensitivity

Let's say CO_2 doubles:

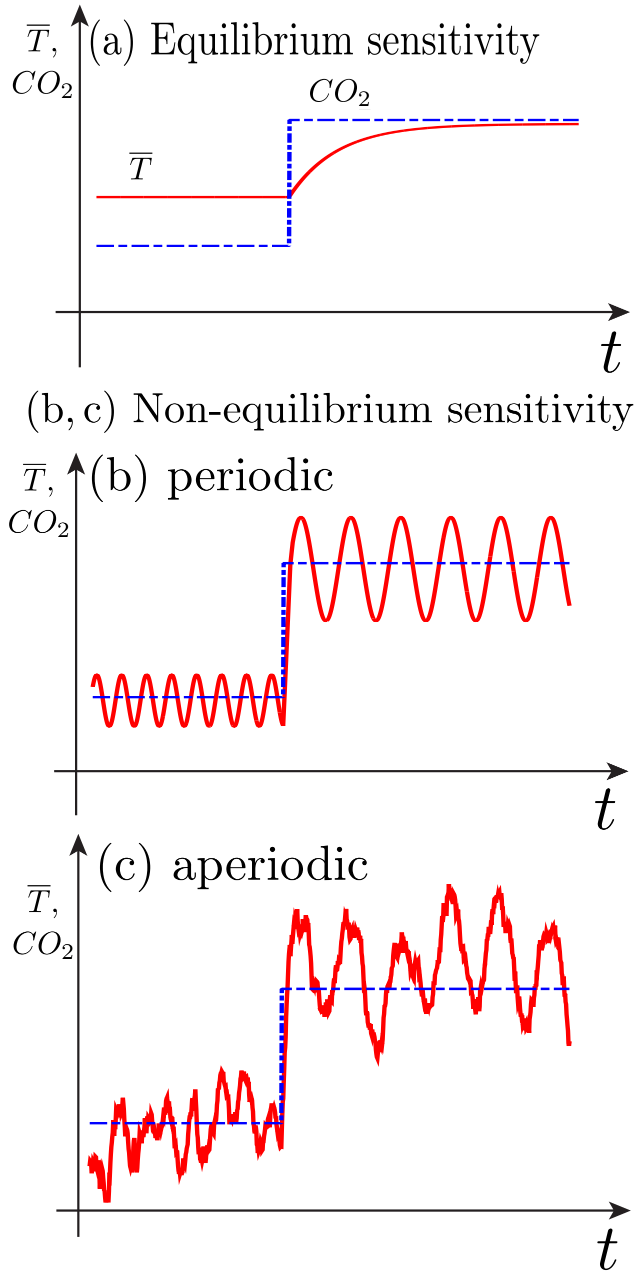
How will "climate" change?

1. Climate is in stable equilibrium (fixed point); if so, mean temperature will just shift gradually to its new equilibrium value.

2. Climate is purely periodic; if so, mean temperature will (maybe) shift gradually to its new equilibrium value. But how will the period, amplitude and phase of the limit cycle change?

3. And how about some "real stuff" now: chaotic + random?

Ghil (in *Encycl. Global Environmental Change*, 2002)



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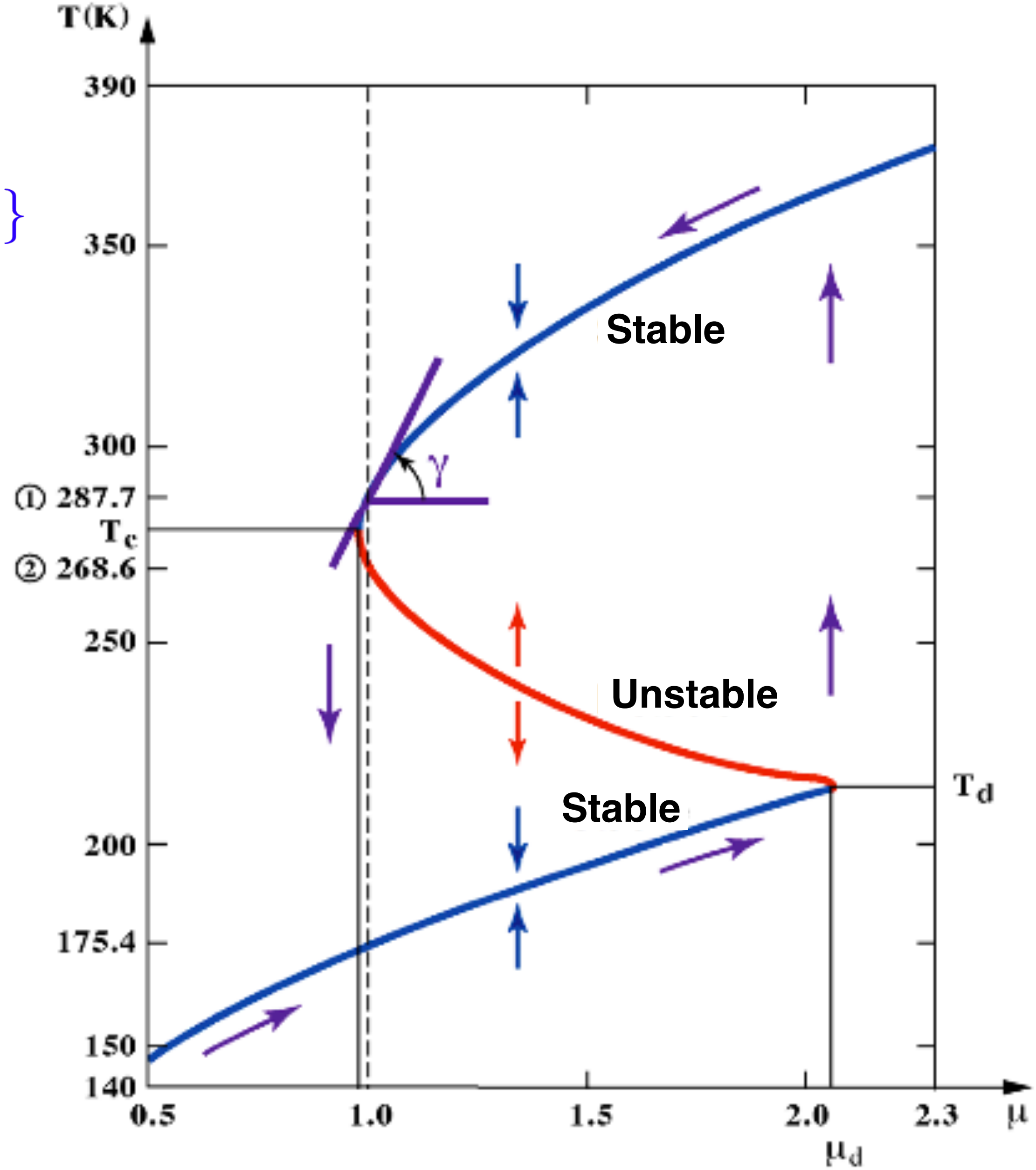
1-D EBM: Bifurcation diagram

$$C(x)T_t = \{k(x,T)T_x\} + \mu Q_0\{1-\alpha(x,T)\} - g(T)\sigma T^4$$

$T_x = 0$ at $x = 0, 1$



Climate sensitivity:

$$\gamma = \frac{dT}{d\mu} \cong 0.01$$

(1K per % of Q)

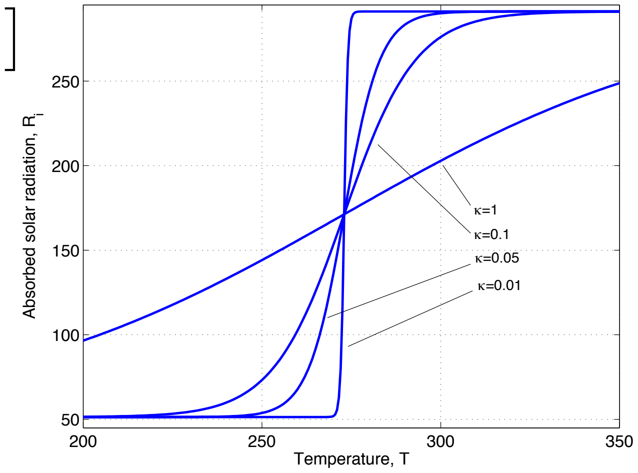
Distance to “tipping points”?

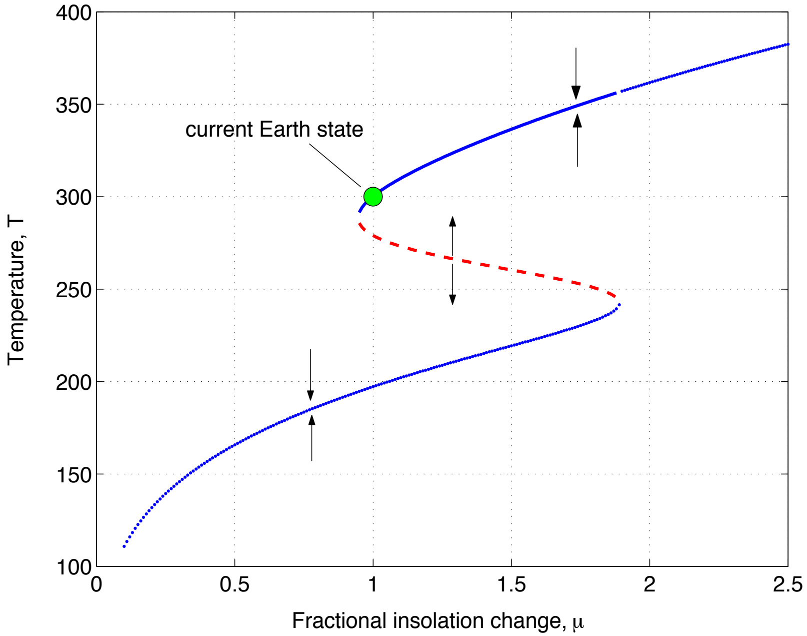
Slightly modified 0-D EBM (Zaliapin & Ghil, *NPG*, 2010)

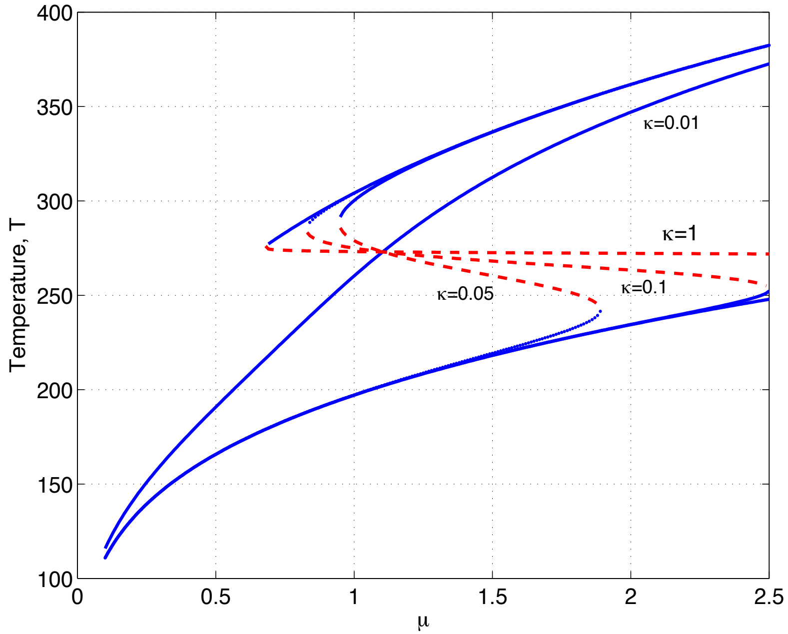
$$c\dot{T} = \mu Q_0 \left(1 - \alpha(T)\right) \sigma T^4 \left[1 - m \tanh\left((T/T_0)^6\right)\right]$$

$$\alpha(T;\kappa) = c_1 + c_2 \frac{1 - \tanh\left[\kappa\left(T - T_c\right)\right]}{2}.$$

T_c is the ice-margin temperature,
while κ is an extra “Budyko-vs.-Sellers” parameter







Time-dependent forcing

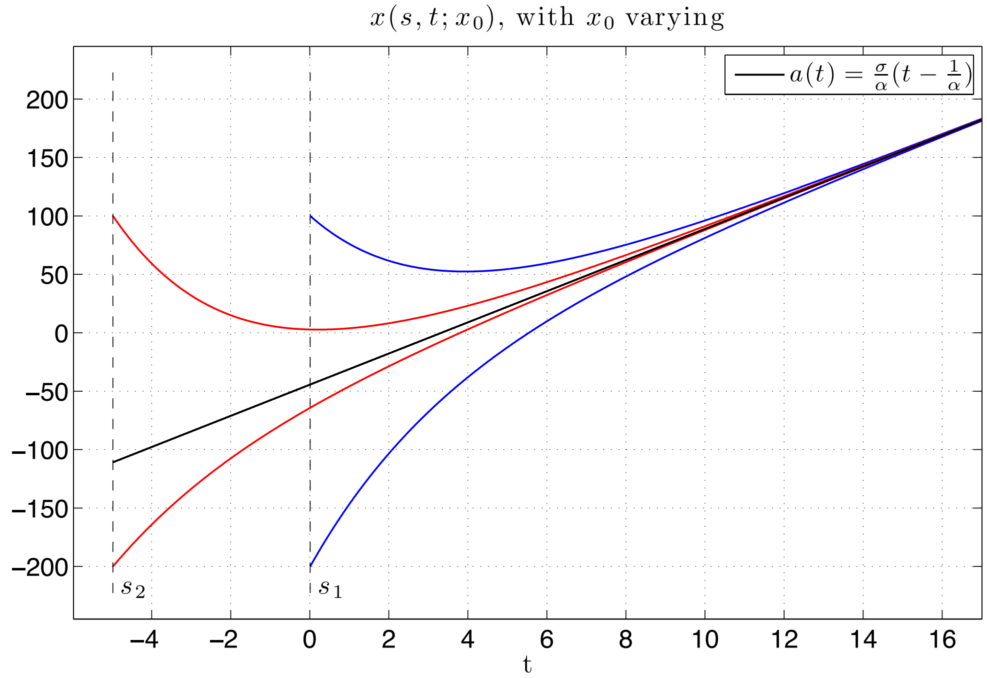
- Much of the theoretical work on the intrinsic variability of the climate system has been done with time-independent forcing and coefficients.
- Mathematically, this relied on autonomous dynamical systems (DDS).
- To address the changes in time of the system's overall behavior — and not just of its mean properties — an important step is to examine time-dependent forcing and coefficients.
- The proper framework for doing so is the theory of non-autonomous and random dynamical systems (NDS and RDS).
- Here is a "super-toy" introduction to **pullback attractors**: what are they?

The pullback attractor of a linear, scalar ODE,

$$\dot{x} = -\alpha x + \sigma t, \ \ \alpha > 0, \ \sigma > 0\,,$$

is given by

$$a(t) = \frac{\sigma}{\alpha}\left(t - \frac{1}{\alpha}\right).$$



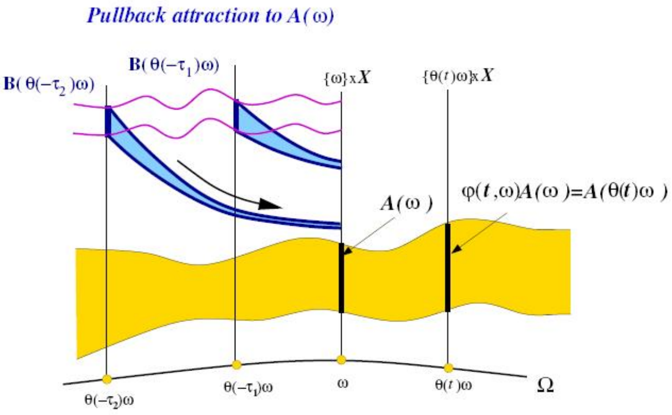
The sources of nonautonomous dynamics

- **Physically open vs. closed systems:** fluxes of mass, momentum & energy between the system & its surroundings are present or not.
- The mathematical framework of **nonautonomous dynamical systems (NDSs)** is appropriate for physically open ones:
  - skew-product flows (G. Sell)
    $\dot{x} = f(x, q)$, $\dot{q} = g(q)$, $x \in \mathbb{R}^d$, $q \in \mathbb{R}^n$, with q the driving force for x.
  - pullback (Flandoli, L. Arnold) or snapshot (C. Grebogi & E. Ott) attractors
    $\mathrm{d}X_t = f(X, q)\,\mathrm{d}t + \sigma(X)\,\mathrm{d}W_t$,
    where W_t is a Brownian motion in $\mathbb{R}^d$ and $\mathrm{d}t \sim (\mathrm{d}W)^2$.
- More generally, studying **explicit time dependence** in forcing or coefficients requires NDSs.
- The term **nonautonomous** is used both for the deterministic case and for a unified perspective on the deterministic & the random case.
- The commonality between the two cases is (i) the independence & (ii) the semi-group property of the driving force, whether $q(t)$ or W_t.
- Likewise, **pullback attractor (PBA)** is used both for the deterministic & the random case, while in the latter case uses more specifically the phrase **random attractor (RA)**.

A random attractor $\mathcal{A}(\omega)$ is both *invariant* and "pullback" *attracting*:

(a) **Invariant**: $\varphi(t,\omega)\mathcal{A}(\omega) = \mathcal{A}(\theta(t)\omega)$.

(b) **Attracting**: $\forall B \subset X$, $\lim_{t\to\infty} \mathrm{dist}(\varphi(t,\theta(-t)\omega)B, \mathcal{A}(\omega)) = 0$ a.s.



Tipping Points – A Classification(*)

- ***B-Tipping or Bifurcation-due tipping***

  – slow change in a parameter leads to the system's passage through a classical bifurcation

- ***N-Tipping or Noise-induced tipping***

  – random fluctuations lead to the system's crossing an attractor basin boundary

- ***R-Tipping or Rate-induced tipping***

  – rapid changes lead to the system's losing track of a slow change in its attractors.

N.B. All three types of tipping involve an **open system**.
We start **with closed systems** & study their **bifurcation structure**.
Then we proceed to open systems & see how that changes things.

(*) Ashwin *et al*. (*PTRSA*, 20012)

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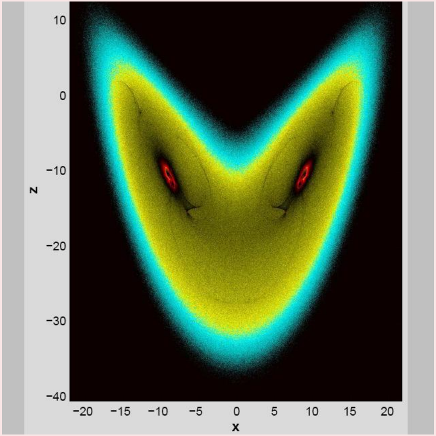
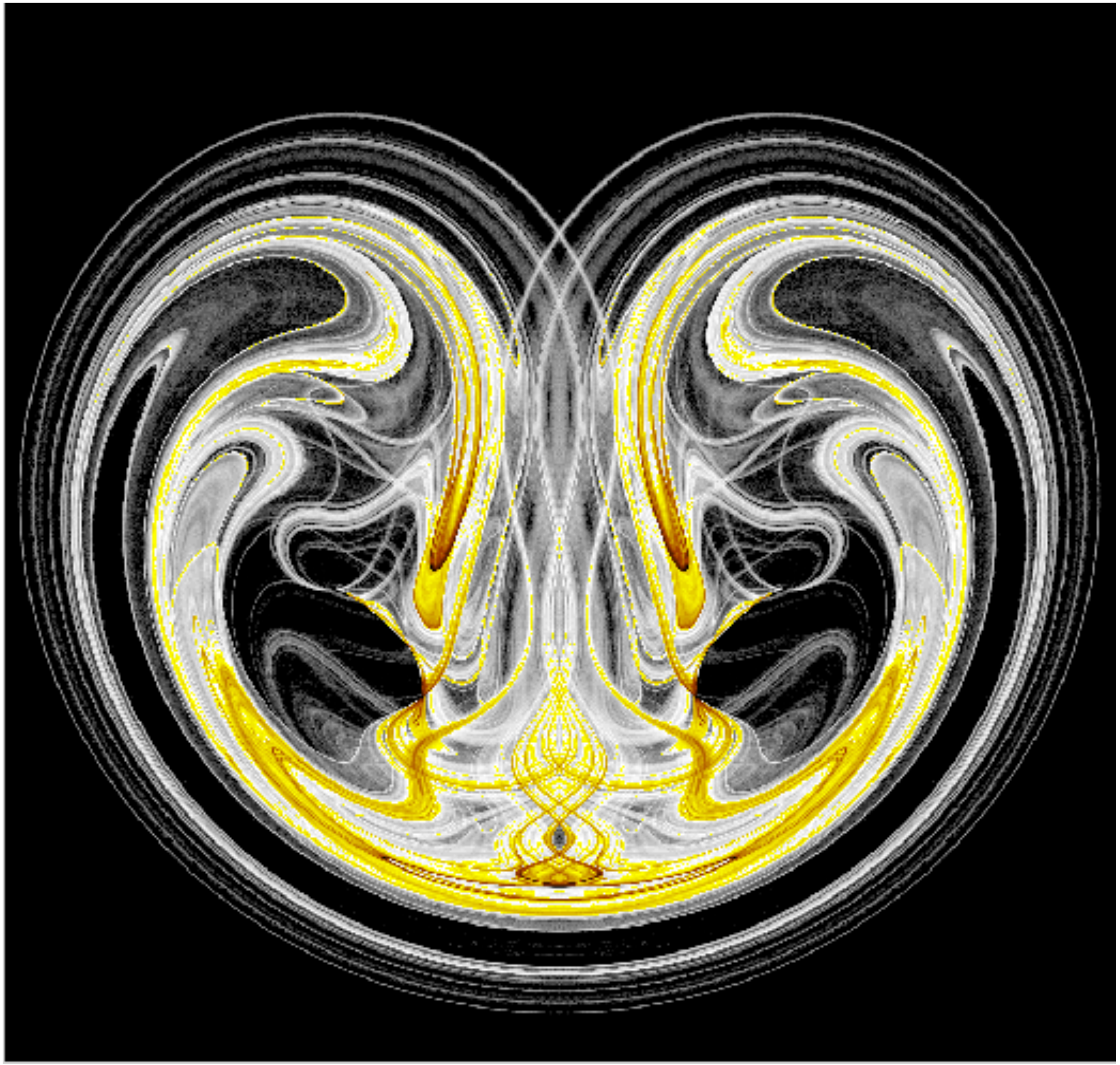
Sample measures supported by the R.A.



- We compute the probability measure on the R.A. at some fixed time t, and for a fixed realization ω. We show a "projection", $\int \mu_\omega(x, y, z)\mathrm{d}y$, with multiplicative noise: $\mathrm{d}x_i$=Lorenz$(x_1, x_2, x_3)\mathrm{d}t + \alpha\ x_i \mathrm{d}W_t; i \in \{1, 2, 3\}$.
- 10 million of initial points have been used for this picture!

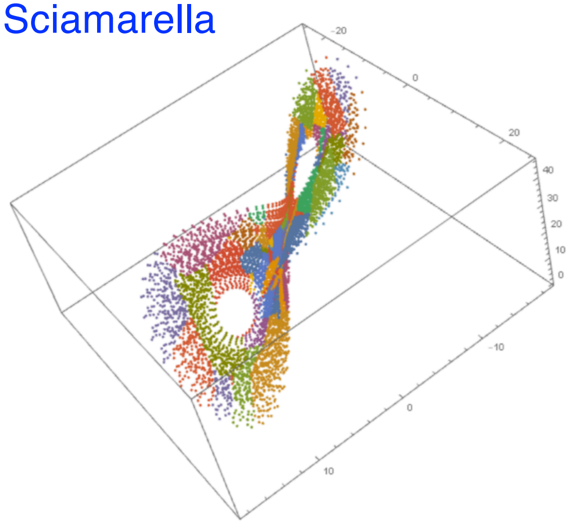


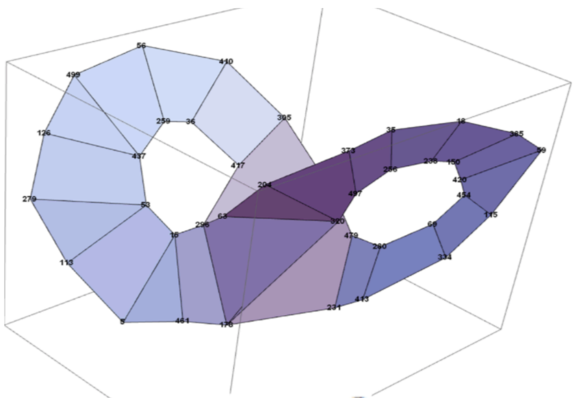
A day in the life of the Lorenz (1963) model's random attractor, or LORA for short;
see SI in Chekroun, Simonnet & Ghil (2011, *Physica D*) or
Vimeo movie: https://vimeo.com/240039610

State-space topology

Joint work with G. Charó, M. Chekroun & D. Sciamarella

The dynamics on a deterministic attractor can be compactly described as the limit of a semi-flow on a branched manifold (BM). The topological description of the BM or template encodes the invariant structure of the attractor in phase space. Reconstructing a BM from data amounts to (1) approximating a cloud of points in phase space by Euclidean closed sets; and (2) forming a cell complex in the sense defined in algebraic topology. The BM can be identified through the homology groups and the orientability chains associated to the cell complex (Sciamarella & Mindlin (*PRL*, 1999; *PRE*, 20010).

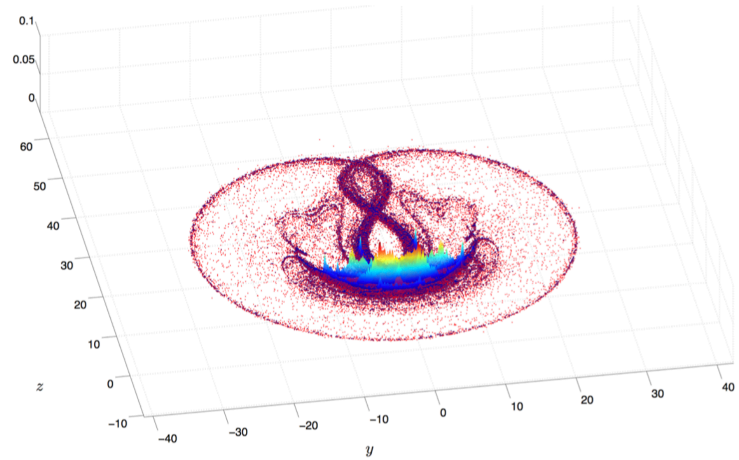


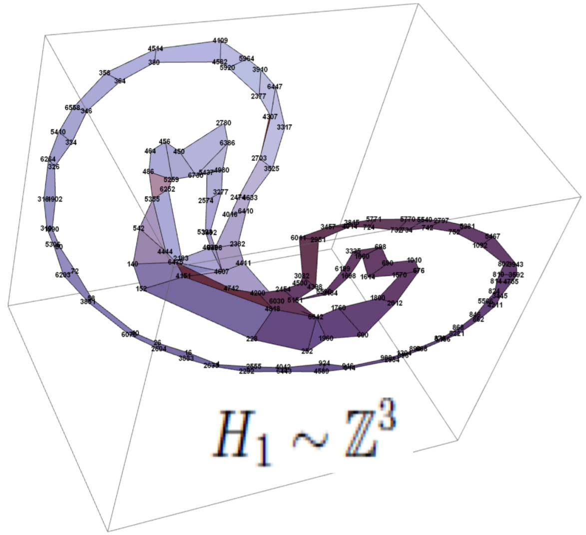


$H_1 \sim \mathbb{Z}^2$

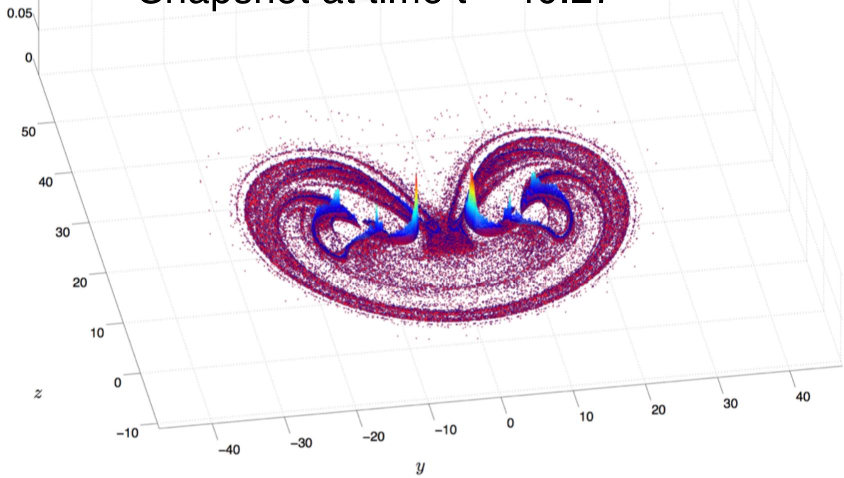
This work examines the topological structure of the snapshots that approximate the global random attractor associated with the stochastically perturbed Lorenz (1963) model. It is shown that — within the framework of random dynamical systems — the BM identification approach used to characterize the topological structure of deterministic chaotic flows from (noisy) time series can be extended to nonlinear noise-driven systems.

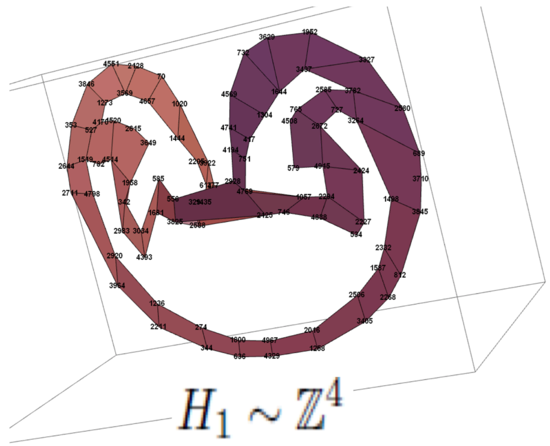
Snapshot at time $t = 40.09$





Snapshot at time t = 40.27



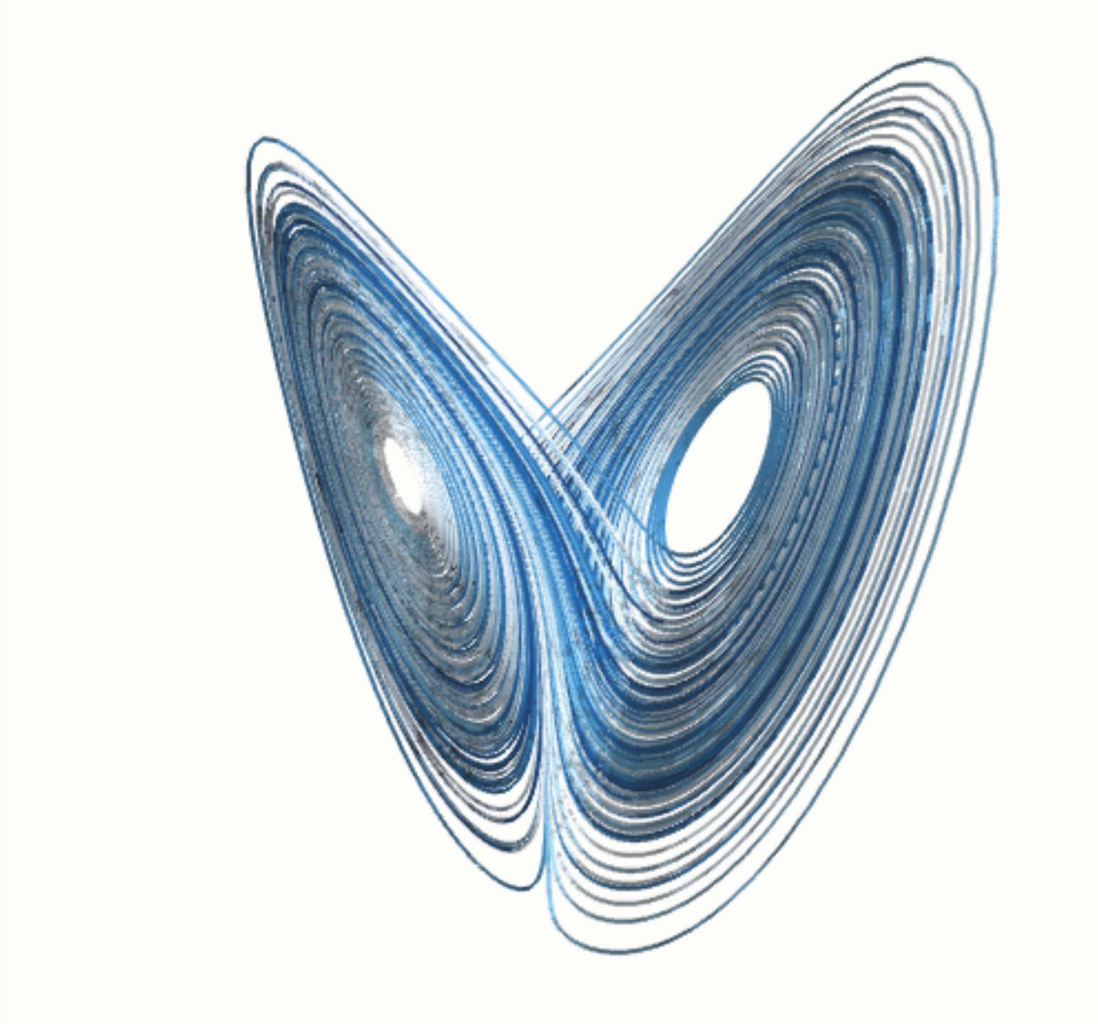


Classical Strange Attractor

Physically closed system, modeled mathematically as autonomous system: neither deterministic (anthropogenic) nor random (natural) forcing.

The attractor is strange, but still fixed in time ~ "irrational" number.

Climate sensitivity ~ change in the average value (first moment) of the coordinates (x, y, z) as a parameter λ changes.



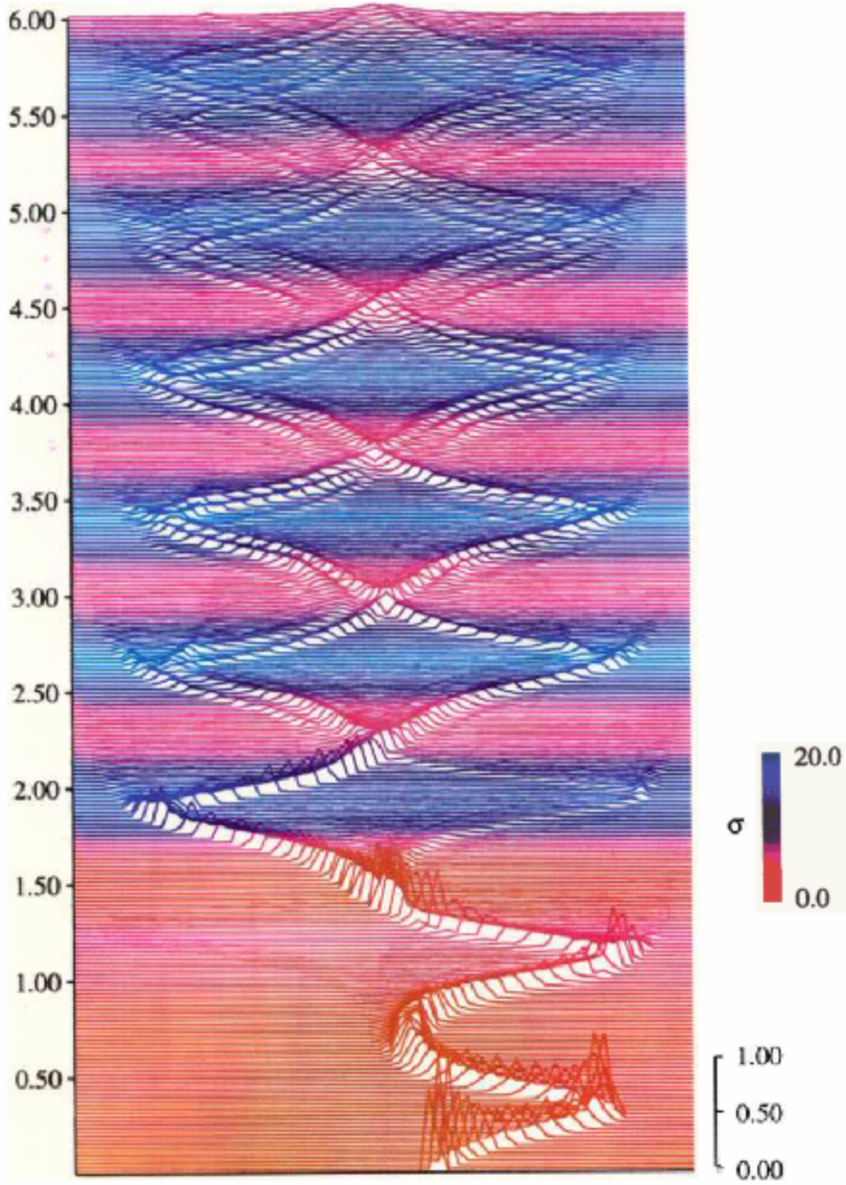
Exponential divergence vs. "coarse graining"

The classical view of dynamical systems theory is:

positive Lyapunov exponent ➔ trajectories diverge exponentially

But the presence of multiple regimes implies a much more structured behavior in phase space

Still, the probability distribution function (pdf), when calculated forward in time, is pretty smeared out



L. A. Smith (*Encycl. Atmos. Sci.*, 2003)

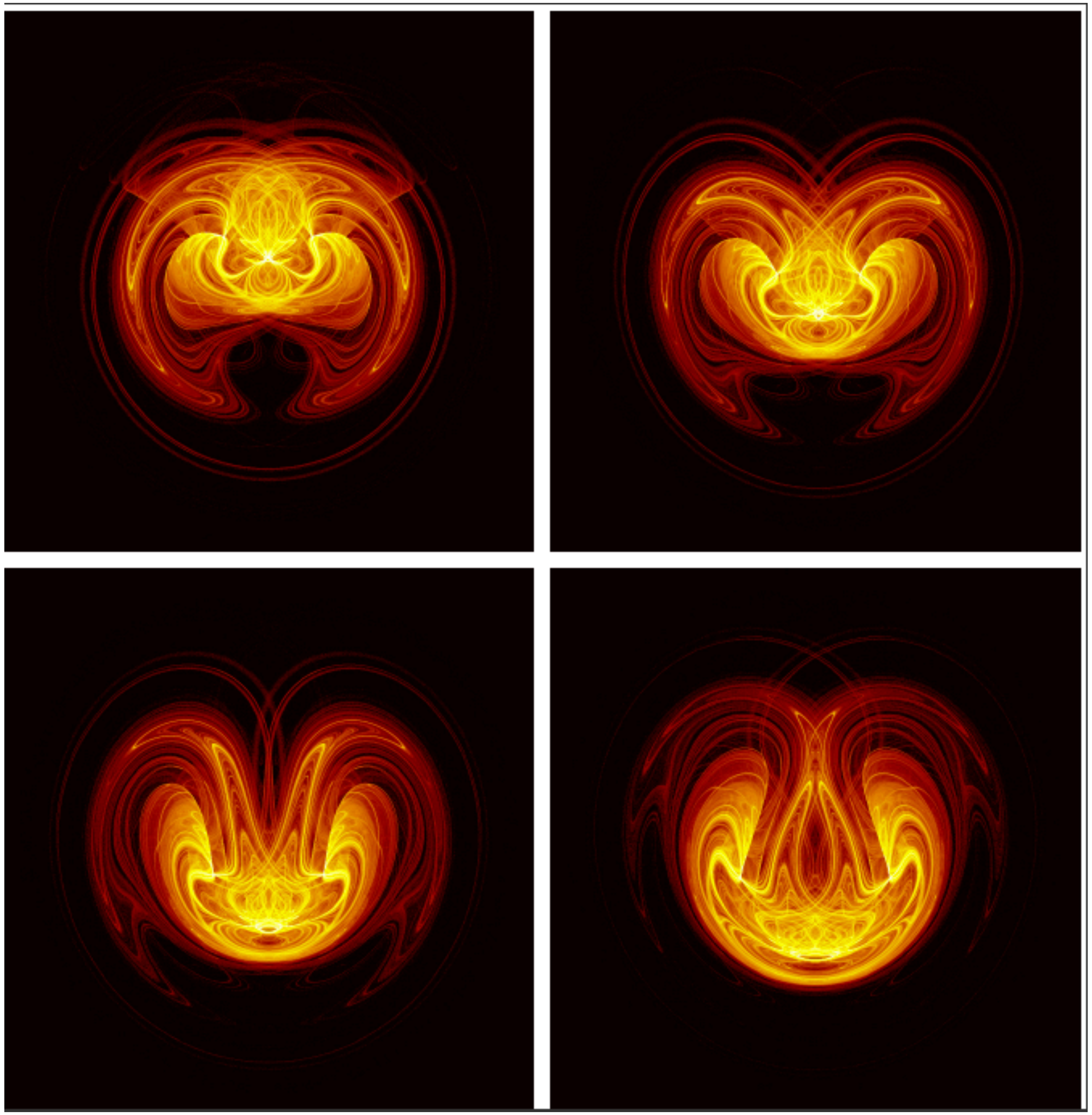
Random Attractor

Physically open system, modeled mathematically as non-autonomous system: allows for deterministic (anthropogenic) as well as random (natural) forcing.

The attractor is "pullback" and evolves in time ~ "imaginary" or "complex" number.

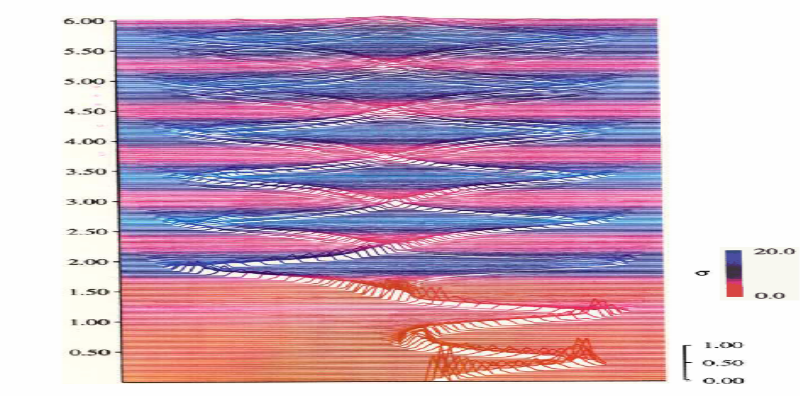
Climate sensitivity ~ change in the statistical properties (first and higher-order moments) of the attractor as one or more parameters (λ, μ, …) change.

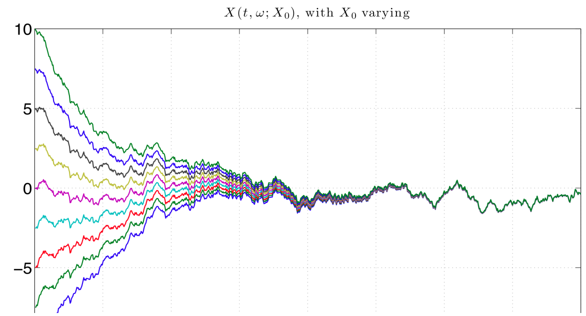
Ghil (*Encyclopedia of Atmospheric Sciences*, 2nd ed., 2012)



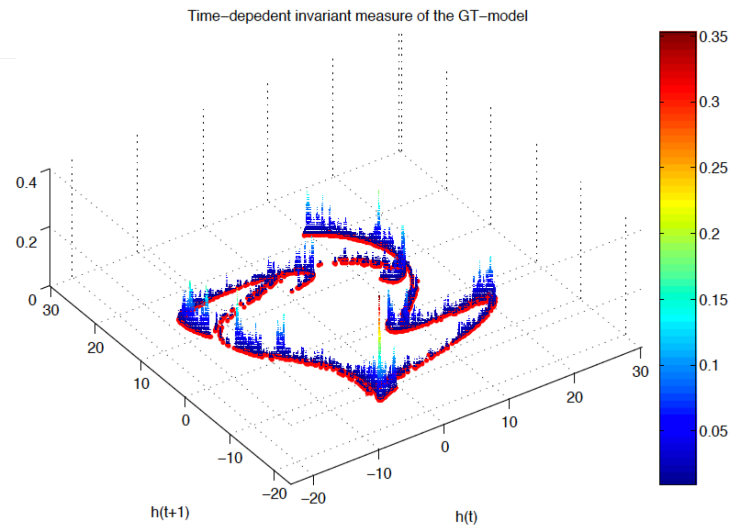
Yet another (grand?) unification

Lorenz (*JAS*, 1963)
Climate is deterministic and autonomous,
but highly nonlinear.
Trajectories diverge exponentially,
forward asymptotic PDF is multimodal.





Hasselmann (*Tellus*, 1976)
Climate is stochastic and noise-driven,
but quite linear.
Trajectories decay back to the mean,
forward asymptotic PDF is unimodal.



Grand unification (?)
Climate is deterministic + stochastic,
as well as highly nonlinear.
Internal variability and forcing interact
strongly, change and sensitivity
refer to both mean and higher moments.

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Concluding remarks

What do we know?

- There are great uncertainties in climate sensitivity & prediction; some are irreducible.
- The climate system is open, & affected by time-dependent forcing,
  both deterministic & stochastic.
- There is a nice general framework for including time-dependent forcing: NDS + RDS.

What do we know less well?

- How does the climate system really work?
- Smooth & rough dependence: Tipping points, crises?
- How do the latter affect the intrinsic variability: higher moments, ExEv's?

What to do?

- Work the model hierarchy, and the observations!
- Explore further non-autonomous and randomly driven models,
  and their tipping points!

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Some general references

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Reserve Slides

Pullback, Snapshot & Random Attractors

The framework of physically open and mathematically non-autonomous dynamical systems

- ***skew-product flows*** (G. Sell) – in **deterministic** systems, referred to usually as ***non-autonomous***
- **mathematical literature – *pullback attractors*** (F. Flandoli, L. Arnold)
- **physical literature – *snapshot attractors*** (C. Grebogi & E. Ott)

N.B. When the forcing is (also) **stochastic**, one talks of ***random attractors***

Applications to the climate sciences

- **pullback and random attractors** (M. Ghil & associates)
- **snapshot attractors** (T. Tél & associates)

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Nature is not deterministic or stochastic:

It depends on what we can, need & want to know
— more or less detail, with greater or lesser accuracy —
larger scales more accurately,
smaller scales less so

But we need both, deterministic and stochastic descriptions.
Knowing how to combine them is necessary, as well as FUN!